Non-diffusive transport in pressure driven plasma turbulence

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20th IAEA Fusion Energy Conference
Vilamoura, Portugal. 1-6 Nov, 2004

Beyond the standard diffusive transport paradigm

•Experimental and theoretical evidence suggests that transport in fusion plasmas deviates from the diffusion paradigm:

$$\partial_t T = \partial_x \left[\chi(x,t) \partial_x T \right] + S(x,t)$$

Examples: • Fast propagation and non-local transport phenomena
• Inward transport observed in off-axis fueling experiments

•Our goal is to develop transport models that overcome the limitations of the diffusion paradigm.

•The models incorporate in a unified way non-locality, memory effects, and non-diffusive scaling.

•To motivate and test the models we consider transport in pressure driven plasma turbulence

Non-diffusive transport

Pressure gradient driven plasma turbulence

$$\left(\partial_t + \tilde{V}\cdot\nabla\right)\nabla_\perp^2\tilde{\Phi} = \frac{B_0}{m_i n_0 r_c}\frac{1}{r}\frac{\partial\tilde{p}}{\partial\theta} - \frac{1}{\eta m_i n_0 R_0}\nabla_\parallel^2\tilde{\Phi} + \mu\nabla_\parallel^4\tilde{\Phi}$$

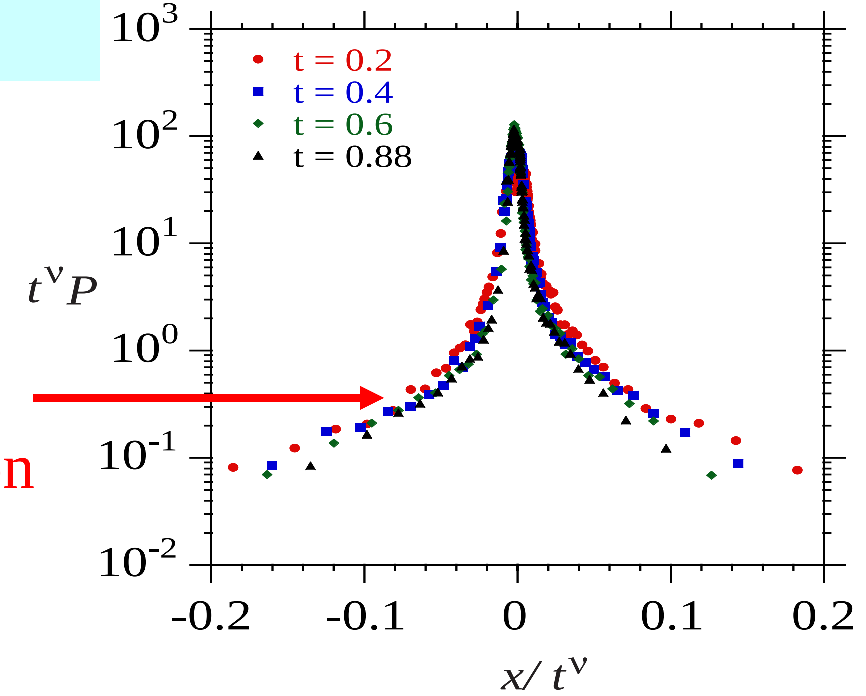
$$\left(\partial_t + \tilde{V}\cdot\nabla\right)\tilde{p} = \frac{\partial\langle p\rangle}{\partial r}\frac{1}{r}\frac{\partial\tilde{\Phi}}{\partial\theta} + \chi_\perp\nabla_\perp^2\tilde{p} + \chi_\parallel\nabla_\parallel^2\tilde{p}$$

$$\frac{\partial\langle p\rangle}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}r\langle\tilde{V}_r\,\tilde{p}\rangle = S_0 + D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\langle p\rangle}{\partial r}\right)$$

Tracers dynamics

$$\frac{d\vec{r}}{dt} = \vec{V} = \frac{1}{B^2}\nabla\tilde{\Phi}\times\vec{B}$$

Probability density function of radial displacements



Super-diffusive scaling

$$\langle\delta r^2\rangle \sim t^{4/3}$$

Levy distribution

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Towards an effective transport model for non-diffusive turbulent transport

- Individual tracers follow the turbulent field

$$\frac{d\vec{r}}{dt} = \tilde{V} = \frac{1}{B^2}\nabla\tilde{\Phi}\times\vec{B}$$

- The distribution of tracers *P* evolves according to

$$\frac{\partial P}{\partial t} + \tilde{V}\cdot\nabla P = 0$$

- The idea is to construct a model that "encapsulates" the complexity of the turbulence field $\tilde{V}$ in an effective flux Γ, and reproduces the observed pdf

$$\frac{\partial P}{\partial t} = -\frac{\partial \Gamma}{\partial x}$$

homogeneous isotropic turbulence → diffusive transport → Gaussian closures → $\Gamma = -\chi\,\partial_x P$

transport in pressure driven plasma turbulence → Non-Gaussian distributions → $\Gamma = ???????$

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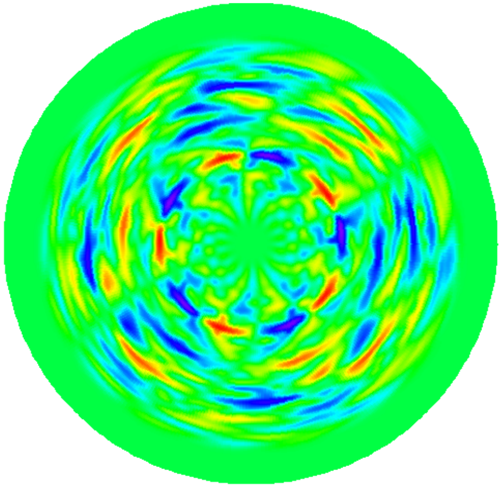
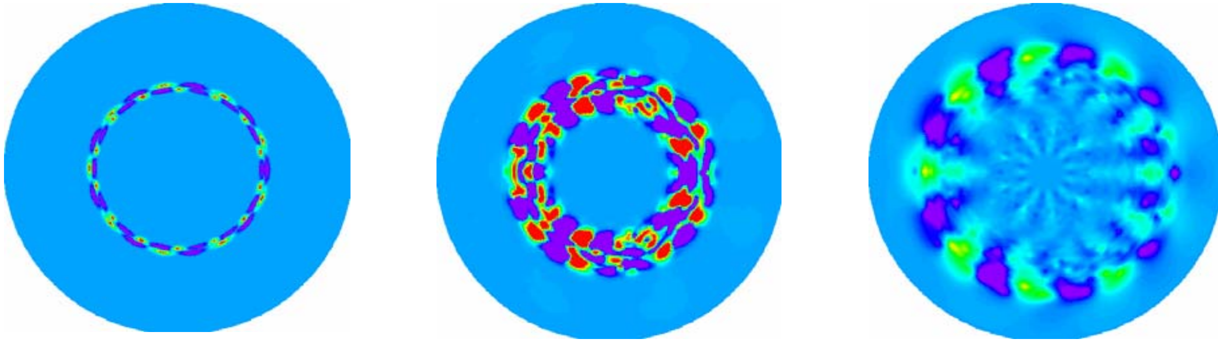
What is the origin of non-diffusive transport?

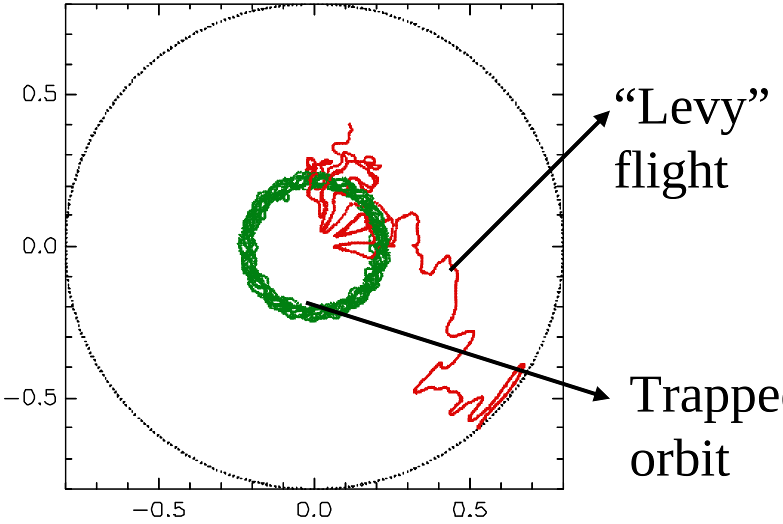
E*x*B flow velocity eddies induce large tracer trapping that leads to temporal non-locality, or memory effects



"Avalanche like" phenomena induce large tracer displacements that lead to spatial non-locality



Tracer orbits



The combination of tracer trapping and flights leads to non-diffusive transport

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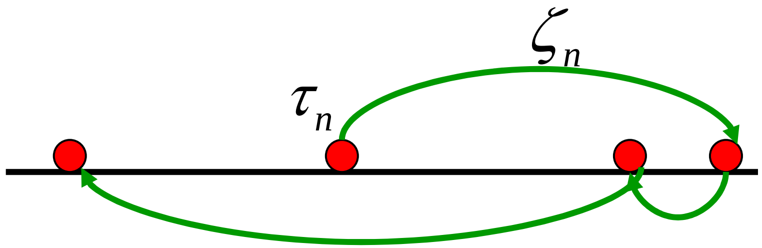
Continuous time random walk model



τ_n = waiting time $\psi(\tau)$ = waiting time pdf

ζ_n = jump $\lambda(\zeta)$ = jump size pdf

Master Equation

(Montroll-Weiss 1965)

$$P(x,t) = \delta(x)\int_t^{\infty}\psi(t')dt' + \int_0^t \psi(t-t')\left[\int_{-\infty}^{\infty}\lambda(x-x')\,P(x',t')\,dx'\right]dt'$$

Contribution from particles that have not moved during *(0,t)*

Contribution from particles located at *x'* and jumping to *x* during *(0,t)*

No memory $\psi(\tau) \sim e^{-\mu\tau}$

No long displacements $\lambda(\zeta) \sim e^{-\zeta^2/2\sigma}$

$$\partial_t P = \partial_x\left[\chi\,\partial_x P\right] + S$$

Standard diffusion model

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Proposed transport model

Long waiting times $\psi(\tau) \sim \tau^{-(\beta+1)}$

Long displacements (Levy flights) $\lambda(\zeta) \sim \zeta^{-(\alpha+1)}$

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}\left[w^- \Gamma_\ell + w^+ \Gamma_r\right]$$

Non-local effects due to avalanches causing Levy flights modeled with integral operators in space.

$$\Gamma_\ell = -\chi \frac{\partial}{\partial t}\int_0^t d\tau \int_a^x dy\, K(x-y;t-\tau)\, P(y,\tau)$$

$$K(x-y;t-\tau) = \frac{1}{(t-\tau)^{1-\beta}(x-y)^{\alpha}}$$

Non-Markovian, memory effects due to trapping in eddies modeled with integral operators in time.

Equivalent formulation using fractional derivatives

$$\frac{\partial^\beta P}{\partial t^\beta} = \chi \frac{\partial^\alpha P}{\partial x^\alpha}$$

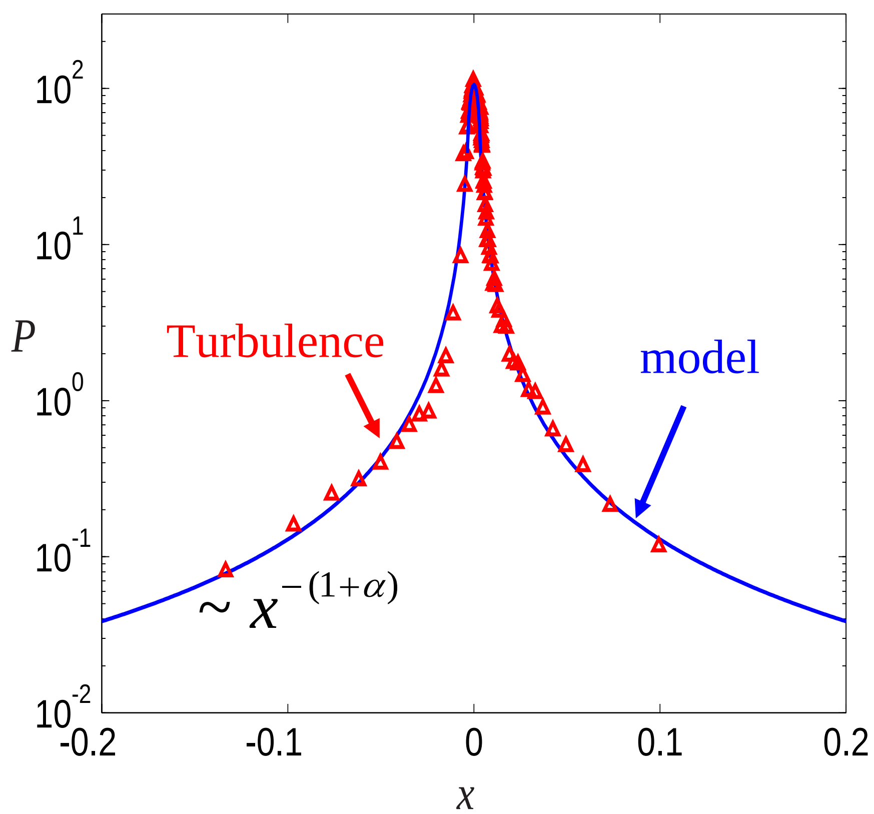
For pressure driven plasma turbulence

$$\alpha = 3/4 \qquad \beta = 1/2$$

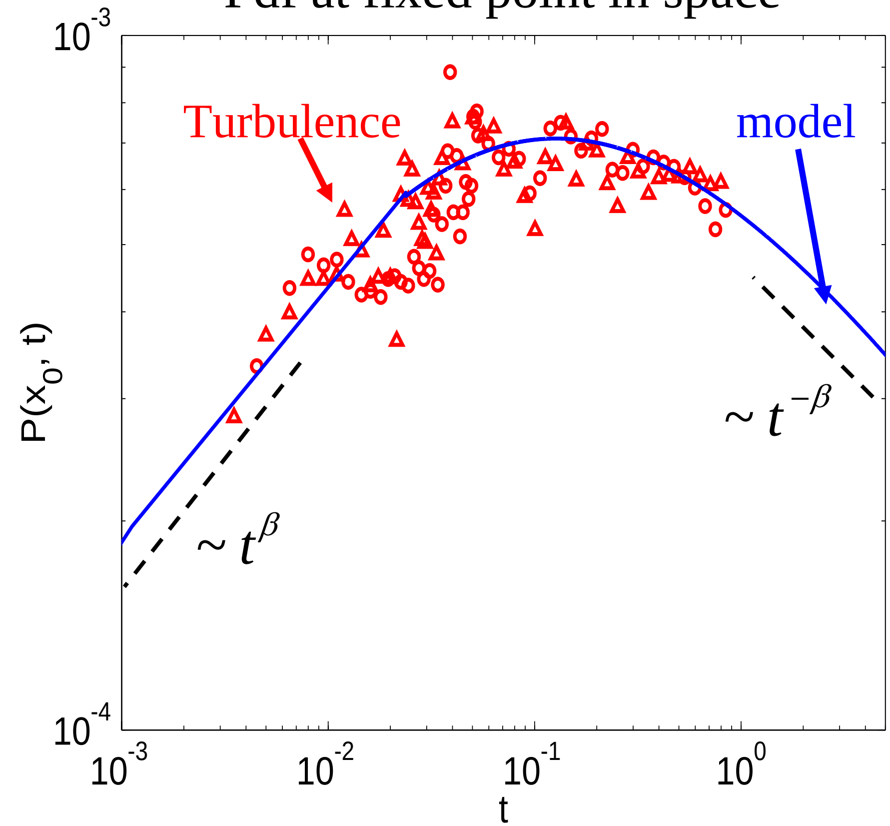
Test of fractional transport model

$$\frac{\partial^\beta P}{\partial t^\beta} = \chi \frac{\partial^\alpha P}{\partial x^\alpha}$$

Levy distribution at fixed time



Pdf at fixed point in space



Algebraic decay in space due to "Levy flights" implies that there is no characteristic transport scale

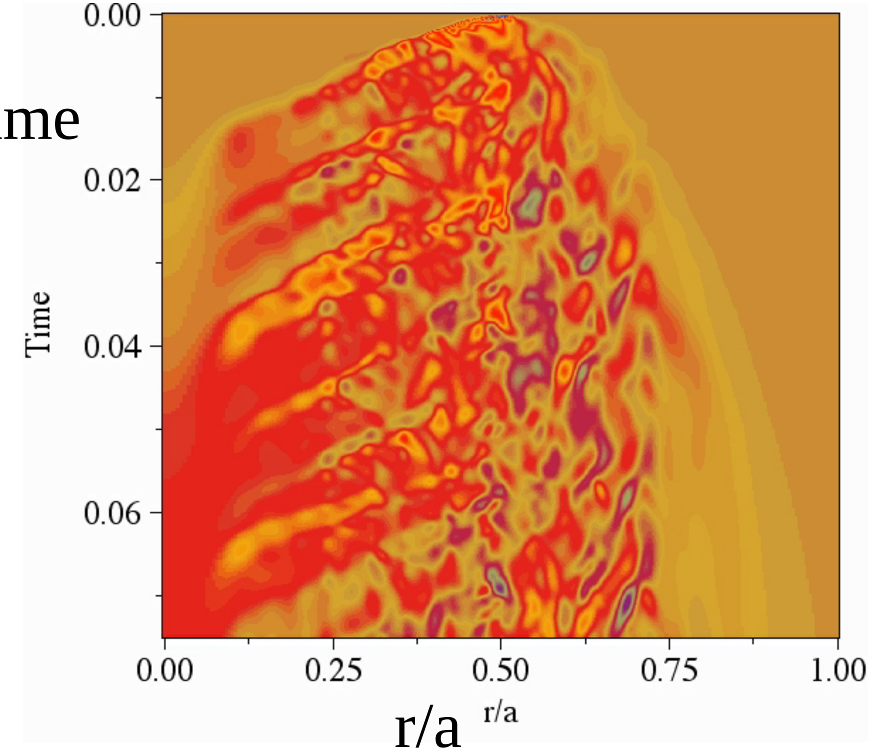
Algebraic scaling in time caused by memory effects

Super-diffusive scaling $\langle \delta r^2 \rangle \sim t^{2\nu}$

Diffusive scaling	Tracers	Model
$\nu = 1/2$	$\nu = 0.63$	$\nu = \beta/\alpha = 2/3 = 0.666$

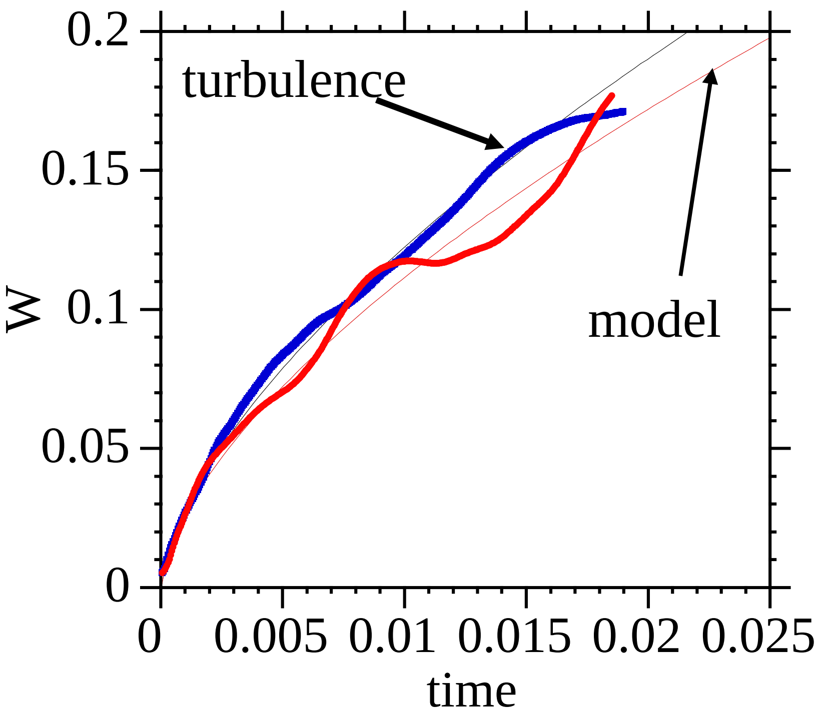
Pressure fluctuations

Time



(8) r/a

$W \sim \langle x^2 \rangle$



Conclusions

- We presented numerical evidence of non-diffusive transport i.e., super-diffusion and Levy distributions, in plasma turbulence.

- We proposed a transport model that incorporates in a unified way space non-locality, memory effects and anomalous diffusion scaling.

- There is quantitative agreement between the model and the turbulence calculations.

- The model represents a first attempt to construct effective transport operators when the complexity of the turbulence invalidates the use of Gausian closures.

$$\left[\partial_t + \tilde{V}\cdot\nabla\right] \Leftrightarrow \left[\partial_t^{\beta} - \chi\,\partial_x^{\alpha}\right]$$

Fractional derivatives

Further details: del-Castillo-Negrete, et al., Phys. of Plasmas, **11,** 3854, (2004).