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Physics of Zonal Flows

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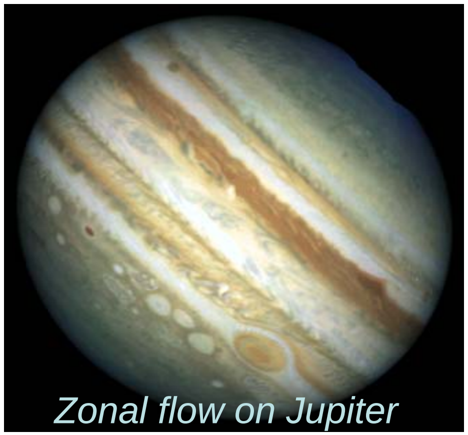
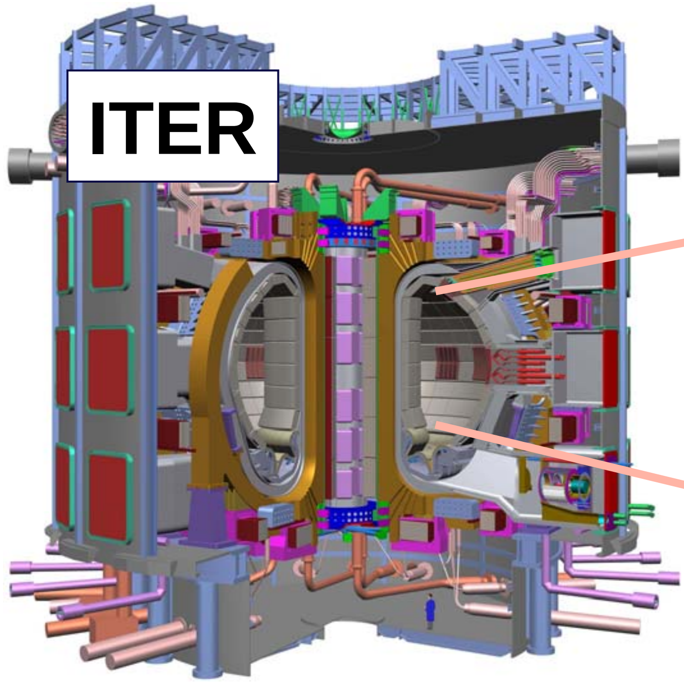
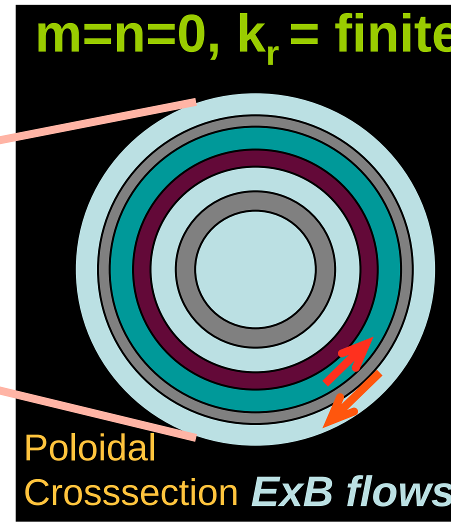
This overview is dedicated to the memory of
Professor Marshall N. Rosenbluth.

Contents

What is a zonal flow?







Drift Waves

↓

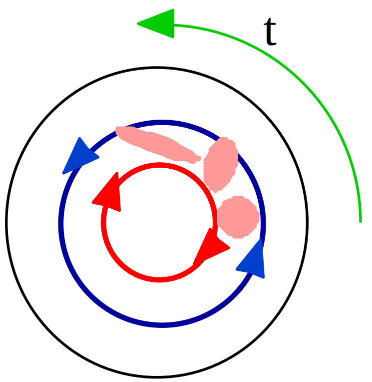
Drift waves
+
Zonal flows

Paradigm Change

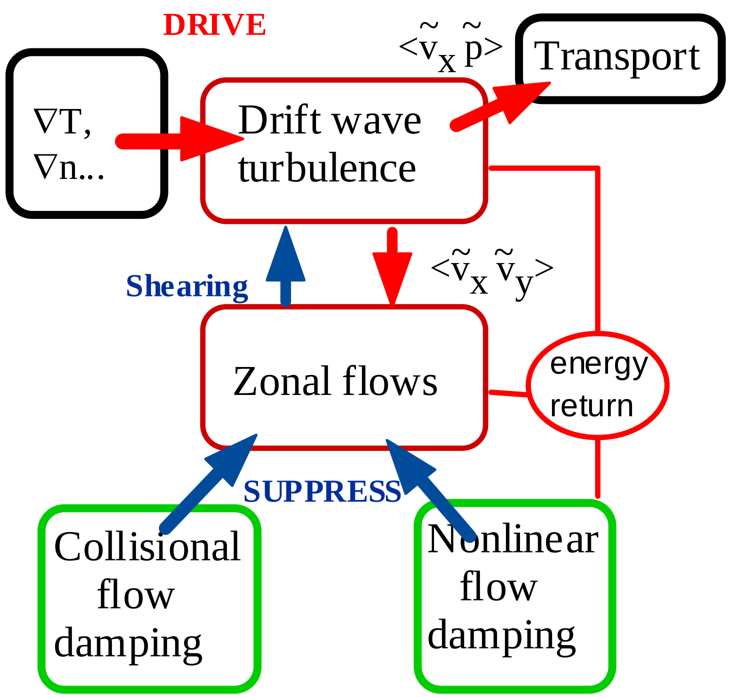
ZFs are "mode", but:

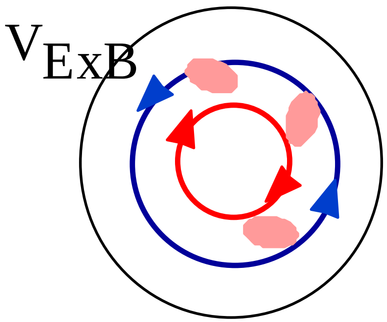
1. Turbulence driven
2. No linear instability
3. No direct radial transport

Basic Physics of a zonal flow

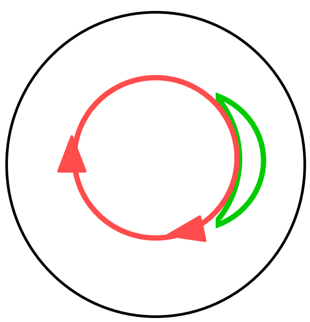


Suppression of DW
by shearing

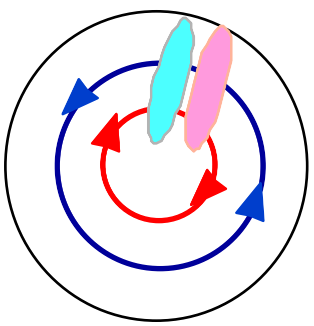




Generation
By vortex tilting



Damping by collisions



Tertiary instability

Self-regulation: Co-existence of ZF and DW

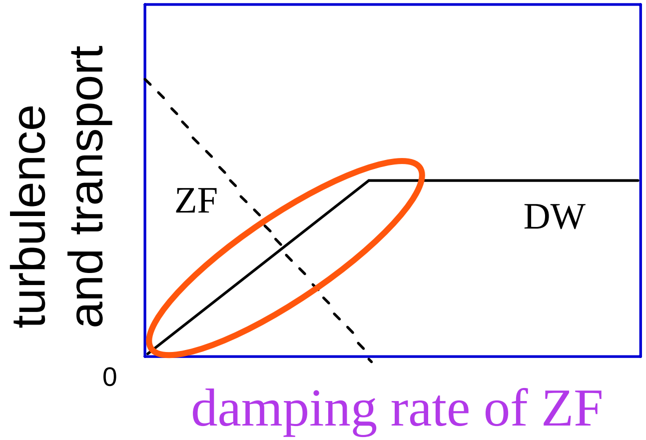
$$\begin{cases} \frac{\partial}{\partial t} W_d = \gamma\left[\nabla p_0, \cdots\right] W_d - \alpha W_d W_{ZF} \\ \frac{\partial}{\partial t} W_{ZF} = \gamma_{damp}\left[\cdots\right] W_{ZF} + \alpha W_d W_{ZF} \end{cases}$$

W_d : drift wave energy

W_{ZF} : zonal flow energy

(ν_{ii}, q, ε geometry) (+ rf, etc.)

$$W_d \sim \frac{\gamma_{damp}}{\alpha}$$



Transport coefficient

$$\chi_i \sim \frac{\gamma_{damp}}{\omega_{eff}} \chi_{gB} \Rightarrow \chi_i = \mathcal{R} \chi_{gB}$$

"$\mathcal{R}$ – Factor"

Confinement enhancement
Includes other reduction effects
(i.e., cross phase)

Why ZFs are Important for Fusion ?

Issues in fusion

1. Self-regulation

$$\chi = \mathcal{R}\chi_{gB}$$

$\tau_E = H\tau_{E,\mathrm{L-scaling}}$

"Intrinsic" H-factor (wo barrier)

$$H \propto \mathcal{R}^{-0.6}$$

Realizability of Ignition

$$\$ \propto \mathcal{R}^{-0.8}$$

$$\$ \propto H^{-1.3}$$

2. Shift for onset of large transport → Operation of ITER near marginality

3. Identify transport control knob → Flow damping ? → P_{LH}- threshold

→ Intermittent flux, e.g., ELMs → Peak heat load

4. Meso-scales and zonal field → Impacts on RWM and NTM → β-limit → Steady state

Assessment I "*What we understand*"

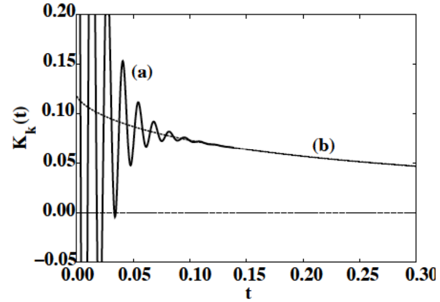
ZFs are UNIVERSAL

	Scales	Transport	Role of ZF	This IAEA Conference
ITG	ρ_i c_s/a	core χ_i, χ_ϕ, D	Very important	Falchetto (TH/1-3Rd), Hahm (TH/1-4), Hallatschek (TH/P6-3), Hamaguchi (TH/8-3Ra), Miyato (TH/8-5Ra), Waltz (TH/8-2), Watanabe (TH/8-3Rb)
TEM/TIM	ρ_i c_s/a	core $\chi_i, \chi_e, \chi_\phi, D$	Very important	Lin (TH/8-4), Sarazin (TH/P6-7), Terry (TH/P6-9)
ETG	ρ_e $v_{\mathrm{Th},e}/a$	core χ_e, χ_J	On-going	Holland (TH/P6-5), Horton (TH/P3-5), Idomura (TH/8-1), Li (TH/8-5Ra), Lin (TH/8-4)
Resistive ballooning/ interchange	$\Delta_{\text{resis. layer}}$	edge, sol, helical edge	Important	Benkadda(TH/1-3Rb), del-Castillo-Negrette (TH/1-2)
Drift Alfven waves at edge	$\sim \rho_i$	edge, sol	Very important	Scott(TH/7-1), Shurygin(TH/P6-8)

Linear Damping of Zonal Flows

Neoclassical process

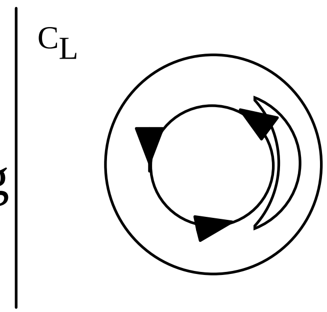
Rosenbluth-Hinton
undamped flow - survives for
$\tau > \tau_{\text{transport}}$



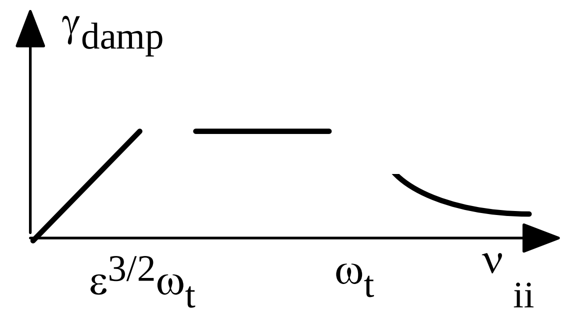
$$\frac{\phi_{\boldsymbol{q}}(t)}{\phi_{\boldsymbol{q}}(0)} = \frac{1}{1 + 1.6\varepsilon^{-1/2} q^2}$$

Role of bananas

Frictional damping



Banana-plateau transition



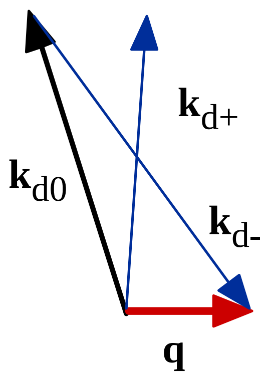
$\gamma_{\text{damp}} \simeq \nu_{ii}/\varepsilon \Rightarrow \chi_i \propto \nu_{ii}$ even in "collisionless" regime

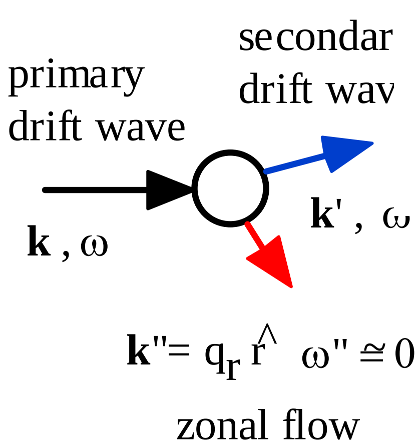
Screening effect if $q_r \rho_{\text{p}} \sim O(1)$

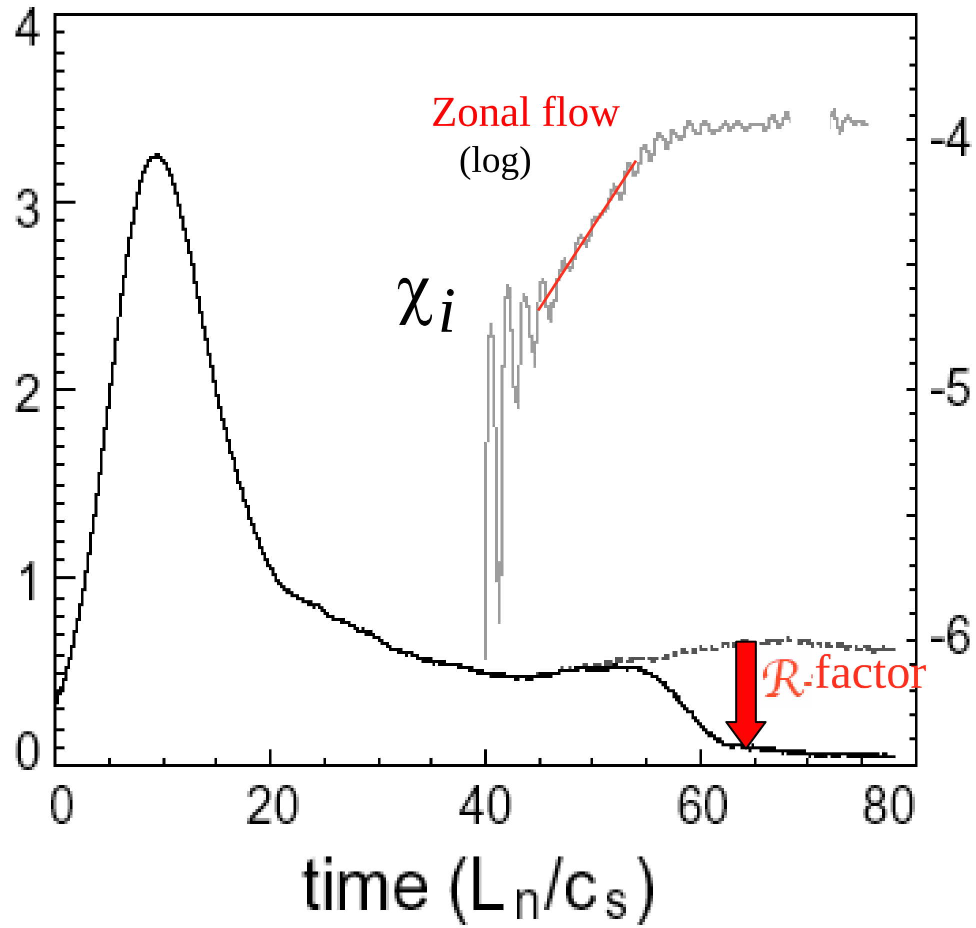
Growth Mechanism

ZFs by

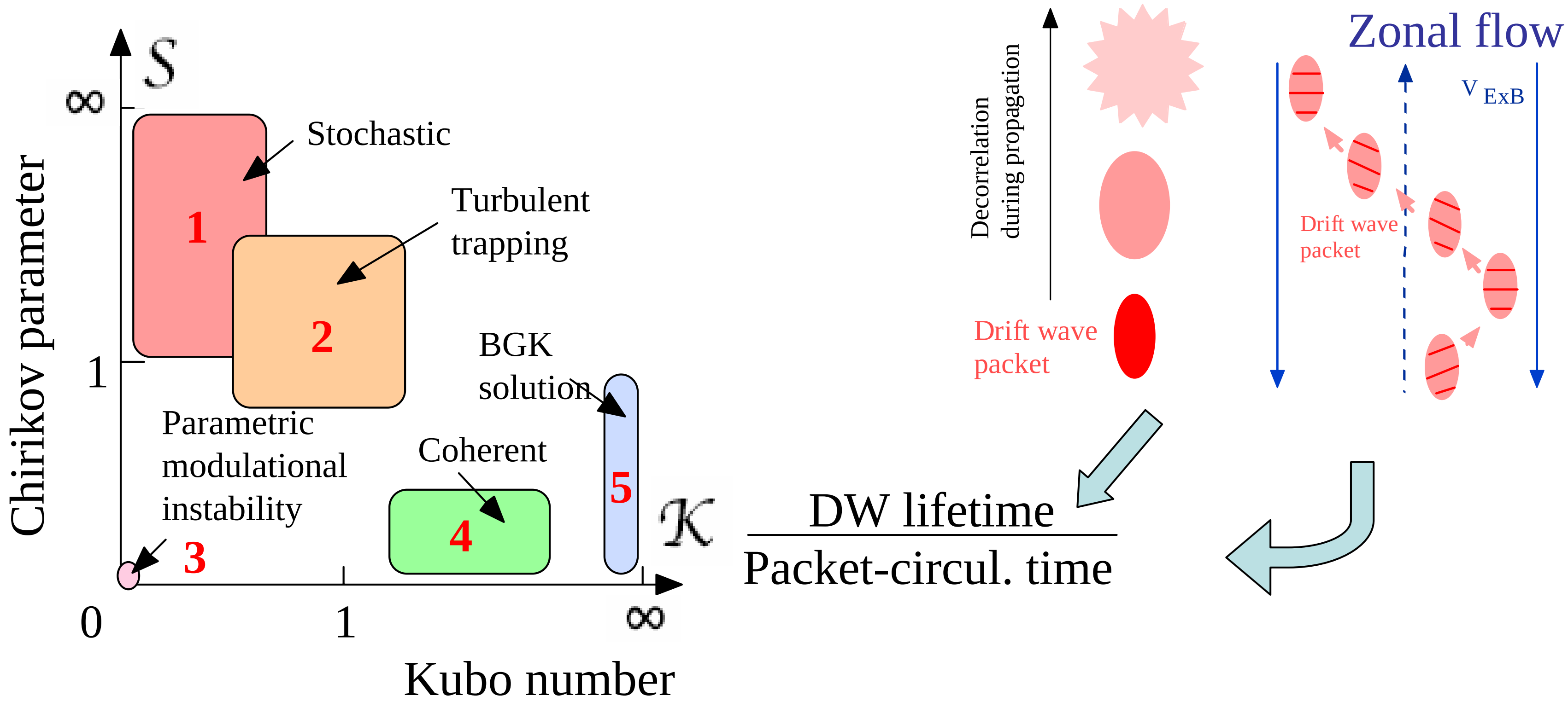
modulational instability







Numerical experiment indicates instability of finite amplitude gas of drift waves to zonal shears



Regime	Keywords	References
1	k_r Diffusion	Zakharov, PHD, Itoh, Kim, Krommes
2	Turbulent trapping	Balescu
3	Single wave modulation	Sagdeev, Hasegawa, Chen, Zonca
4	Reductive perturbation	Taniuti, Weiland, Champeaux
5	DW trapping in ZF	Kaw

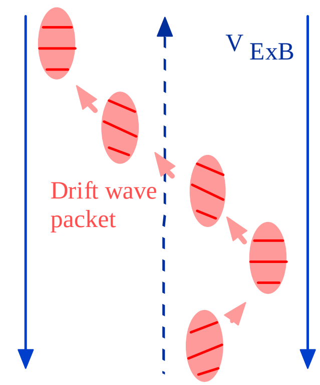
Close Relationship: DW + ZF and Vlasov Plasma

(i) DW + ZF :

$$\frac{dx}{dt} = v_{gx}(\boldsymbol{k}) \qquad \frac{dk_x}{dt} = -\frac{\partial}{\partial x}\left(k_y \tilde{V}_Z\right)$$
$$\frac{dy}{dt} = v_{gy}(\boldsymbol{k}) \qquad \frac{dk_y}{dt} = 0$$

⇒ 'Ray' Trapping

$$\Omega = q_x v_{gx}$$



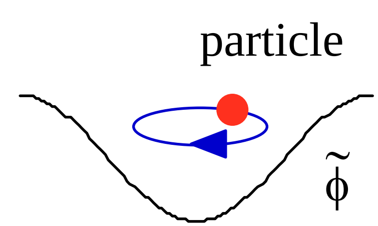
$$\frac{\partial}{\partial t}\tilde{V}_Z + \gamma_{damp}\tilde{V}_Z = -\frac{\partial}{\partial x}\left\langle \tilde{V}_x \tilde{V}_y \right\rangle$$
$$\frac{\partial}{\partial t} N + v_{gx} N - \frac{\partial}{\partial x}\left(k_y V_Z\right)\frac{\partial N}{\partial k_x} = \gamma_k N + C_k(N)$$

(ii) 1D Vlasov Plasma :

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = \frac{e}{m}\tilde{E}$$

⇒ Particle Trapping

$$\omega = k\,v$$



$$\frac{\partial \tilde{E}}{\partial x} = 4\pi n_0 e \int dv \quad f$$
$$\frac{\partial f}{\partial t} + v\frac{d f}{dx} + \frac{e}{m}\tilde{E}\frac{\partial f}{\partial v} = C(f)$$

Note: Conservation energy between ZF and DW

RPA equations

DW $$\frac{\partial}{\partial t}\left|\tilde{V}_{\mathrm{DW}}\right|^2+\sum_{\boldsymbol{k}}\left(\gamma_{\mathrm{L},\boldsymbol{k}}+C_{\boldsymbol{k}}(N)\right)\left|\tilde{V}_{\mathrm{DW},\boldsymbol{k}}\right|^2=\frac{2}{B^2}\sum_{q_x}\int d^2k\,\frac{q_x^2k_\theta^2k_x\left|V_{\mathrm{ZF},q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2}R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

ZF $$\left(\frac{\partial}{\partial t}+\gamma_{\mathrm{damp}}\right)\left|V_{\mathrm{ZF}}\right|^2=-\frac{2}{B^2}\sum_{q_x}\int d^2k\,\frac{q_x^2k_\theta^2k_x\left|V_{\mathrm{ZF},q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2}R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

Coherent equations

DW $$\frac{\mathrm{d}P^2}{\mathrm{d}\tau}=P^2-2PZS\cos(\Psi)$$ (S: beat wave)

ZF $$\frac{\mathrm{d}Z^2}{\mathrm{d}\tau}=-\frac{\gamma_{\mathrm{damp}}}{\gamma_L}Z^2+2PZS\cos(\Psi)$$

$$\left.\frac{\partial}{\partial t}W_{\mathrm{d}}\right|_{\text{by ZF}}=-\left.\frac{\partial}{\partial t}W_{\mathrm{ZF}}\right|_{\text{by DW}}$$

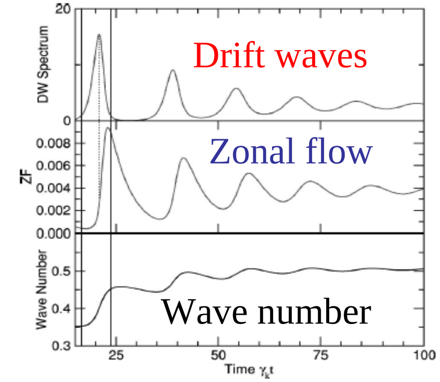
Self-regulating System Dynamics

Simplified Predator-Prey model

DW $$\frac{\partial}{\partial t}\langle N\rangle = \gamma_L\langle N\rangle - \gamma_2\langle N\rangle^2 - \alpha\langle U^2\rangle\langle N\rangle$$

ZF $$\frac{\partial}{\partial t}\langle U^2\rangle = -\gamma_{damp}\langle U^2\rangle + \alpha\langle U^2\rangle\langle N\rangle$$

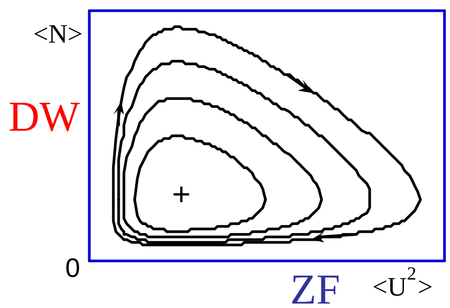
Stable fixed point

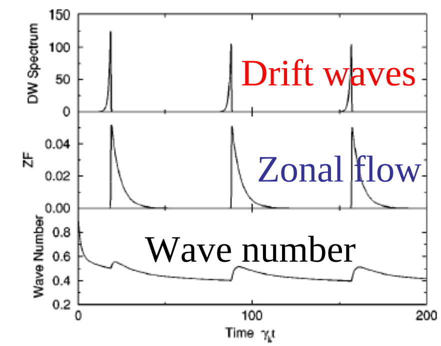


Cyclic bursts

$\gamma_2 \rightarrow 0$

(No self damping)

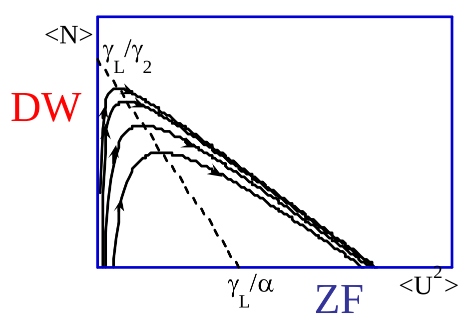


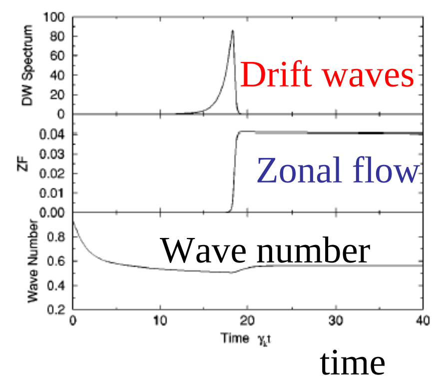


Single burst
(Dimits shift)

$\gamma_{damp} \rightarrow 0$

(No ZF friction)





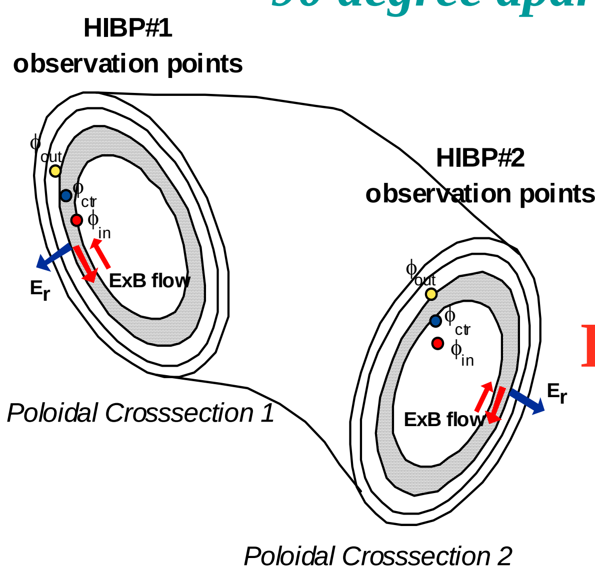
II Current research: "*What we think we understand*"

Zonal flows really do exist

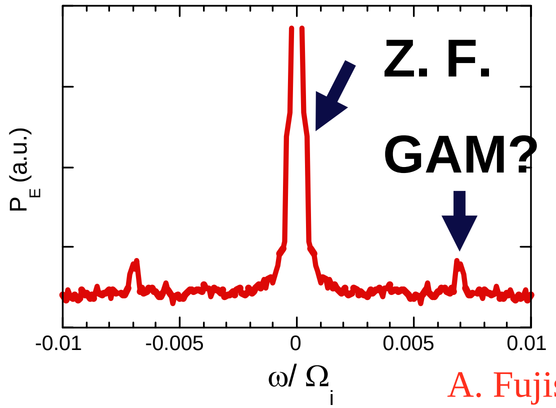
CHS Dual HIBP System

A. Fujisawa et al., EX/8-5Rb

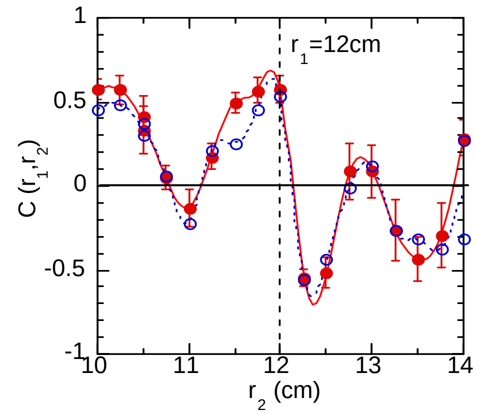
90 degree apart



$E_r(r,t)$



A. Fujisawa et al.,
PRL **93** 165002(2004)



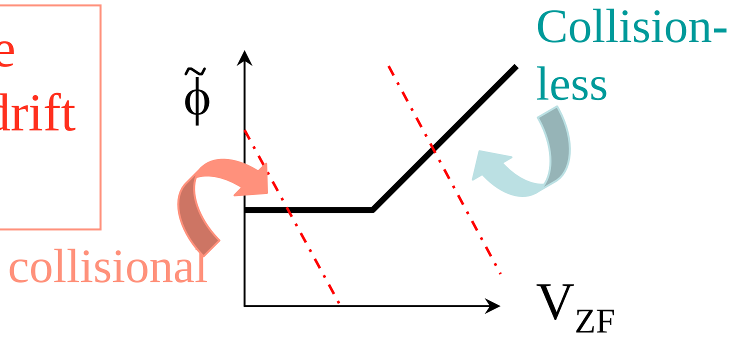
Radial distance

$E_r(r,t)$ {
High correlation on magnetic surface,
Slowly evolving in time,
Rapidly changing in radius.

Candidates for Collisionless Saturation

$$\chi = \frac{\gamma_{nl}^{collisionless}}{\omega_{eff}} \chi_{gB}$$

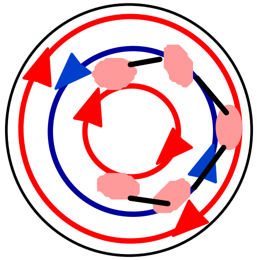
Partial recovery of the dependence of χ_i on drift wave growth rate



Trapping

Plateau in k-space N(k)

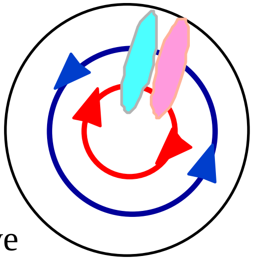
$$\sqrt{V_{ZF}} \propto \omega_{bounce} \simeq \Delta\omega, \gamma_L$$



Tertiary Instability

Returns energy to drift waves

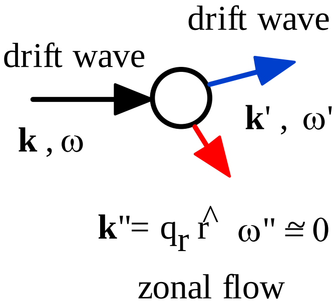
$$V_{ZF} \sim \left| \hat{\phi} + \frac{T_e}{2T_i} \hat{T} \right| q_r$$



Higher Order Kinetics

Analogous to interaction at beat wave resonance

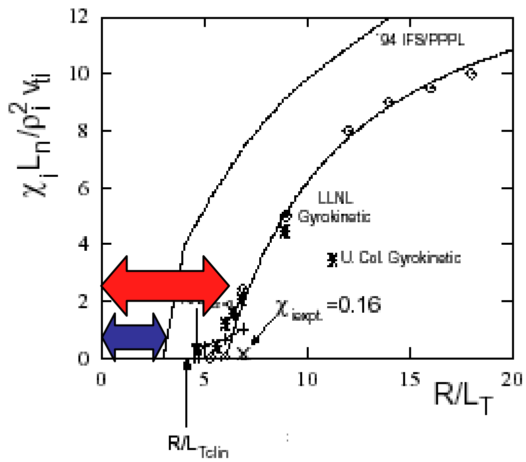
$$V_{ZF} \sim \Delta\omega \, k_\theta^{-1}$$



"Near" Marginality

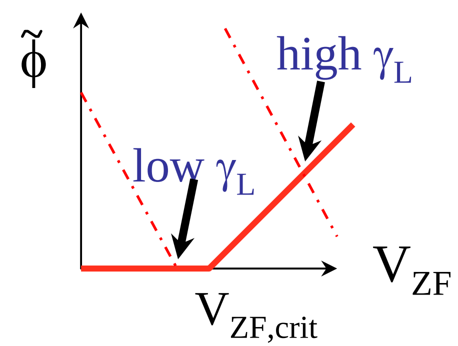
Shift exists

ITER plasma near marginality ⇨
Importance of ZF



Where the energy goes

What limits the "nearly marginal" region:
(Dimits shift)



Mechanisms: Trapping, Tertiary, Higher order nonlinearity,….

An example: $\gamma_{\mathrm{L,\,crit}} \sim q_r^2 k_\theta^{-2} \alpha$

(Higher-order nonlinearity model)

Route to the shift is understood, but "the number" is not yet obtained.

Electromagnetic Effect

Subject	Mechanism for ZF growth	Implications for fusion
ZF generation by finite-β drift waves	Modulational instability of a drift Alfven wave	Transport at high-β, L-H transition
Zonal magnetic field generation	Random refraction of Alfven wave turbulence	Possible Z-field induced transition, NTM,…

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle \Rightarrow \langle \tilde{v}_r \tilde{v}_\theta \rangle - \langle \tilde{B}_r \tilde{B}_\theta \rangle$$

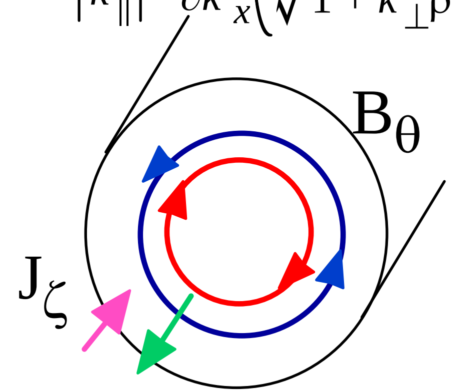
Selected structure
Zonal flow ⇒ Zonal field
β ⟶

$$\frac{\partial}{\partial t} B_\theta = -\eta_{ZF} \nabla^2 B_\theta$$

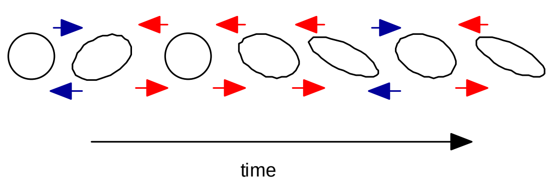
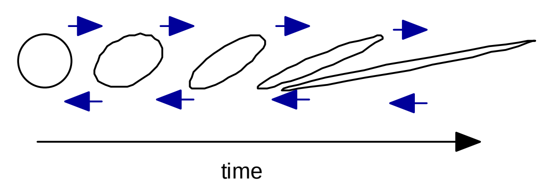
$$\eta_{ZF} = -\frac{4\pi c_s^2 \delta_e^2}{v_{th,e}\left(1 + q_r^2 \delta_e^2\right)} \sum_k \frac{\left(1 + k_\perp^2 \rho_s^2\right)^{5/2}}{2 + k_\perp^2 \rho_s^2} \frac{k_\perp^2 k_y^2}{|k_\parallel|} \frac{\partial^2}{\partial k_x^2} \left(\frac{\langle \omega_k N_k \rangle}{\sqrt{1 + k_\perp^2 \rho_s^2}} \right) f_0$$

⇒ Current layer generation
Corrugated magnetic shear,
Tertiary micro tearing mode ?

⇒ Can seed Neoclassical Tearing Mode
Localized current at separatrix of tearing mode, ……



Distinction between ZF and mean Field $\langle E_r \rangle$

	Zonal Flows	Mean Field $\langle E_r \rangle$
Time	can change on turbulence time scales	changes on transport time scales
Space	oscillating, complex pattern in radius $\sim 20\ \rho_i$	smoothly varying
Stretching Behavior ***k*** of waves	diffusive $\langle \delta k^2 \rangle \propto t$ 	ballistic $\langle \delta k^2 \rangle = t^2\, k^2 V_E'^{\,2}$ 
Drive	Turbulence	equilibrium ∇p, orbit loss, external torque, turbulence, etc.

Interplay of Zonal Flow and Mean $\langle E_r \rangle$

Now: Coupling between DW, ZF, mean $\langle E_r \rangle$ and mean profile

Previous: Coupling between DW, mean $\langle E_r \rangle$ and mean profile

Fluctuations

$$\frac{d\mathcal{E}}{dt} = \mathcal{E}\mathcal{N} - a_1\mathcal{E}^2 - a_2V^2\mathcal{E} - a_3V_{ZF}^2\mathcal{E}$$

Pressure gradient

$$\frac{d\mathcal{N}}{dt} = -c_1\mathcal{E}\mathcal{N} - c_2\mathcal{N} + Q$$

Zonal flow

$$\frac{dV_{ZF}}{dt} = b_1\frac{V_{ZF}\mathcal{E}}{1+b_2V^2} - b_3V_{ZF}$$

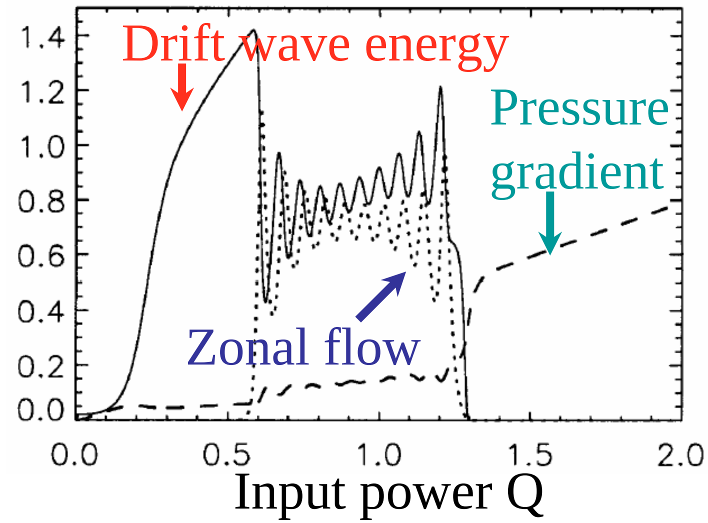
Mean flow

$$\frac{dV}{dt} = \left(V - d\mathcal{N}^2\right) + F_{nonlinear}$$

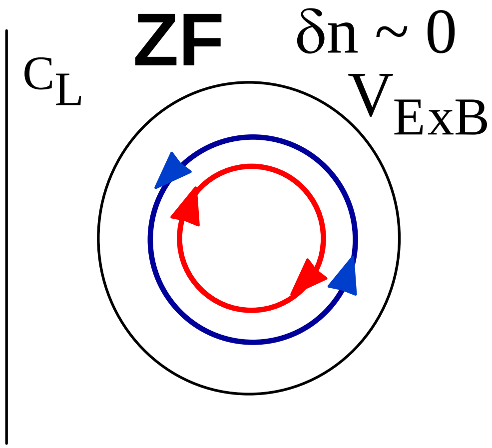
New coupling

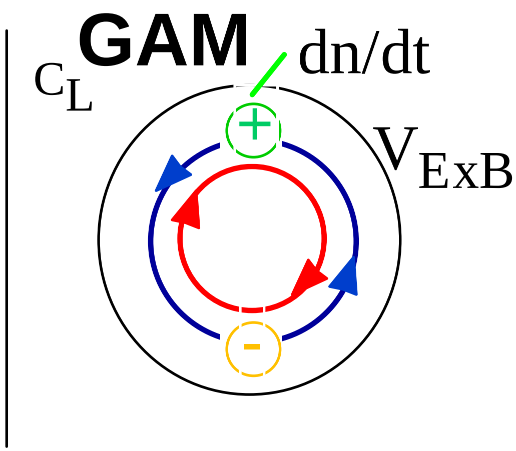
Nonlinearity; orbit loss, etc. in previous model

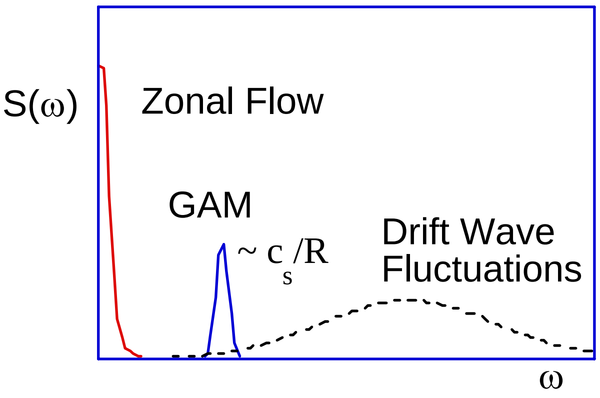
Prediction of bifurcation, dither, hysteresis, …



Zonal Flow and GAM: two kinds of secondary flow





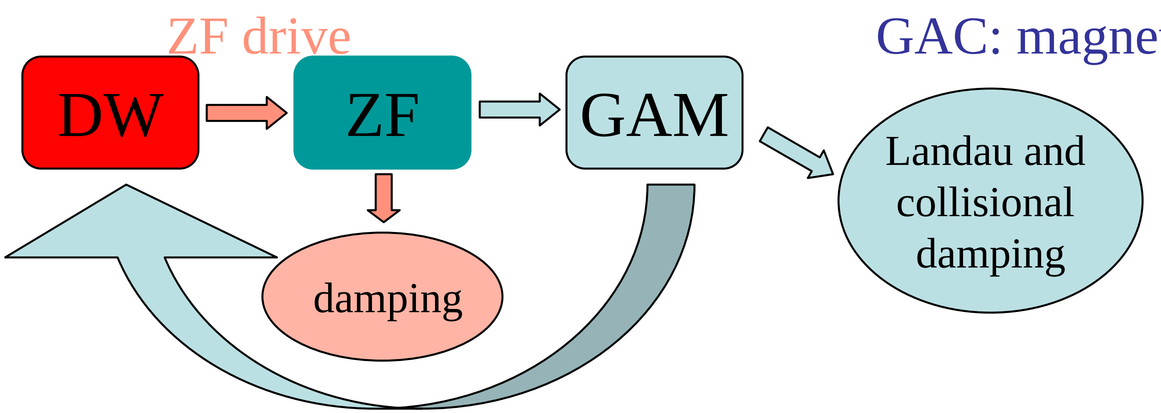


GAMS: important near the edge Lower temperature: ν_{ii} ↑, and c_s/R ↓

⇨ ZF dynamics and GAM dynamics merge

New feature: geodesic acoustic coupling (GAC)

GAC: magnetic field sensitive



Control Knobs

(i) Zonal Flow Damping ⇐ Collisionality, ε, q, geometry, …
n.b. especially stellarators

(ii) External Drive (e.g., RF) ⇐ Choice of wave, Wave polarity, launching, (e.g., studies of IBW), rational surfaces…

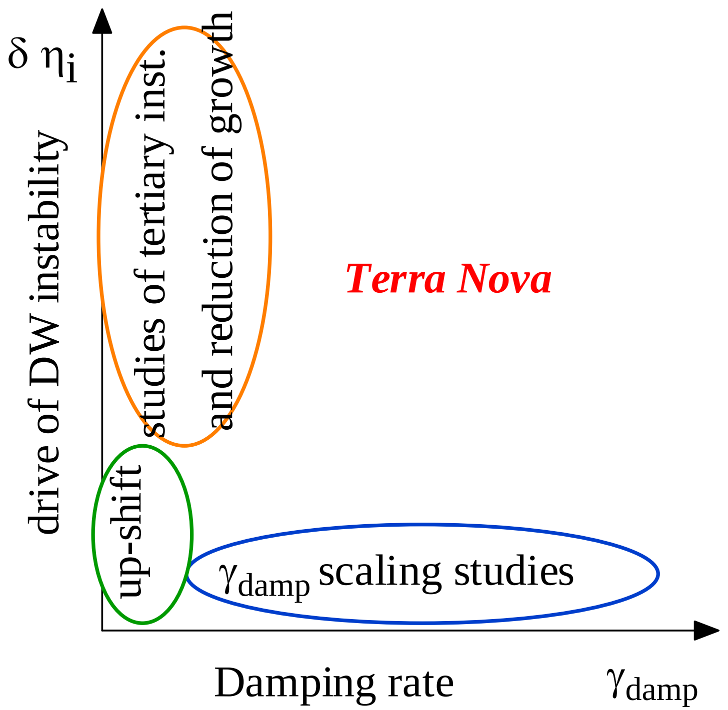
III Future Research: "*What we do not yet understand*"

1. Experimentally convincing link between ZFs and confinement

2. Dominant collisionless saturation mechanism: selection rule ?

3. Quantitative predictability

   (a) $\mathcal{R}$-factor ? - trends ??
   (b) Suppression - γ_E vs γ_{Lin} ?
   (c) How near marginality ?
   (d) Effects on transition ?
   (e) Flux PDF ?



4. Pattern formation competition

   ZF vs. Streamers, Avalanches

5. Efficiency of control

6. Mini-max principle for self-consistent DW-ZF system ?

Summary

1. Paradigm Change: DW and turbulent transport → DW-ZF system and turbulent transport

   Linear and quasi-linear theory → Nonlinear theory

   (Simple way does not work.)

2. Critical for Fusion Devices

   e. g., { $\mathcal{R}$-factor ⇒ (Helps along) Route to ITER; Barrier Transitions }

3. Progress and Convergence of Thinking on ZF Physics

4. Speculations: More Importance for Wider Issues

   e.g., TAE, RWM, NTM; peak heat load problem, etc.

Mini-Max Principle for Self-Consistent DW-ZF System ?

Predator-Pray model

$$\frac{\partial}{\partial t} N = \gamma_L N - \gamma_2 N^2 - \alpha U^2 N$$
$$\frac{\partial}{\partial t} U^2 = -\gamma_{damp} U^2 + \alpha U^2 N$$

Fixed point

$$N_* \equiv \frac{\gamma_{damp}}{\alpha}$$
$$U_*^2 \equiv \frac{\gamma_L}{\alpha} - \frac{\gamma_2 \gamma_{damp}}{\alpha^2}$$

Lyapunov Function

$$F = \left(N - N_* \ln N\right) + \left(U^2 - U_*^2 \ln U^2\right) \;:\; \frac{\partial F}{\partial t} = -\gamma_2 \left(N - N_*\right)^2 < 0$$

Mini-max principle: *F*: minimum

This Lyapunov function is an extended form of Helmholtz free energy

$$N - N_* \ln N \Rightarrow E_{DW} - T_{eff,D} S_{DW}$$
$$U^2 - U_*^2 \ln U^2 \Rightarrow E_{ZF} - T_{eff,Z} S_{ZF}$$

$$S_{DW} = k_B \left(\ln N + 1\right) \qquad S_{ZF} = k_B \left(\ln U^2 + 1\right)$$
$$T_{eff,D} = k_B^{-1} N_* \qquad T_{eff,Z} = k_B^{-1} U_*^2$$

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Research on Structural Formation and Selection Rules in Turbulent Plasmas

At Kyushu, NIFS, Kyoto, UCSD, IPP

Plasma Turbulence - Itoh Project -

Principal Investigator: Sanae-I. ITOH

Focus

Search for mechanisms of structure formation in turbulent plasmas
Selection rule among realizable states through possible transitions

Members and this IAEA Conference

M. Yagi: TH/P5-17 Nonlinear simulation of tearing mode and m=1 kink mode based on kinetic RMHD model

A. Fujisawa: EX/8-5Rb Experimental studies of zonal flows in CHS and JIPPT-IIU

A. Fukuyama: TH/P2-3 Advanced transport modelling of toroidal plasmas with transport barriers

K. Hallatschek: TH/P6-3 Forces on zonal flows in tokamak core turbulence

P. H. Diamond: OV/2-1, This talk

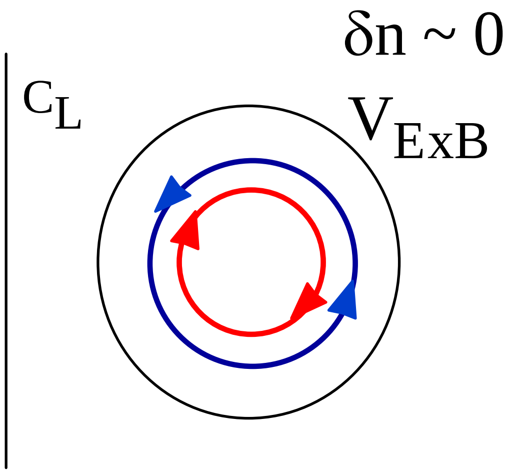
Other member collaborators

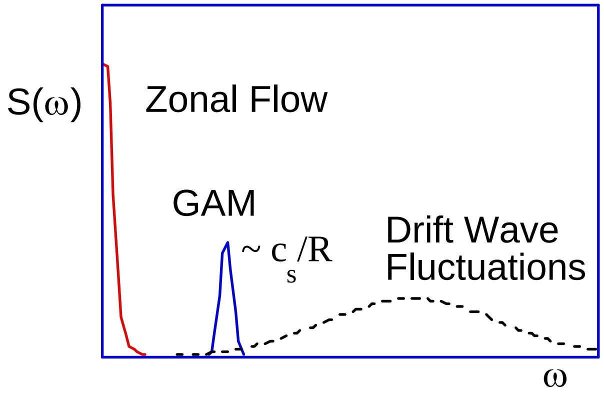
Y. Kawai, S. Shinohara, K. Itoh

NIFS UC San Diego IPP

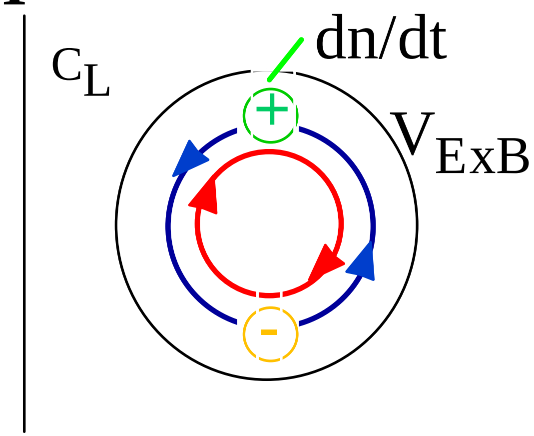
Zonal Flow and GAM: two kinds of secondary flow

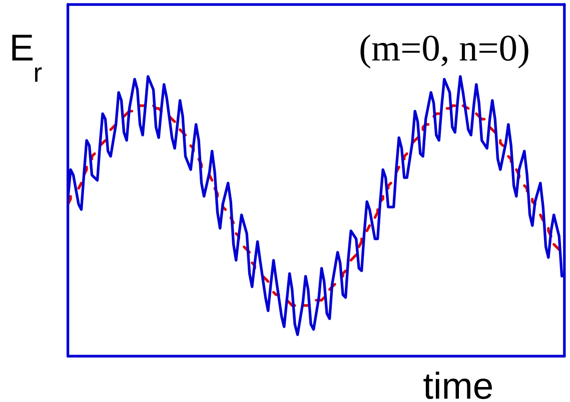
ZF





GAM



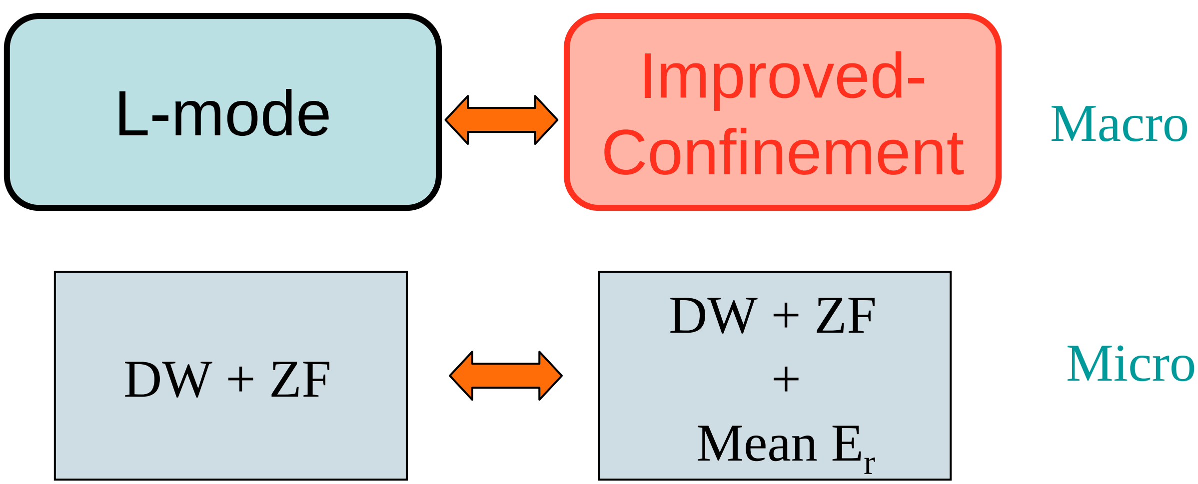


Impact on Transport

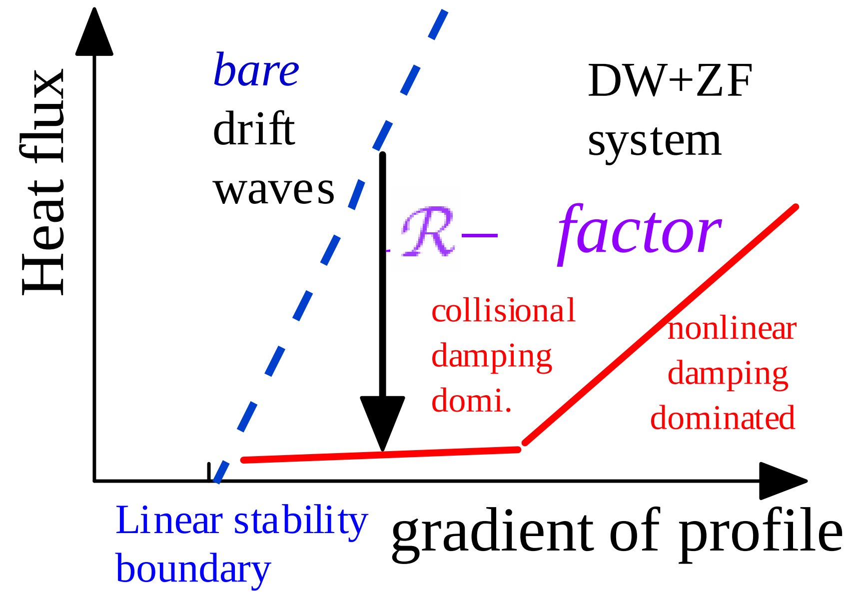
$$Q_r = \langle \tilde{p} \tilde{V}_r \rangle \propto |\tilde{\phi}|^2 \sin\alpha$$

- Shearing ⇒ Suppress fluctuation amplitude
- Cross phase ⇒ Suppress flux, as well

L-mode and Improved-confinement modes



	Theoretical form	User's formula
Old picture *bare* drift waves	D_{mix}	D_{gB}
New picture DW-ZF system	$f\left(\frac{\gamma_{\text{damp}}}{\omega_{\text{eff}}}\right) D_{\text{mix}}$	$\frac{\gamma_{\text{damp}}}{\omega_{eff}} D_{\text{gB}}$

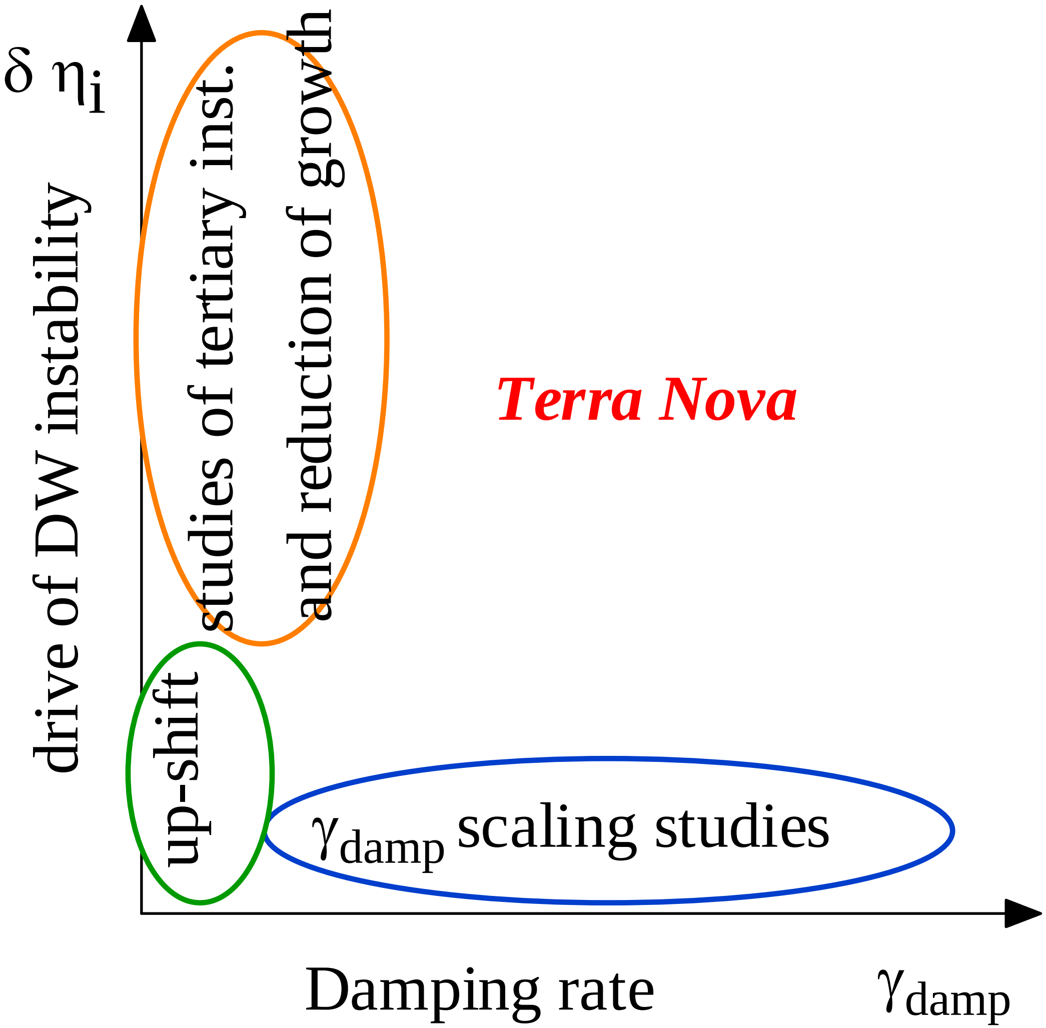


$$\mathcal{R} = \frac{\gamma_{\text{damp}}}{\omega_*}$$

Further issues

α-particle feedback ⇨ zonal flow effects on TAE

zonal field ⇨ feeding neoclassical tearing mode


δ ηi
drive of DW instability
studies of tertiary inst.
and reduction of growth
Terra Nova
up-shift
γdamp scaling studies
Damping rate
γdamp