THEORY AND THEORY-BASED MODELS FOR THE PEDESTAL, EDGE STABILITY AND ELMs IN TOKAMAKS

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MOTIVATION

• After L-H Transition, the plasma develops a steep pressure gradient

• Enhancement factor H in energy confinement for discharge in very strongly correlated with height of pressure pedestal

• What is the maximum sustainable pressure gradient ?

Paper addresses

• Equilibrium of H-mode (Guzdar, Mahajan, Yoshida)
• Stability of pedestal to non-curvature driven modes (Dorland, Rogers)
• Integrated modeling of pedestal and ELMs (Bateman, Kritz, Pankin, Voitsekhovitch, Onjun, Snyder, McElhenny, MacDonald)

THEORY FOR H-MODE EQUILIBRIUM TWO-FLUID HALL-MHD EQUATIONS

S. M. Mahajan and Z. Yoshida, PoP 7, 635, 2000

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \quad \text{Continuity}$$

$$\frac{\partial \vec{A}}{\partial t} = \left(\vec{V} - \underbrace{\frac{\nabla \times \vec{B}}{n}}_{\uparrow\ \text{Hall term}} \right) \times \vec{B} - \nabla\phi - \frac{\nabla p_e}{n} \quad \text{Ohm's Law } (m_e = 0)$$

$$\frac{\partial}{\partial t}\left(\vec{V} + \vec{A}\right) = \vec{V} \times \left(\vec{B} + \nabla \times \vec{V}\right) - \nabla\left(\frac{V^2}{2} + \phi\right) + \frac{\nabla p_i}{n} \quad \text{Ion Momentum}$$

$$p / n^\gamma = 1 \quad \text{Equation of State}$$

Normalizations

$B \to B/B_0$, $n \to n/n_0$, $x \to x/\lambda_i$, $t \to tV_A/\lambda_i$

$V \to V/V_{A,}$ $A \to A/\lambda_i B_0$, $\phi \to \phi c/V_A \lambda_i B_0$

EQUILIBRIUM-2

$$\vec{B} = \vec{B}_0 + \vec{B}_s$$

$$\frac{B_s}{B_0} \sim \frac{V}{V_A} \sim \beta << 1$$

$$\nabla \times (\vec{V} x \vec{B}_0) = 0 \Rightarrow \vec{V} x \vec{B}_0 = \nabla \Pi_i$$

$$\nabla \times (\vec{V}_e x \vec{B}_0) = \nabla \times \left[\left(\vec{V} - \frac{\nabla \times \vec{B}_s}{n} \right) x \vec{B}_0 \right] = 0 \Rightarrow \left(\vec{V} - \frac{\nabla \times \vec{B}_s}{n} \right) = -\nabla \Pi_e$$

$\Pi_{e/i}$ potential functions

$$\frac{\partial \vec{A}_s}{\partial t} = \left(\vec{V} - \frac{\nabla \times \vec{B}_s}{n} \right) \times \vec{B}_s - \nabla(\phi + \Pi_e) - \frac{\nabla p_e}{n}$$

$$\frac{\partial}{\partial t}\left(\vec{V} + \vec{A}_s\right) = \vec{V} \times \left(\vec{B}_s + \nabla \times \vec{V}\right) - \nabla \left(\frac{V^2}{2} + \phi - \Pi_i \right) + \frac{\nabla p_i}{n}$$

EQUILIBRIUM-3

$$\frac{\partial \vec{\Omega}_j}{\partial t} - \nabla \times \left(\vec{U}_j \times \vec{\Omega}_j \right) = 0$$

$j = 1$, Ohm's Law (electrons)

$j = 2$, Ion momentum

Generalized vorticities

$$\vec{\Omega}_1 = \vec{B}_s, \quad \vec{\Omega}_2 = \vec{B}_s + \nabla \times \vec{V}$$

Generalized flow

$$\vec{U}_1 = \vec{V} - \frac{\nabla \times \vec{B}_s}{n} \qquad \vec{U}_2 = \vec{V}$$

Double Beltrami conditions

$$\Omega_j = \mu_j U_j$$

Such a condition for fluid vorticity was derived by Beltrami. In plasma physics force-free magnetic field states are the Woltjer-Chandrashekar states(1956-1958) or Taylor states (1974)

EQUILIBRIUM-4

Double Beltrami conditions

$$\vec{B}_s = \mu_1(n\vec{V} - \nabla\times\vec{B}_s)$$

$$\vec{B}_s + \nabla\times\vec{V} = \mu_2 n\vec{V}$$

Generalized Bernoulli condition

$$p + \vec{B}_s\cdot\vec{B}_0 = C$$

$$p = p_e + p_i$$

Equation of state

$$p/n^\gamma = 1$$

1 D radial (x) equations

y $\rightarrow$ poloidal, z $\rightarrow$ toroidal

$$\frac{dB_{s,y}}{dx} - nV_z + \frac{1}{\mu_1}B_{s,z} = 0$$

$$\frac{dB_{s,z}}{dx} + nV_y - \frac{1}{\mu_1}B_{s,y} = 0$$

$$\frac{dV_y}{dx} + B_{s,z} - \mu_2 nV_z = 0$$

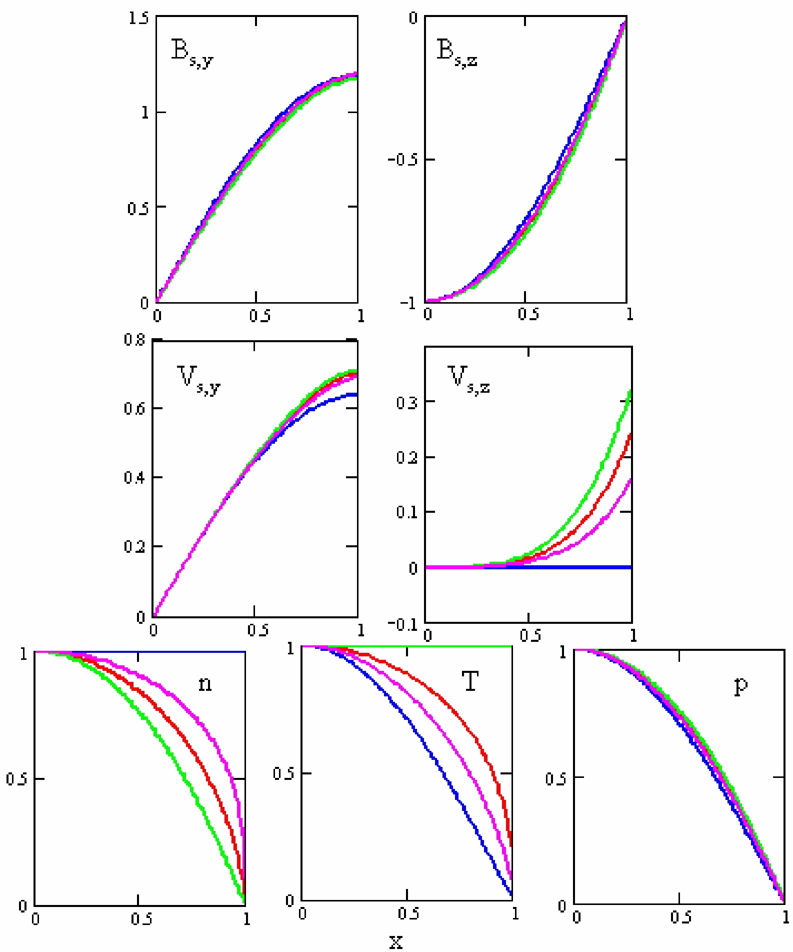
$$\frac{dV_z}{dx} - B_{s,y} + \mu_2 nV_y = 0$$

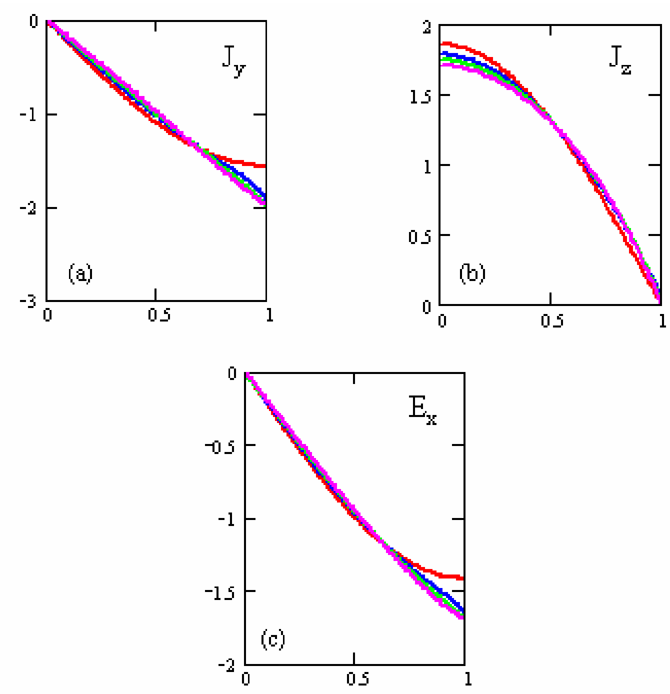
$$n^{1/\gamma} + B_{s,z} = 0$$

Complete system of steady state equations

EQUILIBRIUM-5

Edge profiles for the magnetic fields, B_y, B_z, flows, v_y, v_z, density, temperature Pressure, plasma currents J_y, J_z and radial electric field E_x





γ

1

5/3

3

n=const

EQUILIBRIUM-6

For constant density case

$$\nabla \mathrm{x}\left(\nabla \mathrm{x}\vec{B}_s\right) + c_1 \nabla \mathrm{x}\vec{B}_s + c_2 \vec{B}_s = 0$$

$$c_1 = (1/\mu_1 - \mu_2), \quad c_2 = 1 - \mu_2/\mu_1$$

$$(\mathrm{curl} - \Lambda_+)(\mathrm{curl} - \Lambda_-)\vec{B}_s = 0$$

$$\Lambda_\pm = \frac{1}{2}\left\{\frac{1}{\mu_1} - \mu_2 \mp \left[\left(\frac{1}{\mu_1} + \mu_2\right)^2 - 4\right]^{1/2}\right\}$$

$$B_{s,z} = -B_* \mathrm{Cos}\left(\frac{\pi x}{2\lambda_i}\right) \qquad B_{s,y} = B_*\left(1 + \frac{4}{\pi^2}\right)\mathrm{Sin}\left(\frac{\pi x}{2\lambda_i}\right)$$

$$V_y = \frac{2B_*}{\pi}\mathrm{Sin}\left(\frac{\pi x}{2\lambda_i}\right) \qquad p + \frac{B_{s,z}B_0}{4\pi} = 0$$

EQUILIBRIUM-7

$$\frac{8\pi q^2 R}{B_0^2}\frac{dp}{dx} = \alpha_c$$

$$\Delta_{ped} = \lambda_i$$

$$\beta_{ped} = \frac{2\alpha_c}{\pi}\frac{\lambda_i}{q^2 R}$$

$$\lambda_i = \frac{\sqrt{2}\rho_{pi}}{\sqrt{\beta_{pi}}}$$

$$\beta_{ped} = \frac{2}{q^2}\left(1+\frac{T_e}{T_i}\right)^{1/3}\left(\frac{\alpha_c}{\pi}\right)^{2/3}\left(\frac{\rho_{pi}\varepsilon}{R}\right)^{2/3}$$

$$\frac{\Delta_{ped}}{R} = \left(\frac{\pi}{\alpha_c}\right)^{1/3}\left(1+\frac{T_e}{T_i}\right)^{1/3}\left(\frac{\rho_{pi}\varepsilon}{R}\right)^{2/3}$$

$$V_y = \sqrt{\frac{\beta_{ped}}{2}}c_s \sim V_z$$

$$\frac{cE_x}{B_0} = -\frac{\pi}{4}\sqrt{\frac{\beta_{ped}}{2}}c_s\left(1+\frac{4}{\pi^2}\right)\mathrm{Sin}\left(\frac{\pi x}{2\lambda_i}\right)$$

$$\Delta_{ped}(m) = \frac{0.023}{Z}\sqrt{\frac{A_H}{n_{ped20}}}$$

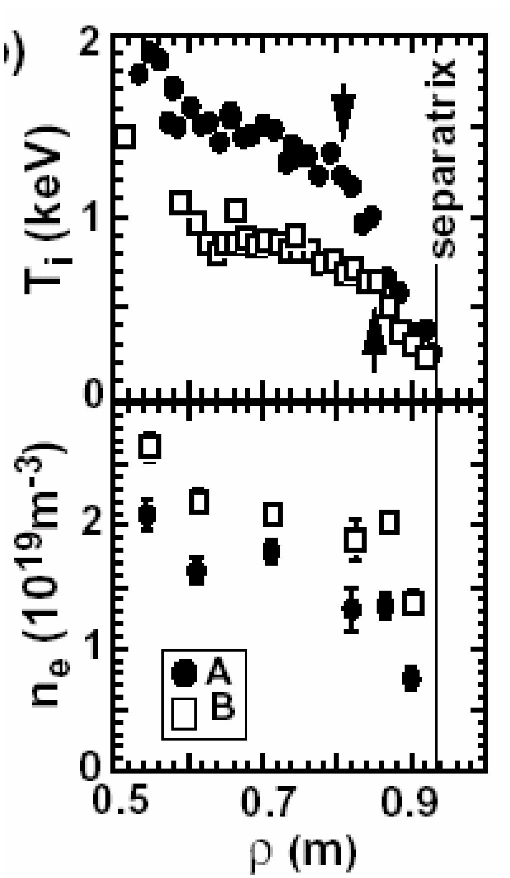
$$T_{e,ped}(keV) + T_{i,ped}(keV) = 0.36\frac{\alpha_c A_H^{1/2} B_T^2(T)}{Zq^2 R(m) n_{ped20}^{3/2}}$$

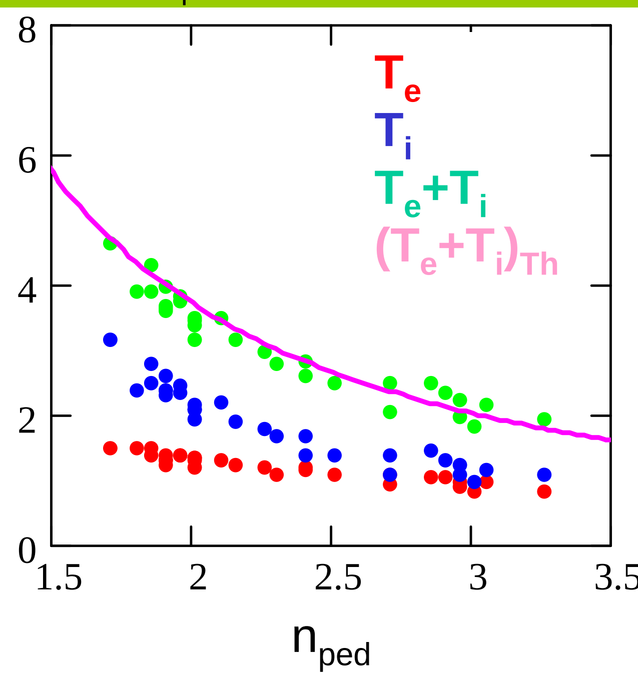
EQUILIBRIUM-8
COMPARISON WITH DATA FROM JT-60U

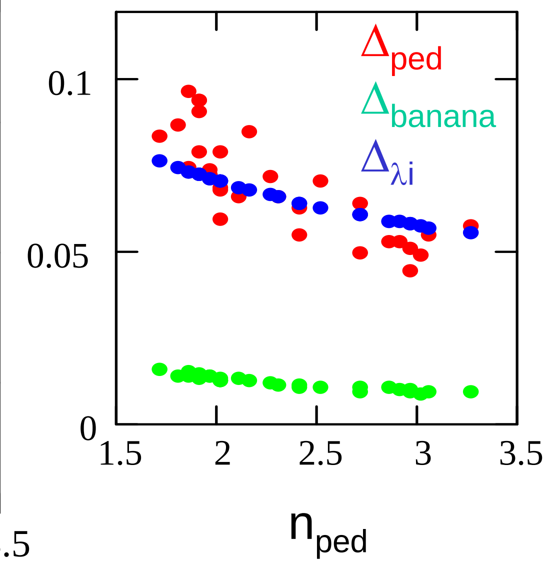
Kamada et al., PPCF 41, 1371 (1999)

H. Urano et al., 27th EPS CCFPP, Budapest 12-16 June 2000, ECA Vol 24B 956-959 (2000)   Courtesy Urano and Kamada

JT-60U I_p=1.8 MA, B_T =3.0 T, κ=1.48-1.55,   δ=0.16-0.19







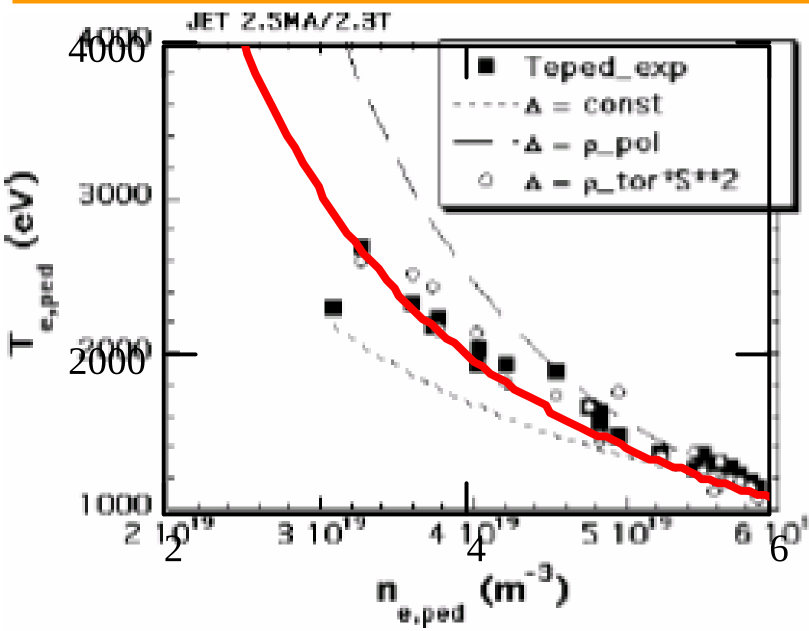
$(T_e^{ped}+T_i^{ped})=10.5/(n_{ped})^{1.5}$ $a_c/q^2=0.22$

$q=3.5$, $a_c=2.7$ (low elongation and triangularity)

EQUILIBRIUM-9
COMPARISON WITH DATA FROM JET

M Sugihara et al., 26th EPS CCFPP, Maastricht 14-18 June 1999,
ECA Vol 23J 1449-1452 (1999)
Data from JET, Lingertat et al., J. Nucl. Mat, 266-269, 124 (1999)

$T_i = T_e$ (assumed) discharges have with high elongation and triangularity



$$T_{e_{ped}}(\mathrm{keV}) = \frac{15.8}{n_{e_{ped}}^{3/2}(10^{19}\,\mathrm{m}^{-3})}$$

$$\frac{\alpha_c}{q^2} = 0.9$$

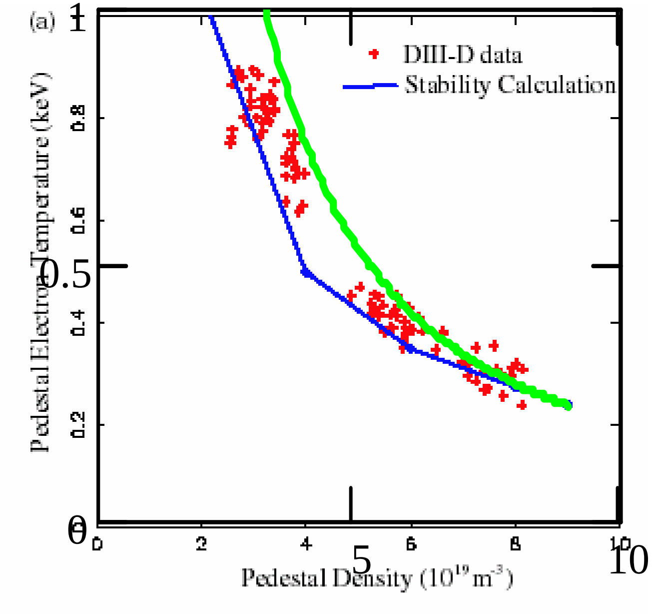
For $\alpha_c = 7$

$q = 2.5$

EQUILIBRIUM-10
COMPARISON WITH DATA FROM DIII-D

Snyder et al., Nuc. Fusion 44, 320 (2004)

DIII-D, B_T=2 T, I_p=1.225 MA, R=1.685, a=0.603, κ=1.77, δ=0.0



$$T_{e_{ped}}(\text{keV}) = \frac{6.0}{n_{e_{ped}}^{3/2}(10^{19}\,\text{m}^{-3})}$$

$$\frac{\alpha_c}{q^2} = 0.31$$

$$q = 3,\ \alpha_c = 2.8$$

STABILITY TO NON-CURVATURE DRIVEN MODES IN THE PEDESTAL

Main Focus:

• What are the dominant linear instabilities in the H-mode edge pedestal ?

Main Results:

• Strong E_r shear of H-mode pedestal is not always stabilizing:

Even without magnetic curvature, there are at least three linear instabilities that are potentially *Destabilizing* by *ExB* shear and plasma gradients

• These modes all *require* finite curvature in V_E and/or plasma gradients to be unstable

• Potentially stronger than curvature driven modes since $L_p \ll R$

• Not present in simulations with spatially constant plasma gradients and/or spatially constant *ExB* shear

PEDESTAL STABILITY-2

Methods of Analysis

•Gyrokinetic GS2 simulations and analytic calculations of a simple slab model for H-mode that includes:

-- spatially varying *ExB* shear and plasma gradients with typical magnitudes and scales for pedestal $V_E \sim V_{*e} \sim -V_{*I}$ and pedestal widths $\Delta \sim (10\text{-}20)\rho_i$

-- Magnetic shear

-- Electromagnetic and kinetic effects (eg Landau damping , FLR)

But excludes (for now)

-- Magnetic curvature

-- Poloidal variation and othe rtoroidal geometry effects

--Parallel flows

PEDESTAL STABILITY-3

Summary: Three Main Modes-I

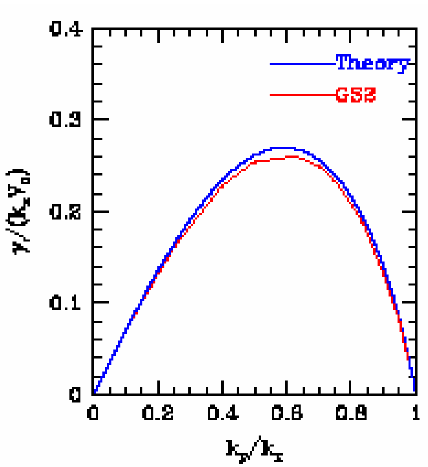
1. Kelvin-Helmholtz Instability

-- Driven by shear in *ExB* velocity V_E
-- Magnetic shear and V_{*i} are stabilizing

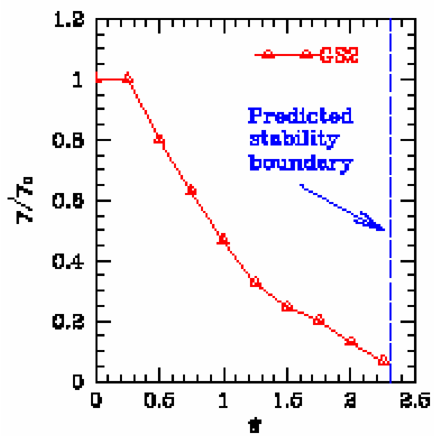
Very near marginal stability for typical H-mode parameters. More detailed analysis that includes toroidal effects is currently underway

$k_y\rho_i=0.2$, $T_i=T_e$, $s=0$, $\beta_e=10^{-4}$

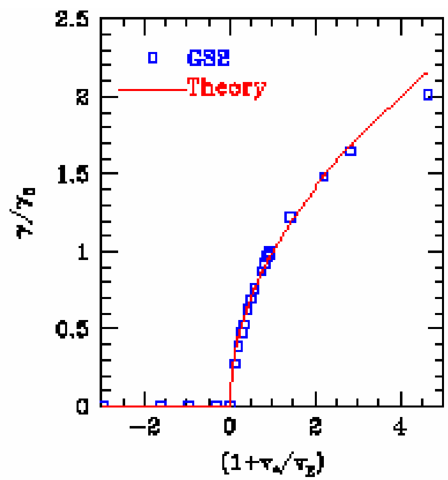
$k_y\rho_i=0.2$, $T_i=T_e$, $q=3.5$, $\phi_0=1.5\times10^{-3}$, γ_0 growth rate for s=0



KH mode γ vs k_y/k_x



KH mode γ vs s



KH mode γ vs V_{*i}

IAEA Meeting, *November 2004*

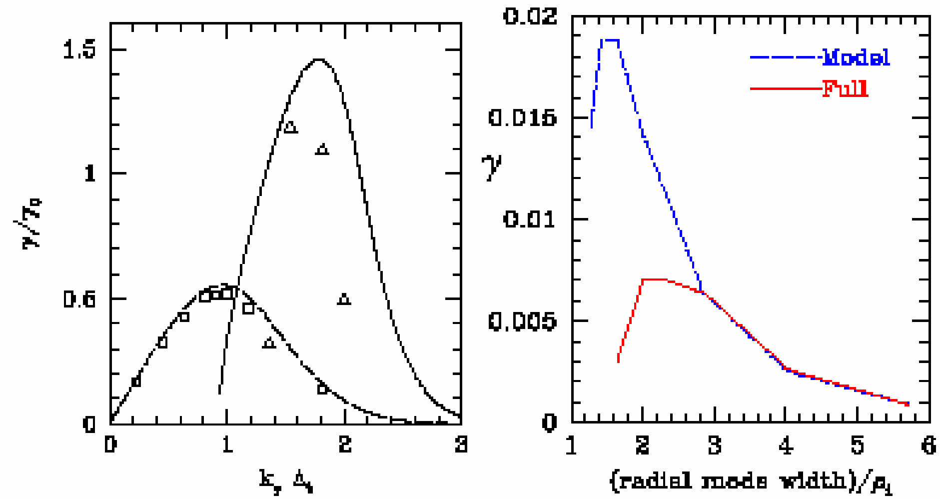
PEDESTAL STABILITY-4

Summary: Three Main Modes-II

2. “Tertiary” Mode

-- An adiabatic, electrostatic mode with high $k_{||}$ and $k_{\perp} > 1/\Delta$
-- Driven by $dT_i dx$
-- Insensitive to magnetic shear
-- FLR effects stabilizing

Also near marginal stability for typical H-mode pedestals due to FLR effects. Further work on to include toroidal effects in progress



$k_y\rho_i$=0.2, T_i=T_e
γ_0= $\Delta_t V_E''/2$

Tertiary Mode γ vs (a) k_y for $\eta_i = \infty$ (squares), η_i=3.8 (triangles) and (b) $\Delta_t \sim (L\rho_s)^{1/2}$

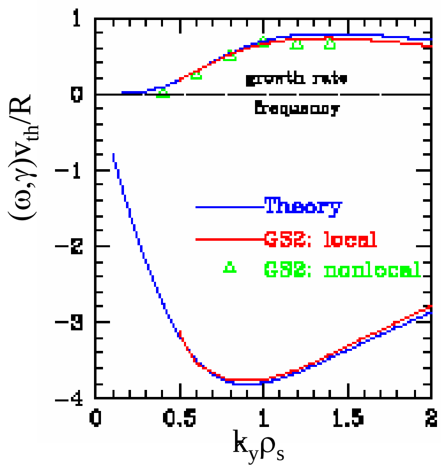
PEDESTAL STABILITY-5

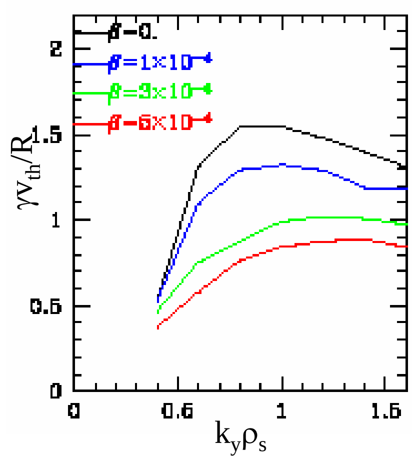
Summary: Three Main Modes-III

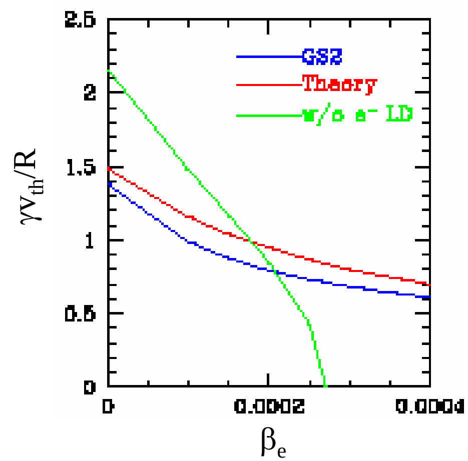
3. Drift-wave

Aside from the *ExB* shear, similar to the mode discussed in literature decades ago
-- Driven by plasma gradients and (for typical H-mode parameters) electron Landau damping
-- $k_\perp \rho_s \sim (0.2 - 5)$ and $k_{||} \sim \sqrt{\beta} k \rho_s / \Delta$
-- *Requires* spatial variations in *ExB* shear or plasma gradients (like those typical of the pedestal) to be linearly unstable in the presence of magnetic shear
-- Electromagnetic effects are stabilizing

Robustly unstable for for all parameters relevant to edge pedestals. Strong candidate for driving transport in the H-mode edge for toroidal and linear devices







$k_x\rho_i$=0.2, T_i=T_e

β_e=4x10^{-4}, V_E= 222.4

V_{*in}=-399.6, V_{*iT}=-16.5

Pedestal and ELM Models Used in Integrated Modeling

- **Objective:**
  - Develop model for integrated simulations, which includes pedestal and core
    - Height, width and shape of the pedestal
    - Frequency and effect of ELMs
    - Effect of pedestal and ELMs on the core plasma profiles
- **Progression of increasingly sophisticated models that have been developed and/or used by Lehigh group:**
  - Pedestal height model developed and used to provide boundary conditions in simulations of evolution of temperature and density profiles in H-modes
    - Model calibrated against pedestal data
    - Resulting density and temperature profiles compared with experimental profiles
  - Pedestal and ELM model used to investigate physics of the ELM cycle
    - Results compared with JET data
  - Pedestal and ELM model developed to investigate pedestal formation and evolution of profiles during the ELM cycle
    - Model calibrated with DIII-D data and results compared with DIII-D and JET data

Static Pedestal Model Used to Provide Boundary Conditions

- Separate models applied for the core and the H-mode pedestal
  - PEDESTAL model used is available as NTCC Module Library Module http://w3.pppl.gov/NTCC
    - Models described in papers by T. Onjun *et al.* Phys. Plasmas 9 (2002) 5018 and G. Cordey *et al.,* 2002 IAEA, Nucl. Fusion 43 (2003) 670
  - Multi-Mode or GLF23 model used for core transport
    - These simulations do not have time-dependent model for ELM cycle
- Simulations with pedestal model boundary conditions have been carried out and compared with data
  - Profiles agree with data as well as when experimental data boundary conditions are used
    - G. Bateman *et al.,* Phys. Plasmas 10 (2003) 4358
- Simulations of burning plasma experiments
  - Predicted performance of burning plasma experiment depends on the choice of core and pedestal models
    - G. Bateman, *et al.,* Plasma Phys. Control. Fusion 45 (2003) 1939.
    - J. Kinsey, *et al.,* Nucl. Fusion 43 (2003) 1845.

Model in which MHD Criteria are Used to Initiate ELM crashes

- **Implemented by V. Parail *et al.* in the JETTO code and used by the Lehigh group and researchers at JET**
  - **V. Parail *et al.*, Plasma Physics Reports 29 (2003) 539**
  - **Two criteria to trigger ELMs: Pressure-driven ballooning mode and current-driven peeling mode**
    - **ELM triggered by ballooning mode if normalized pressure gradient, α, exceeds the critical pressure gradient, α_c, anywhere in pedestal**

$$\alpha \equiv -\frac{2\mu_0 q^2}{\varepsilon B_T^2}\left(\frac{dp}{d\rho}\right) > \alpha_c(s,\kappa,\delta)$$

    - **ELM triggered by peeling mode if the current density, $J_{||}$, satisfies**

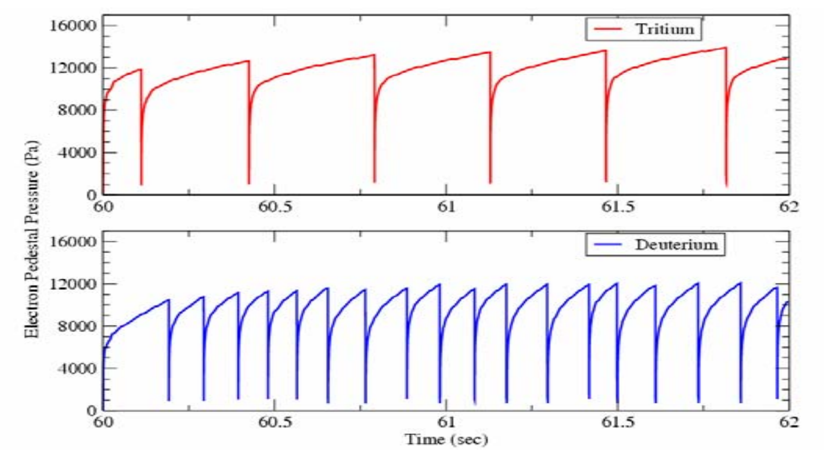
$$\sqrt{1-4D_m} + C < 1 + \frac{1}{\pi q'} \oint \frac{\mu_0 J_{\|} B}{R^2 B_p^3} dl$$

Details in article by H. R. Wilson *et al*., Nucl. Fusion, 40 (2000) 713

  - **Criteria calibrated using HELENA and MISHKA MHD stability codes**
    - **Trigger criteria affect frequency of ELMs**

Results of Dynamic Pedestal and ELM Model

- Neoclassical transport used in the pedestal
  - Determines height of temperature and density pedestal
- Transition from 1st to 2nd stability region observed in triangularity scan
  - T. Onjun *et al.,* Phys. Plasmas 11 (2004) 3006
- Pedestal height increases with heating power
  - T. Onjun *et al.,* Phys. Plasmas 11 (2004) 1469
- Transition from ballooning to peeling mode ELM crash trigger observed in simulations of some discharges
  - T. Onjun *et al.,* to appear in Phys. Plasmas (2005)
- Effect of isotope mass on H-mode pedestal and ELMs
  - In collaboration with V. Parail and JET Team



	JET Discharge	Deuterium 43154	Tritium 43003
Pedestal Pressure (kPa)	Simulation	12	14
	Experiment	10	14
ELM Frequency (Hz)	Simulation	9	3
	Experiment	6 - 18	3 - 5

Pedestal and ELM Model Developed at Lehigh

- Single transport model used in plasma core and pedestal
  - Local self-consistent calculation of plasma profiles from magnetic axis to the base of the pedestal
  - Currently implemented in the ASTRA code
- Model predicts the height, width, and shape of the H-mode pedestal and frequency of the ELMs
  - Started with model developed by G. Pacher, H. Pacher, G. Janeschitz, *et al.,* Nucl. Fusion 43 (2003) 188
  - Initial development and application of model by A. Pankin, I. Voitsekhovitch *et al.,* submitted to Plasma Physics and Controlled Fusion (2005)
- Transport model consists of a combination of
  - ITG/TEM ion modes (Weiland) and resistive ballooning (Guzdar/Drake)
  - ETG driven by short wavelength electron drift modes (Horton/Jenko)
  - Neoclassical transport (NCLASS)
- Flow shear stabilization of local anomalous transport
  - Modes with different wavelengths stabilized at different rates
- ELM crashes triggered by pressure-driven ballooning or current-driven peeling mode analytic instability criteria

Transport Model Used for Core and Pedestal

- **Ion drift mode turbulence within the pedestal suppressed by *ExB* flow shear, but not the short wavelength ETG mode turbulence**

$$\chi^{i} = \frac{\chi^{i}_{\mathrm{ITG/TEM}}}{1+C_{ITG/TEM}(\omega_{E\times B}\tau_{ITG/TEM})^{2}} + \frac{\chi^{i}_{\mathrm{RB}}}{1+C_{RB}(\omega_{E\times B}\tau_{RB})^{2}} + \chi^{i}_{\mathrm{neoclassical}}$$

$$\chi^{e} = \frac{\chi^{e}_{\mathrm{ITG/TEM}}}{1+C_{ITG/TEM}(\omega_{E\times B}\tau_{ITG/TEM})^{2}} + \frac{\chi^{e}_{\mathrm{RB}}}{1+C_{RB}(\omega_{E\times B}\tau_{RB})^{2}} + \chi^{e}_{\mathrm{ETG}}$$

$\chi^{i,e}$	**ion or electron thermal diffusivities**
$\chi_{ITG/TEM}$	**ion drift mode (ITG/TEM) diffusivity without flow shear (Weiland)**
χ_{RB}	**resistive ballooning mode diffusivity without flow shear**
$\omega_{E\times B}$	**flow shear rate**
$\tau_{ITG/TEM}$	**turbulence correlation time for ion drift mode**
τ_{RB}	**turbulence correlation time for resistive ballooning mode**
$C_{ITG/TEM,RB}$	**calibration coefficients**
$\chi^{i}_{neoclassical}$	**neoclassical ion thermal diffusivity (NCLASS)**
χ^{e}_{ETG}	**ETG mode electron thermal diffusivity**

Scaling of Pedestal Temperature and ELM Frequency

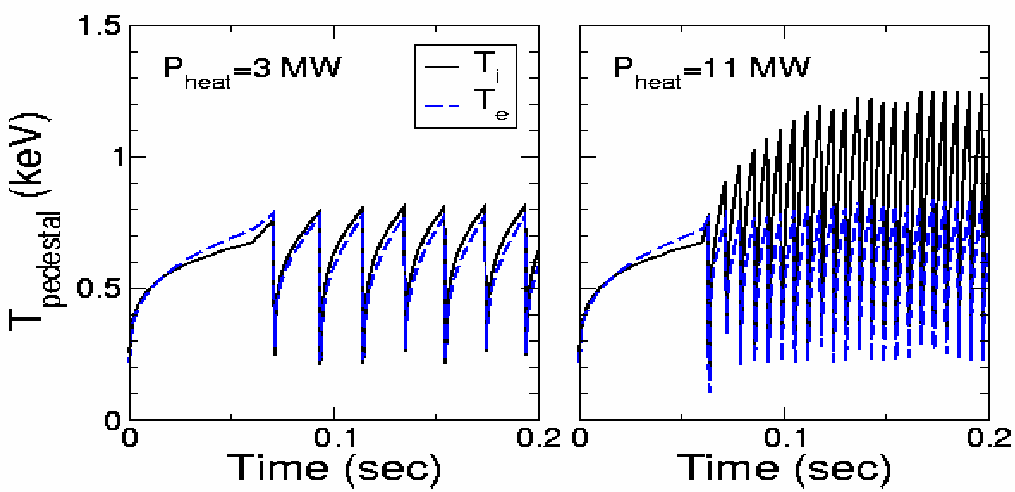
- Transport model calibrated using experimental data for DIII-D 98889 provided by Tom Osborne at GA
  - Noise reduced by overlaying data from consecutive ELM cycles
- Once flow shear large enough, the critical normalized pressure gradient for triggering ELM crashes (α_c) is most important calibration constant
- Simulations follow current profile diffusion, including bootstrap current
  - Prescribed particle density and power density profiles in these simulations
- Model in ASTRA code has been used to study scaling of pedestal temperature and ELM frequency
  - Many of the trends are consistent with experimental data
  - Pedestal temperature decreases with pedestal density:

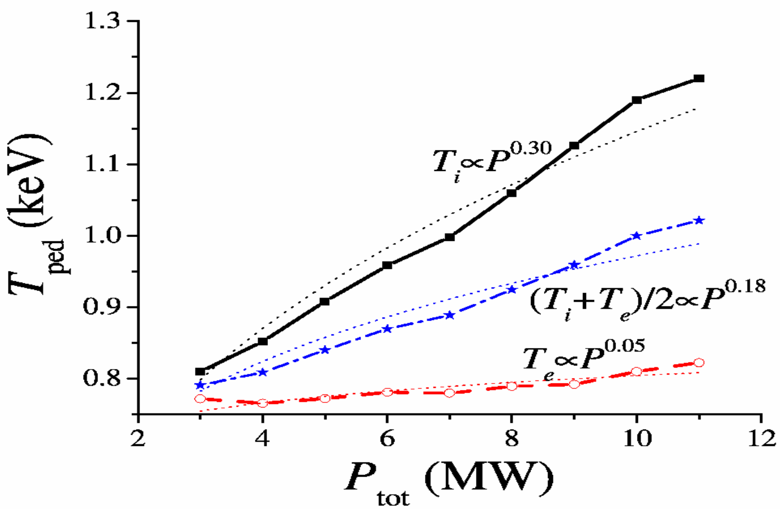
$$T_{ped} \sim 1/n^{\alpha}_{ped} \; ; \; \alpha > 1$$

  - Pedestal temperature increases slowly with increasing toroidal magnetic field (with fixed plasma current)
  - ELM frequency decreases with magnetic field

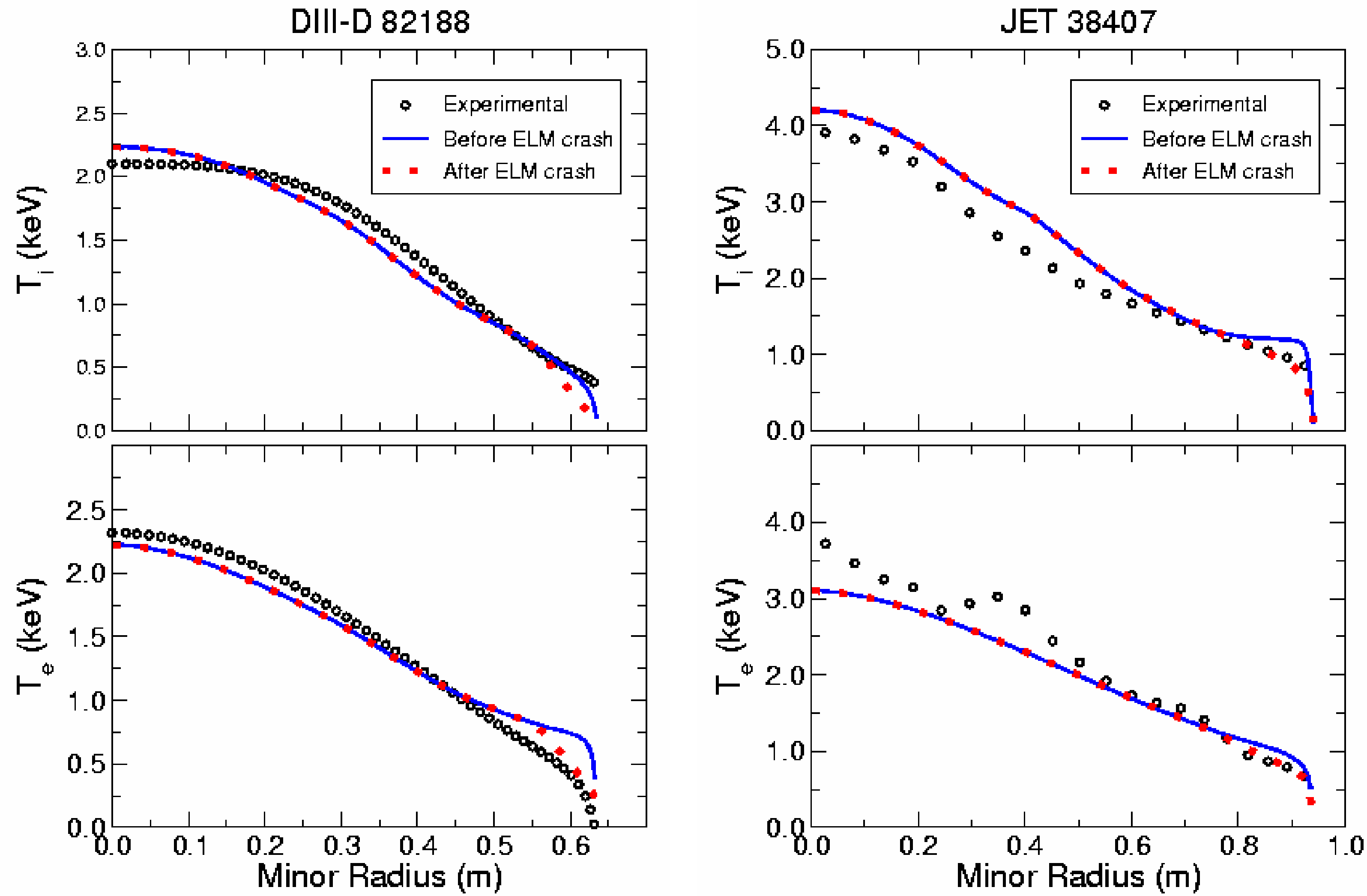
Effect of Heating Power on Pedestal and ELMs

- **Simulations carried out for scan of discharges with different auxiliary heating power**
- **ELM frequency and pedestal temperature increase with power**
  - **Consistent with data [J.G. Cordey *et al.*, Nucl. Fusion 43 (2003) 670]**
  - **Effect caused by changes in pedestal current density and resulting magnetic shear**
  - **Pedestal temperature increases with heating power because of time-dependent inductive effects as current density rebuilds in pedestal between ELM crashes**





Simulation of T_i and T_e Profiles



- **Lehigh model for pedestal and ELMs used in ASTRA simulations of DIII-D and JET discharges**

LEHIGH UNIVERSITY

CONCLUSIONS

Equilibrium

• Double-Beltrami H-mode equilibrium with constraint for pressure gradient determined by ideal ballooning mode stability yields unique prescription for pressure pedestal width and height.

• Comparison with experimental data shows promise

Stability

• Linear drift-wave eigenmodes robustly unstable in the H-mode edge region despite ExB and magnetic shear effects, hence potentially main driver of transport in the (H-mode) edge of toroidal or linear devices.

• Linear modes **not** stabilized by magnetic shear (as is widely believed) because eigenmode well-localized radially by ExB shear and profile variation.

• Result verified by GS2(gyrokinetic) simulations and analytic calculations and consistent with past work on the universal instability in the literature going back many decades.

Integrated Modeling

• The Lehigh University Fusion Research Group developed and tested theory-based edge models in the JETTO and ASTRA codes to predict the height of the H-mode pedestal as well as the frequency of ELM crashes.

• Simulations using these edge models, validated using experimental data from JET and DIII-D, reproduce experimentally observed scalings as a function of heating power, magnetic field strength, plasma density, triangularity, and isotope mass