Interferometry & Polarimetry

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Interferometry

- Basic equations and principles
- Standard interferometry
- Choice of wavelength
- Vibration compensation
- Omodyne vs Etherodyne detection

Inversion methods

- Abel inversion and semianalytical methods
- Function parametrization

Advanced systems

- Scanning interferometer
- Poloidal Interferometer/Polarimeter
- Densitometer (Tangential Interferometer/Polarimeter)
- Differential interferometer
Tecniques to measure electron density in a plasma

- **Interferometry**
  Probing beam crosses the plasma and is phase-shifted

- **Reflectometry**
  Probing beam is reflected at a radius where density exceeds a critical value

- **Polarimetry**

- **Ellypsometry**
  Polarization state depends on electron density & magnetic field

  Refraction index depends on electron density
Interferometry for electron density measurement

General reference:
\[ \mathbf{E}(\mathbf{r},t) = E_0 \cos (k \cdot \mathbf{r} - \omega t + \phi_0) \]

\[ \omega = 2\pi c / \lambda \]

Dispersion relation:
\[ k = k(\omega) = N(\omega) \omega / c \]
\[ k = \omega / c \quad \text{in air / vacuum} \]

e.m. wave phase shift over distance \( d \) (spatial dependence):
\[ \varphi = k \cdot \mathbf{r} = kd = N(\omega) \cdot \omega / c \cdot d = 2\pi / \lambda \cdot N(\omega) \cdot d \]
Basic Principles

The basic equation for interferometry and polarimetry is the Appleton-Hartree* dispersion relation, which gives the refractive index $N$ for EM waves propagating through a magnetized plasma under the “cold plasma” assumption (neglecting ion motion):

$$N^2 = 1 - \frac{X[1 - X]}{1 - X - \frac{1}{2}Y^2 \sin^2 \theta \pm \sqrt{\left(\frac{1}{2}Y^2 \sin^2 \theta\right)^2 + \left[1 - X\right]^2Y^2 \cos^2 \theta}}$$

Where:

- $X = \left(\frac{\omega_p}{\omega}\right)^2$
- $Y = \frac{\omega_{ce}}{\omega}$
- $\omega$ frequency of the wave
- $\theta$ angle between the wave vector $k$ and the magnetic field $B$
- $e, m_e =$ charge and mass of the electron
- $n_e =$ electron density
- $\varepsilon_0 =$ permittivity of vacuum.

Interferometry - 1

Dispersion relations for a fully ionized magnetized plasma
(wave propagation in a “cold” plasma)

Ordinary wave

\[ N_{\text{ord}} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \]

Extra-ordinary wave

\[ N_{\text{strad}} = \left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2 - \omega_{ce}^2}\right)^{1/2} \]

Circ.pol. ordinary wave

\[ N_{\pm} = \left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega}{\omega \pm \omega_{ce}}\right)^{1/2} \]

\[ \omega_p = \left(\frac{n e^2}{\varepsilon_0 m_e}\right)^{1/2} \]

Plasma frequency

\[ n = 10^{19} \div 10^{21} \text{m}^{-3} \rightarrow \omega_p = 30 \div 300 \text{ GHz} \]

\[ \omega_{ce} = \frac{e B}{m_e} \]

Ion cyclotron frequency

\[ B = 0.1 \div 10 \text{ T} \rightarrow \omega_{ce} = 3 \div 300 \text{ GHz} \]
In toroidal confinement devices the direction of B changes along the beam path. Hence in principle different dispersion relations would be valid. Unless the probing wavelength is short enough so that:

\[ \omega^2 \gg \omega_p^2 \quad \omega^2 \gg \omega_{ce}^2 \]

\[
N_{\text{ord}} = \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2} \quad N_{\text{strad}} = \left( 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2 - \omega_{ce}^2}{\omega^2 - \omega_p^2 - \omega_{ce}^2} \right)^{1/2} \quad N_{\pm} = \left( 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega}{\omega \pm \omega_{ce}} \right)^{1/2}
\]

\[
N = \left( 1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2} \approx 1 - \frac{\omega_p^2}{2 \omega^2}
\]
In general an EM wave travelling along a path in medium with non uniform refraction index $N$, experiences a phase change given by:

$$\varphi = \frac{2\pi}{\lambda} \int N(x) \, dx$$

Where:
- $\lambda$: wavelength of measuring beam
- $N(x)$: Refraction index along beam path

An interferometer measures the phase difference between a measuring beam (crossing the plasma) and a reference (vacuum) beam:

$$\phi(t) = \frac{2\pi}{\lambda} \int_{L} N_v - N(x,t) \, dx = \frac{2\pi}{\lambda} \int_{L} 1 - N(x,t) \, dx$$
Global measured phase shift due also to:

- Constant phase due to difference path $\Delta$ (not exactly equalized beam path lengths):
  \[ \phi_{\text{geom}} = \frac{2\pi\Delta}{\lambda} \]

- Phase change due to mechanical vibrations of optics:
  \[ \phi_{\text{vib}}(t) = \frac{2\pi\delta(t)}{\lambda} \]

\[ \phi_{\text{tot}}(t) = \phi_{\text{pl}}(t) + \phi_{\text{vib}}(t) + \phi_{\text{geom}} \]
Mechanical vibrations - 1

\[ \phi_{pl}(t) = 2.82 \cdot 10^{-15} \lambda \int_{L} n(x,t) \, dx \]

\[ \phi_{vib}(t) = \frac{2\pi \delta(t)}{\lambda} \]

\[ \frac{\phi_{vib}}{\phi_{pl}} = \frac{2.2 \cdot 10^{15} \delta}{nL} \frac{1}{\lambda^2} \]

Interferometers usually mounted on vibration isolated optical benches, typically shaped as a “C” frame around the plasma

- \( n = 10^{20} \text{ m}^{-3} \)
- \( L = 1 \text{ m} \)
- \( \delta = 5 \mu\text{m} \)
- \( \frac{\phi_{vib}}{\phi_{pl}} \leq 1\% \)
- \( \lambda \geq 100 \mu\text{m} \)
Mechanical vibrations - 2

RFX interferometer granite optical bench
Dimensions: 3 x 2 x 0.2 m
Weight: 3 t
Two-color vibration compensation - 1

\[
\begin{align*}
\phi_1 &= \frac{2\pi \delta}{\lambda_1} + K \lambda_1 \int n(x)dx \\
\phi_2 &= \frac{2\pi \delta}{\lambda_2} + K \lambda_2 \int n(x)dx
\end{align*}
\]

- \( \lambda_2 \ll \lambda_1 \) (e.g. visible) \( \Rightarrow \phi_2 \sim 2\pi \delta/\lambda_2 \)
- or more in general:

\[
\int n(x)dx = \frac{\phi_1 \lambda_1 - \phi_2 \lambda_2}{L} K \left( \frac{\lambda_2^2 - \lambda_1^2}{\lambda_2 - \lambda_1} \right)
\]

\[
\delta = \frac{\lambda_1 \lambda_2}{2\pi} \left( \frac{\lambda_2^2 - \lambda_1^2}{\lambda_2 - \lambda_1} \right) (\lambda_1 \phi_2 - \lambda_2 \phi_1)
\]

Caveat: vibrations of optics not shared by the two systems at different \( \lambda \) are not compensated (unless a LO chord is used, see following)!
Choice $\lambda_2$ (compensation wavelength):

Best if $\lambda_2 \sim 1/5 \div 1/2 \lambda_1$

if $\lambda_2 / \lambda_1$ is smaller there can be dispersion problems:
- alignment of main and compensation beams (crucial for optimum compensation) can be degraded by refraction due to crossing of transmissive optics and plasma

if $\lambda_2 / \lambda_1$ is larger the noise due to difference measurement becomes large:
- Can be easily seen by propagating the $\phi_1$ & $\phi_2$ errors on $\int n$

$$\int n(x)dx = \frac{\phi_1 \lambda_1 - \phi_2 \lambda_2}{K \left( \lambda_1^2 - \lambda_2^2 \right)}$$

*Note: vibration compensations systems relax specification for optical bench and actually make it possible to use in-vessel optics!*
Choice of laser wavelength - 1

1. Plasma “transparency” and Linearization of $N$ : $\omega >> \omega_p$
   typically in experiments $n_{\text{max}} < 10^{21}$ m$^{-3}$ => $\omega_{p\text{max}} \sim 300$ GHz

2. Laser beam diameter $d$, both for space resolution and window access (in particular in harsh burning plasma environment):
   \[ d \propto \lambda^{1/2} \]
   \[ d = 2 \sqrt{\lambda Z_0 / \pi} \] *

3. Beam refraction $\alpha$ due to density (refraction index) gradients:
   \[ \alpha \propto \lambda^2 \]
   \[ \alpha = 8.97 \times 10^{-16} n_0 \lambda^2 \]

4. Measuring accuracy:
   \[ \phi_{\text{meas}} \propto \lambda \]

5. Sensitivity to mechanical vibrations
   \[ \phi_{\text{vib}} \propto 1/\lambda \]

6. Easy and stable alignment:
   \[ \propto \lambda \]

=> short $\lambda$ better
$\lambda < 1$ mm

=> short $\lambda$ better
$\lambda < 200\div300$ $\mu$m

=> long $\lambda$ better
but not too much to avoid fringe jumps!

* Gaussian beam propagation $Z_0$ is distance between focusing mirrors
### Available laser sources $\lambda \sim 3 \div 400 \, \mu m$ (MIR ÷ FIR)

<table>
<thead>
<tr>
<th>Spectral Range</th>
<th>Laser Source</th>
<th>$\lambda$ (µm)</th>
<th>Power (mW)</th>
<th>Detector</th>
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<tbody>
<tr>
<td>MIR</td>
<td>He-Ne</td>
<td>3.39</td>
<td>1-10</td>
<td>InAs</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td>InAs</td>
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<tr>
<td>CO</td>
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<td>1-2 (W)</td>
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<td>InSb</td>
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<td>HgCdTe</td>
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<tr>
<td>CO$_2$</td>
<td>10.6</td>
<td>1-100 (W)</td>
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<tr>
<td>FIR</td>
<td>CH$_3$OD</td>
<td>47-57</td>
<td>0.1-0.5W</td>
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<td>CH$_3$OH</td>
<td>119</td>
<td>0.1-0.5W</td>
<td>DLATGS</td>
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<td></td>
<td></td>
<td></td>
<td>(deuterated L-alanine doped triglycine sulphate)</td>
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<tr>
<td></td>
<td>CH$_2$F$_2$</td>
<td>185</td>
<td>0.1-0.5W</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DCN</td>
<td>195</td>
<td>0.1-1W</td>
<td>InSb He-cooled</td>
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<tr>
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<td>HCN</td>
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<tr>
<td></td>
<td>HCOOH</td>
<td>433</td>
<td>0.1-1W</td>
<td></td>
</tr>
</tbody>
</table>
Choice of laser wavelength - 3

**Characteristics of a MIR interferometer:**

- $\phi_{pl} \propto \lambda$  
  =>$\text{measures } n_e > 10^{20} \text{ m}^{-3} \text{ without fringe losses}$

- $\alpha_{\text{beam}}(\nabla n) \propto \lambda^2$  
  =>$\text{little refraction by large } \nabla n_e$

- $\Delta \omega 40\div2000 \text{ MHz}$  
  =>$\text{high time resolution: bandwidth } \sim \text{MHz}$

- $\phi_{\text{beam}} \propto \lambda$  
  =>$\text{High spatial resolution } (\phi_{\text{beam}} \leq 1 \text{ cm})$

- **Requires two-color vibration compensation!**

**Characteristics of a FIR interferometer :**

- $\phi_{pl} \propto \lambda$  
  =>$\text{High resolution } \sim n_e = 10^{17} \text{ m}^{-3} , \text{ may suffer from fringe jumps}$

- $\Delta \omega 1\div5 \text{ MHz}$  
  =>$\text{good time resolution: bandwidth } \sim 100\text{kHz}$

- **may not need vibration compensation**

- $\alpha_{\text{beam}}(\nabla n) \propto \lambda^2$  
  =>$\text{may suffer refraction by large } \nabla n_e$
Beam aperture diameter at CRR must obey:

\[ D_{\text{CRR}} \geq 2.2 \times d_{\text{je}} \quad d = 2 \sqrt{\frac{\lambda Z_0}{\pi}} \quad \alpha_m Z_0 \leq 0.25 d \]

Plasma cut-off frequency: \[ n_{\text{coff}} = 1.115 \times 10^{15} / \lambda^2 \, [\text{m}^{-3}] \]

Beam bending angle:

\[ \alpha_m = \arcsin \left( \frac{n_0}{n_{\text{coff}}} \right) \approx \frac{n_0}{n_{\text{coff}}} = 8.972 \times 10^{-16} n_0 \lambda^2 \]

Preferable choice of probe beam wavelength:

\[ \lambda \leq \frac{0.583 \times 10^{10}}{\sqrt{Z_0 n_0^2}} \, [\text{m}] \]

The optimum wavelength for the polarimeter:

\[ \lambda_{\text{opt}} \leq 110 \, \mu\text{m} \]
Homodyne detection

How to measure $\phi$?

Electric field on detector:

\[ E_r = E_{0r} \cos(\omega t) \]
\[ E_m = E_{0m} \cos(\omega t + \phi) \]

Interference signal:

\[ I = \varepsilon_0 (E_r + E_m)^2 \]

Electrical voltage on detector:

\[ V \propto I_d = <I>_{\tau_d} \]
\[ \tau_d \sim \text{ns} >> T_{\text{wave}} = \frac{2\pi}{\omega} \]
\[ = \varepsilon_0 <E_{0r}^2 \cos^2(\omega t) + E_{0m}^2 \cos^2(\omega t + \phi) + E_{0r} E_{0m} [\cos(2\omega t - \phi) + \cos(\phi)]>_{\tau_d} \]

\[ V(t) = \varepsilon_0 \frac{E_{0r}^2 + E_{0m}^2}{2} + \varepsilon_0 E_{0r} E_{0m} \cos \phi(t) \]
Problems with homodyne detection:

\[ V(t) = \varepsilon_0 \frac{E_{0r}^2 + E_{0m}^2}{2} + \varepsilon_0 E_{0r} E_{0m} \cos \phi(t) \]

- For an absolute calibration a total phase change $> \pi$ is necessary
  -> could be partially overcome with forced phase change in vacuum shots, ...

- sign of $\phi$ is not determined &
- inversion of $\cos \phi$ function critical around $\pi$
  -> quadrature detection can be arranged (explained later in talk)

- $E_{0r}$ and $E_{0m}$ can change due to laser emission fluctuations or mechanical vibrations
  -> call for real time measurement of $E_{0r}$ and $E_{0m}$
Heterodyne detection

\[ V(t) = \varepsilon_0 \frac{E_{0r}^2 + E_{0m}^2}{2} + \varepsilon_0 E_{0r} E_{0m} \cos(\Delta \omega t + \phi(t)) \]

**Modulation:**
- mechanical (moving mirrors, Veron wheel) \( \rightarrow \) 10-100 kHz
- twin laser cavity \( \rightarrow \) MHz
- acusto-optic \( \rightarrow \) 10-10^3 MHz

**Demodulation:**
Mixing with “carrier” LO signal at modulation frequency \( \Delta \omega \) \( \rightarrow \) only zero crossings matter and detected \( \phi(t) \) completely independent from interference signal amplitude
Same principle as AM and FM radio broadcast, \( \Delta \omega \) must be \( \gg \) frequency bandwith of \( \phi(t) \)!
Frequency shift techniques

- Cylindrical Rotating Grating Rotating
- Twin (FIR) laser cavity
- Acousto-Optic Modulator for MIR laser beam
Cylindrical Rotating Grating

The first-order angle of diffraction $\theta_1$ for a plane electromagnetic wave ($\lambda_0 = 2\pi/k_o, \omega_0 = ck_o$) incident at angle $\theta_i$, on an infinite plane grating with groove spacing $d = 2\pi/K$ is given by

$$\sin \theta_1 = \sin \theta_i + \frac{K}{k_o}$$

The equation accounts successfully for the reflection behavior of a Gaussian beam incident on the edge of a circular scanning grating wheel. When the wheel is rotated, the radiation is Doppler shifted in angular frequency by

$$\Omega = KR\omega$$

where $R$ is the wheel radius, $\omega$ is the wheel angular rotation frequency. Typical $\Omega$ values do not exceed 10-100 kHz
The emission frequency of a FIR laser can be tuned by finely adjusting the cavity length so that the cavity mode is slightly off the maximum laser Gain bandwidth. So 2 twin laser cavities can be adjusted to emit at different $\lambda$ to be used for etherodyne detection with a frequency difference $(\omega_1 - \omega_2 \sim 2\text{MHz})$. 

Frequency shift by twin FIR laser cavity

The diagrams illustrate the frequency response of the laser output power over wavelength for two cavities with different cavity lengths, resulting in a frequency difference $\omega_1 - \omega_2$.
Frequency shift by twin FIR laser cavity

- The frequency shift can be increased by Stark-tuning the laser cavity: the gain curve is shifted by applying a transverse DC voltage to one of the cavities, so that it operates $\sim 5$ MHz away from the other laser.

Cavity scan from Stark-tuned laser with and without applied dc electric field
NSTX interferometer polarimeter

- Two FIR laser beams (linearly polarized) are combined and transformed into left- and right-hand circularly polarized probe beams with a small frequency difference ($\omega_1 - \omega_2 \sim 2\,\text{MHz}$)
- Third laser is Stark-tuned: the gain curve is shifted by applying a transverse DC voltage to the cavity, and operates $\sim 5\,\text{MHz}$ away from the other two lasers.

Cavity scan from Stark-tuned laser with and without applied dcelectric field
Acousto-optic modulators, also called Bragg cells, use acousto-optic effect to diffract and shift the frequency of light using sound waves. They can be used at mid-infrared \( \lambda \).

Light reflected by flat sound wave into first diffraction order up to 90% deflection efficiency.

\[
\Delta \Theta = 2 \Theta_B
\]

\[
2 \Lambda \sin \Theta_B = m\lambda
\]

- \( \Lambda = \frac{v_s}{\Delta \omega} \) equivalent grating step
- \( v_s \) sonic speed in crystal
- \( \Delta \omega \) acoustic wave frequency
- \( m \) (typ. = 1) order of diffraction
- \( \lambda \) beam wavelength

Typically for CO\(_2\) laser: Ge crystal

\( \Delta \omega = 20\text{-}80\text{MHz} \)
Demodulation & quadrature phase detection

Quadrature Phase Comparator

Detector (e.g. phoconductor)

\[ \varepsilon_0 E_r E_m \sin[\omega_1 t + \phi_1(t)] \]

Automatic Gain Control Amplifier

\[ \sin[\omega_1 t + \phi_1(t)] \]

Quadrature Phase Comparator

\[ \sin \phi_1(t) \]

\[ \cos \phi_1(t) \]

ADC
Advantages:

• Absolutely calibrated and accurate measurement (in heterodyne systems it depends only on $\lambda$)
• Continuous measurement with high time resolution
• Line-integral real-time measurement is well suited for feedback density control
Interferometry Summary - 2

Drawbacks:

• Line integral measurements call for inversion codes to get local density profile

• Measurement is “relative” to a constant $\varphi_{\text{geom}}$ to be measured before or after the pulse

=> temporary loss of signal due to fringe “jumps” or laser malfunction may result in complete loss of measurement.
Interferometers on existing experiments are typically FIR or MIR (vibration compensated).

To obtain the density profiles with good accuracy they can be:

- multichord (with several beam splitters and mirrors for each measuring chord)
- expanded beams (in some cases tomographic) using large optics to expand the laser beam and detector arrays
Interferometers on existing experiments - 1

- Schematic drawing of the multichord FIR (HCN laser) interferometer / polarimeter on TEXTOR
- Line-integrated $n_e$ and Faraday rotation angles measured along 9 vertical and 1 horizontal chords
Interferometers on existing experiments - 2

RFX multichord MIR interferometer. Two independent two-color modules: CO$_2$/HeNe laser (8 chords) e CO$_2$/CO lasers (5 chords)
Sample RFX multichord interferometer measurements

Module B chords use in vessel mirrors

Plasma current

Module A

Module B
Interferometers on existing experiments - 3

**Expanded beam**

- MIR (CO₂/HeNe) on Alcator C-MOD

- FIR on TEXT-Upgrade (tomographic)
Getting local density profile from line integral measurements
The problem: reconstruct local density profiles starting from the measured line integral profile

\[ I(s, \phi) = \int_{-\infty}^{+\infty} f(s, \ell) \, d\ell \]

\( f(x,y) \) is univocally determined if one has \( I(s, \phi) \) for

\(-\infty < s < +\infty\)

\( 0 < \phi < 2\pi \)

In practice some “reasonable assumptions are needed before computing \( f(x,y) \)
Inverting line integral profiles - 2

For a smooth cylindrically symmetric plasma, it is possible to compute the local profile from line-integral measurements by Abel inversion:

\[
I(x) = 2 \int_{x}^{a} \frac{n(r) r}{\sqrt{r^2 - x^2}} \, dr
\]

\[
n(r) = \frac{1}{\pi} \int_{r}^{a} \frac{dI(x)}{dx} \frac{dx}{\sqrt{x^2 - r^2}}
\]

function \( I(x) \) fitted using measurements at various \( x \)

More in general for plasmas that exhibit distortions from cylindrical symmetry, numerical inversion procedures have been developed which still utilize the Abel inversion integral. See for example:

Other widely used inversion methods are:

- **Function parametrization**
  
  *Aims at avoiding unphysical features cause by “blind” inversion techniques by limiting the class of acceptable solutions*

- **Use of neural networks**
  
  *Particularly suited when a large database of similar data is available and data from different diagnostics can be used for the network learning*

- **Tomographic reconstructions**
  
  *can be used when a high number of closely spaced measuring chords, preferably probing the plasma from different angles, are available*
Inverting line integral profiles - 4

An example of Function parametrization

- A “reasonable” analytical function (of the geometrical coordinates or, better, of the flux function) is chosen to model the density profile. To avoid overfitting, the function must have a number of free parameters $a_i$ smaller than the number of measurements

$$n(x,y) = n(x,y,a_1,a_2,a_3...) \text{ or } n(\rho) = n(\rho,a_1,a_2,a_3...)$$

- For each possible free parameter set the “model” $I_t(x)$ can be computed and compared to the experimental measurement to compute a mean square deviation $\chi^2$

$$\chi^2 = \sum_{i} \left\{ \frac{1}{\sigma_i} [I_i - I_t(x_i)]^2 \right\}$$

- The $\chi^2$ is minimized by exploring the parameter space and imposing $\partial \chi^2 / \partial a_i = 0$
Inversion of RFX interferometer data - 1

14 interferometer chords, 11 different impact parameters

“reasonable” $n_e$ profile: smooth function centrally peaked or with central “hollow” profile
Inversion of RFX interferometer data - 2

Function chosen:

\[ n(\rho) = n_0 - (n_0 - n_a - n_1)\rho^\alpha - n_1\rho^\beta \]

\[ \rho = r/(a-|\Delta|) \]

5 free parameters: \( \alpha, \beta, \Delta, n_0, n_1 \)

- for \( \alpha, \beta \) e \( \Delta \) a complete search is performed over a given grid of values
- for \( n_0 \) e \( n_1 \) a direct minimisation of \( \chi^2 \) is done
Sample time evolution of inverted profiles with pellet injection
Advanced systems:

- **Scanning interferometer:**
  
  the FTU scanning interferometer

- **Polarimetry (Faraday rotation & Cotton-Mouton effect):**
  
  ITER poloidal polarimeter

- **tangential polarimeter interferometer:**
  
  ITER Tangential Polarimeter Interferometer
Scanning the probing beam through the plasma

Advantages

• High space resolution
• Simple alignment, small number of optics
• High power on detector
• Complete profile at each scan: no need of a “zero” point before/after pulse (zero is obtained by edge pass) => fringe “jumps” no longer a problem

Drawback

Profile measurement is not at the same time (tolerable is scan is fast enough)
FTU Scanning beam interferometer

Optical Scheme

FTU setup
Scanning interferometer measurement on FTU

Line density measures for a multi-pellet discharge.
Sawtooth measured by scanning interferometer.

a) 33 chords line density profile: before and after a sawtooth and line density variation (black diamonds)

b) Computed density profile before and after

c) Temperature variation measured by ECE during same time interval.
Two sample design cases

- Poloidal interferometer polarimeter for ITER
- The (“densitometer”) tangential interferometer polarimeter for ITER
**Concept of interferometry-polarimetry (I)**

**Faraday rotation &**

**Cotton-Mouton effects**

The **Faraday rotation** is change of the polarization vector of the probing beam caused by the presence of a magnetic field parallel to the direction of propagation.

The **Cotton-Mouton** effect is an elliptisation of the polarization state of the inpinging linearly polarized beam caused by the presence of a magnetic field perpendicular to the direction of propagation.
Concept of interferometry-polarimetry (II)

The state of polarization can be described by the Stokes vector $s(z)$. The evolution along the line of sight (z-direction) is given by:

$$\frac{d\vec{s}(z)}{dz} = \vec{\Omega}(z) \times \vec{s}(z)$$

$$\begin{pmatrix}
\Omega_1 \\
\Omega_2 \\
\Omega_3
\end{pmatrix} = \frac{\omega_p^2}{2c\omega} \begin{pmatrix}
e^2/m^2 (B_x^2 - B_y^2) \\
e^2/m^2 (2B_xB_y) \\
e/m(2\omega B_z)
\end{pmatrix}$$

- $\Omega_3$ rotation angle (Faraday effect)
- $\Omega_1, \Omega_2$ ellipticity (Cotton–Mouton effect)

Faraday rotation

$$\alpha(x) = 2.62 \times 10^{-13} \lambda^2 \int_{Z_1}^{Z_2} n_e(x, z) B_{\parallel}(x, z) dz$$

Cotton–Mouton effect

$$\alpha_{CM} = 2.45 \times 10^{-11} \lambda^3 \int_{z} n_e B_{\perp}^2 dz$$

Design of 48, 57 \(\mu\)m Poloidal Polarimeter for ITER

- Control of the current density profile becomes a paramount issue for the modern tokamak experiments.

- Polarimetry can provide information on the density and magnetic field distribution inside plasma (current profile), utilizing the Faraday effect in a magnetized plasma.

\[
\alpha_F = 2.63 \times 10^{-13} \lambda^2 \int n_e B_{p\parallel} \, dz
\]

- In order to evaluate \(B_{\parallel}\) from the rotation angle, the electron density is necessary. This can be done by interferometry or using the Cotton–Mouton effect since the transverse field for a poloidal beam is the well known toroidal field \(B_t\)

\[
\alpha_{CM} = 2.46 \times 10^{-11} \lambda^3 \int n_e B_t^2 \, dz
\]

\[
\alpha_{int} = 2.82 \times 10^{-15} \lambda \int n_e(z) \, dz
\]

Both \(n_e\) & \(B_{\parallel}\) must be measured along same chord simultaneously.
Calculated Faraday rotation angles (double pass through the plasma) for a horizontal fan of chords (right top) and the corresponding ellipticity values (right center); with q-profile, pressure profile and electron density profile on the left.

Very small ellipticity (Cotton-Mouton effect)
Design of 48, 57 mm Poloidal Polarimeter for ITER (III)

Two general approaches exist to evaluate plasma current profile

**Polarimeter - polarimeter**

- $B_{||}$ polarimeter (Faraday)
- $n_e$ polarimeter (Cotton–Mouton)

**Polarimeter - interferometer**

- $B_{||}$ polarimeter (Faraday)
- $n_e$ interferometer

**Advantages**

- There are no fringe jumps in principle
- Lots of application on major tokamaks (JT-60U, JET, TotaSupra, RTP,NSTX,MST…)

**Drawbacks**

- Complicated channeling and calibration due to coupling of Faraday rotation and Cotton-Mouton effect.
- Despite promising theoretical and numerical results there is very limited experimental support. 1ch pure Cotton–Mouton polarimeter (W7-AS)
- Fringe jumps
- Mechanical vibrations
- Small Faraday rotation in the core plasma region

**Shorter wavelength laser, with smaller refraction and two color interferometer solve the problems**
Tangential Interferometer / Polarimeter (TIP) for ITER

5 chords for "real-time" measurement to be used in $n_e$ feedback control

TIP gives “double” density measurements:

$$\Phi = 2.82 \times 10^{-15} \lambda \int n_e(z)dz$$
$$\Psi_F = 2.62 \times 10^{-13} \lambda^2 \int n_e(z)B_{||}(z)dz$$

Magnetic field known along tangential chords

- Double-pass system utilizing retro-reflectors
- Retro-reflectors mounted in recessed plugs in shield wall
- Optical labyrinth through shield wall
Faraday Rotation and Interferometer Phase Shifts

Two $\lambda$ considered:

| MIR 10.6/5.3 $\mu$m | & | FIR 57/47 $\mu$m |

Density profiles approximated by:

$$n_e = d \times 10^{20} \left( 1 - \left( \frac{r}{a_o} \right)^b \right) \text{ m}^{-3}$$

Low-Density: Steady State
d=1, b=10, $n_e = 1 \times 10^{20}$ m$^{-3}$

High-Density: Gas Injection, Pellets, etc.
d=10, b=2, $n_e = 1 \times 10^{21}$ m$^{-3}$

- Faraday rotation $< 2\pi$ for 10.6 $\mu$m
  but $> 2\pi$ for 57/47 $\mu$m
  Polarimetry phase shift has potential for fringe skips at 57/47 $\mu$m.

- Interferometry phase shifts large (10$^2$-10$^3$ fringes) at all $\lambda$'s

- At 10.6 $\mu$m only for $n_e > 10^{22}$ m$^{-3}$, Faraday phase shifts $> 2\pi$ and refraction may be large
Two-Color Interferometers meet all but the lowest \( n_e \) range

Two-color interferometer resolution given by:

\[
\delta \phi = \text{Phase Measurement Error} \\
\lambda = \text{Wavelength}
\]

\[
\delta n_e L \propto \frac{\delta \phi}{(\lambda_1 - \lambda_2)}
\]

<table>
<thead>
<tr>
<th>( \lambda_1 ) (( \mu \text{m} ))</th>
<th>( \lambda_2 ) (( \mu \text{m} ))</th>
<th>Error (degree)</th>
<th>Path Length (m)</th>
<th>Required Resolution ( nL ) (m(^{-2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.59</td>
<td>5.3</td>
<td>1</td>
<td>20</td>
<td>2.0E+19</td>
</tr>
<tr>
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<td>9.27</td>
<td>1</td>
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<td>2.0E+19</td>
</tr>
<tr>
<td>12.1</td>
<td>9</td>
<td>1</td>
<td>20</td>
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<tr>
<td>57.2</td>
<td>47.6</td>
<td>1</td>
<td>20</td>
<td>2.0E+19</td>
</tr>
</tbody>
</table>

Obtainable with Required Bandwidth

Central Chord

At lowest specified density of \( n_e = 1 \times 10^{18} \text{ m}^{-3} \), no wavelength fulfills 1% line-density requirement, even for central chord.
Faraday Rotation Polarimeter meets all $n_e$ range

<table>
<thead>
<tr>
<th>$\lambda$ (μm)</th>
<th>BT (T)</th>
<th>Path Length (m)</th>
<th>$n_e=1E20$ m$^{-3}$ Required Resolution (degree)</th>
<th>$n_e=1E18$ m$^{-3}$ Required Resolution (degree)</th>
<th>Phase Resolution at 1 kHz (degree)</th>
<th>Phase Resolution at 60 Hz (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.59</td>
<td>5.3</td>
<td>20</td>
<td>0.62</td>
<td>0.006</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>47.6</td>
<td>5.3</td>
<td>20</td>
<td>12</td>
<td>0.13</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>57.2</td>
<td>5.3</td>
<td>20</td>
<td>18</td>
<td>0.18</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

10.59 μm polarimetry meets requirements in $n_e \sim 10^{19}$ m$^{-3}$ range with 1 kHz bandwidth

- over all ranges, 10.59 μm phase shift is $< 2\pi = \text{no fringe jumps}$
- can be used to correct 2-Color Interferometer for fringe errors

Assuming 0.1 degree resolution, 47/57 μm polarimetry meets requirements even at minimum density, BUT

- even at moderate densities $\sim 10^{20}$ m$^{-3}$, fringe jumps are possible
Refraction negligible at 10.6 µm

- Density profile refraction effects at 10.6 µm are negligible
- For high-density case (10^{21} m^{-3}) displacements several cm at 57/47 µm
- Radial displacement doubled by retro-reflector
- Path length change introduces negligible line-density errors at all λs
- Refraction likely dominated by ELMs, pellets, and H-mode pedestals - 3D calculations to be carried out
Control of Plasma Facing Mirror Degradation

Plasma facing mirrors in ITER will be subject to a variety of deleterious effects - a concern for first mirrors and retro-reflectors

Problems Include:

- Erosion due to charge exchange neutrals
- Impurity deposition
- Both affect reflectivity and polarization dependence

Mitigate deleterious effects by:

1. Reducing Solid Angle ($d\Omega$)
2. Temperature Control
3. Choice of Material

44 cm recess $\Rightarrow$ ~ 1/340 reduction in $d\Omega$

5 µm Erosion* is Reduced to 15 nm = Acceptable

*V.S. Voitsenya et al., RSI, 76, 083502 (2005)
Multiple First Mirror Design to improve mirror protection from neutron flux

- Individual mirror for each chord
- Single small penetration through the wall
- Better protection for mirrors
Mitigate deleterious effects by:

1. Reducing Solid Angle ($d\Omega$)
2. Temperature Control
3. Choice of Material

- Evidence from DIII-D* suggests deposition can be drastically reduced by keeping mirrors in 100-150°C range
- Temperature control is already required to reduce mirror distortion
- Several materials have acceptably low sputtering rates and minimal phase shifts between S&P, i.e. Tungsten, Rhodium, Molybdenum

*D.L. Rudakov et al., RSI 77, 010F126 (2006)
Laser Beam Diameter < 3.5 cm at 10.59 µm

- Gaussian beam propagation using ZEMAX
- Beam diameter ~ λ^{1/2}
- Area of hole in wall, solid angle, and mirror erosion ~5x larger for 57 µm
- Longer λ does not help mirror erosion problem
Finite Temperature Effects

• Finite $T_e$ will cause apparent reduced density using cold plasma interpretation

• At 15 keV effect is 4.4% for Interf. and 6% for Polarimetry

**Analytic Approx. by Mirnov:**

**Interfer.**  \[ \phi = c_I \int n_e \, dl - \frac{3}{2} \frac{c_I}{m_e c^2} \int n_e T_e \, dl \]

**Faraday**  \[ \alpha = c_F \int n_e B_\parallel \, dl - \frac{2c_F}{m_e c^2} \int n_e T_e B_\parallel \, dl \]

Summary TIP for ITER

• CO₂ laser 10.6/5.3 μm based TIP meets ITER requirements
  – Provides 2 independent density measurements (no fringe skips)
  – well-developed technology
  – negligible refractive effects
• First mirror survivability issues can be resolved
  – solid angle reduction
  – temperature control
  – Longer wavelengths require larger solid angles -> little advantage
• Use multiple separate first mirrors and a common first wall penetration to maximize redundancy and minimize exposure
• Feedback alignment system required for each chord
• Thermal effects can contribute 5-10% error unless properly treated
More advanced concepts

- Differential interferometer
- Fizeau Interferometer
Differential Interferometer scheme

- **Differential Interferometer measures phase difference between closely-spaced laser beams**
- **Little sensitive to path length changes and immune to fringe counting errors**

\[
\begin{align*}
\phi(x) &= r \lambda \int \frac{\sqrt{a^2 - x^2}}{\sqrt{a^2 - x^2}} n_e(r) \, dz, \\
n_e(r) &= -\frac{1}{\pi r \lambda} \int \frac{\partial \phi(x)}{\partial x} \frac{dx}{\sqrt{x^2 - r^2}}
\end{align*}
\]

- Beam diameter $\sim 1$ cm
- $\delta \sim 1$ mm

\[
\omega_1 - \omega_2 \rightarrow \Delta \phi(x) = \int n \, dl - \int n \, dl
\]

\[
\omega_1 - \omega_3 \rightarrow \phi(x_1) = \int_{x_1} n \, dl
\]

*Ding et al. RSI 77, 10F105 (2006)*
Differential Interferometer results

- $d\phi/dx$ from standard interferometer matches differential interferometer and gives the same equilibrium profile

\[ n_e(r) = -\frac{1}{\pi r_c \lambda} \int_r^a \frac{\partial \phi(x)}{\partial x} \frac{dx}{\sqrt{x^2 - r^2}} \]
FIZEAU EFFECT

Fizeau effect is the relativistic phase shift of an EM wave due to movement of a dielectric medium. [H. Fizeau, Ann. Chim. Phys. (3)57,385(1859)].

Consider a laser beam propagating in a moving plasma with refractivity index $N'$, length $L'$, and velocity $v$:

The phase shift is relativistically invariant and to lowest order in $\beta = v/c$, the shift measured by an interferometer is

$$\Delta \phi = \Delta \phi' = \omega' \Delta t' - k' \Delta x' = -\omega' c L' (\beta + N')$$

Where primed and unprimed variables refer to the moving plasma and laboratory frames, respectively

FIZEAU EFFECT

By using the Lorentz transformation and considering a small Doppler shift ($\omega' = \omega - \beta \omega$), the phase shift in laboratory frame is:

$$\Delta \phi \sim \frac{\omega}{c} \left[ - L' N - \beta L' \left( 1 - N - \omega N \frac{\partial N}{\partial \omega} \right) \right]$$

First term is phase measured by a standard interferometer.

The second term, caused by plasma motion, can be measured by a Fizeau interferometer (i.e., with counter-propagating beams along same optical path).