The Stability of Internal Transport Barriers to MHD Ballooning Modes and Drift Waves: a Formalism for Low Magnetic Shear and for Velocity Shear

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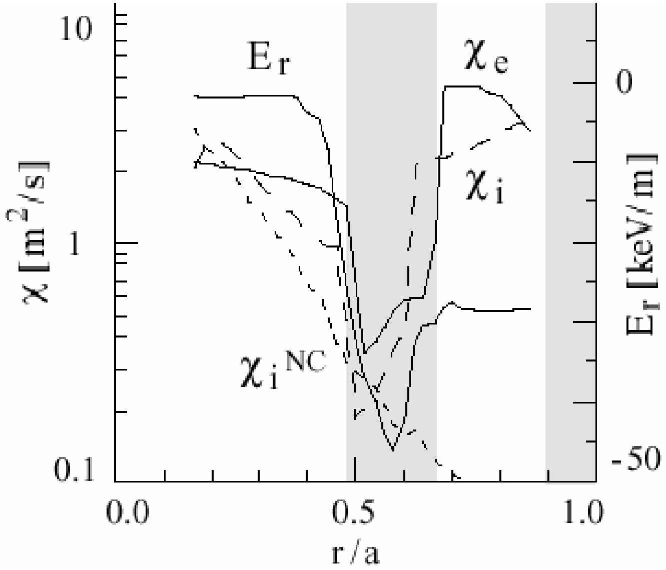
INTRODUCTION

- Transport Barriers improve confinement
  - – important to assess their stability
- Transport Barriers are often associated with low magnetic shear and sheared radial electric fields



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- The standard tool for stability studies - the ballooning theory - fails in these circumstances
- We present a complementary approach to overcome this
- Applications to high-n MHD ballooning modes and micro-instabilities

LIMITATIONS ON BALLOONING THEORY

Theory requires A/B << 1,

A: breaking of degeneracy between mode resonant surfaces (mrs), m = nq (r_m), due to profile variation

$$A = \frac{\Delta\omega}{\omega} \sim \frac{1}{\omega}\frac{\partial\omega}{\partial x}\Delta x \quad \sim \frac{1}{L}\cdot\frac{1}{nq'}$$

B: toroidal coupling between modes on adjacent surfaces

$$B = C^{T} \sim \frac{a}{R}\exp\left(-\frac{1}{|s|}\right)$$ (for MHD)

- At low magnetic shear, s, theory fails, unless n $\rightarrow \infty$ fast enough!

- Toroidal rotation shear $\Rightarrow$ Doppler-shift, $\omega \rightarrow \omega + n\Omega' x$

$$A = \frac{1}{\Omega}\cdot\frac{d\Omega}{dx}\cdot\frac{1}{q'} \equiv \frac{d\hat{\Omega}}{dq} \;\; (\text{ie } \hat{\Omega}_q)$$

$\Rightarrow$ Theory fails unless $\hat{\Omega}_q \leq 0(1/n)$

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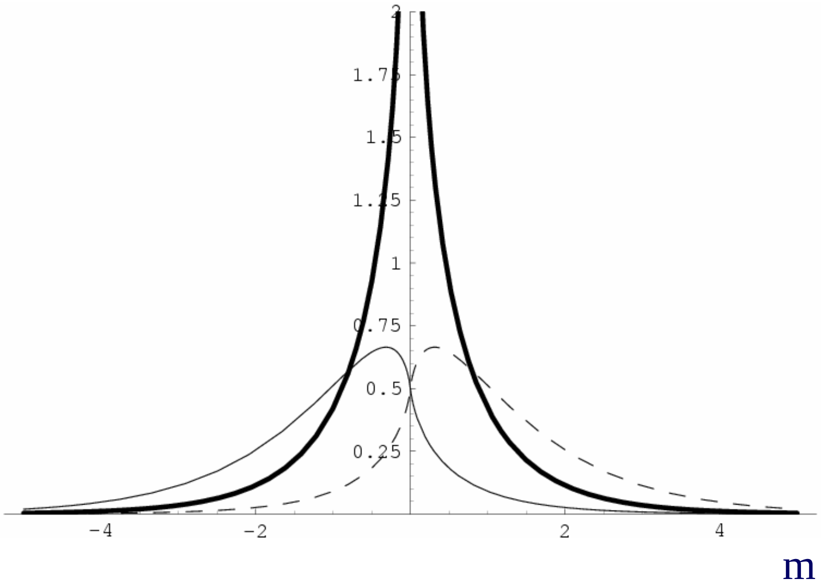
RECURRENCE RELATION AT LOW S

- Alternative approach: recurrence relation between amplitudes of 'modelets' located at each mrs
  - – not simple Fourier modes



- For low s, gives eqn for eigenvalue λ and spectrum, a_m

$$(\lambda - \lambda_m)a_m + C_m^T(a_{m+1} + a_{m-1})/2 = 0$$

- Can deduce coefficients from local ballooning space dispersion relation as a function of x and ballooning parameter k.

$$\lambda - \lambda(x) + C^T(x)\cos k = 0$$

  - – let $x \leftrightarrow m/nq' \equiv x_m$, $e^{ikm} \leftrightarrow a_m$

- Use analytic solutions of ballooning eqn at low s; in general

$\Rightarrow C^T \sim \exp(-1/|\varepsilon|)$, $\varepsilon \propto s^p$

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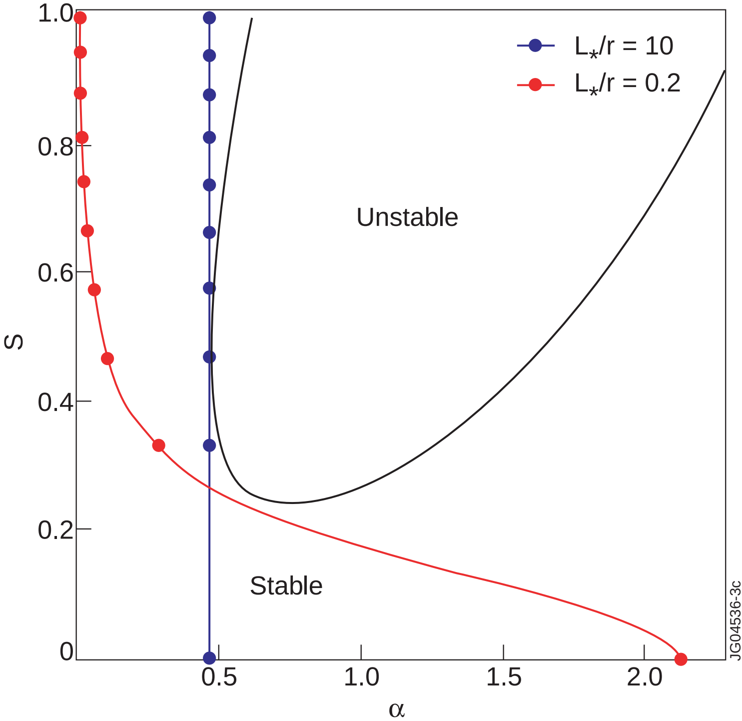
APPLICATION (1): HIGH-N MHD BALLOONING

- s - α equilibrium with favourable average curvature $\propto$ a/R (1 - 1/q^2)

$$[\gamma - \gamma_m] a_m + C(\alpha_m, s_m)[a_{m+1} + a_{m-1}]/2 = 0$$

$$\gamma_m = \gamma_0(\alpha_m, s_m); \qquad C \propto \alpha_m \exp(-1/|s_m|)/s_m$$

s-α diagram



- Near ITB at q_{min}:
  $\alpha = \alpha_{max}\, \mathrm{sech}^2 (x/L_*)$
  $q = q_{min} (1 + \mu\, x^2/2a^2)$

- Discrete mrs, x_m
  lie on trajectory in s - α space
  controlled by α_{max}, L_*, μ
  – avoid unstable region
  – or step across using discreteness;
  easier if q_{min} is low order rational

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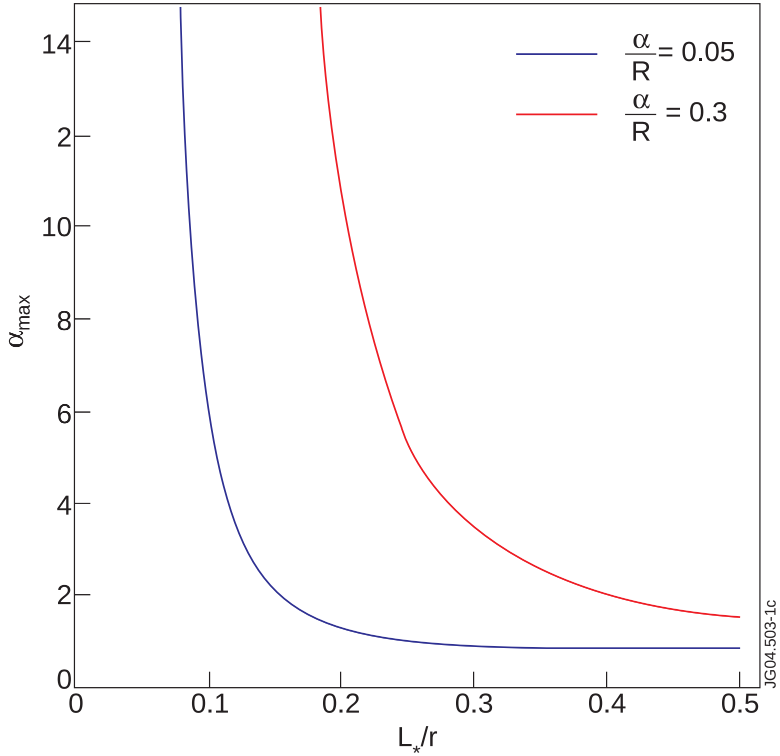
- Always unstable if $a/R \rightarrow 0$, but can map out stable operating diagram for (α_{max}, L_*/r) at finite a/R



Maximum stable α, α_{max}, as a function of the barrier width L_*, ($\mu = 1$)

- Can sustain high pressure, $\Delta p \propto \alpha_{max} L_*$, if barrier narrow

- Self-consistent bootstrap current J_{BS} (α, a/R)) modifies shear

  $\Rightarrow$ stable regimes even if $s \neq 0$ when barrier absent

APPLICATION (2): DRIFT WAVES - TOROIDAL ITG

- Long wavelength, $b_s = k_\perp^2 \rho_s^2 << 1$, flat density *(Romanelli & Zonca, Phys Fluids B**5** (1993) 4081)*

  $\Rightarrow$ parameter $b_T = b_s (R/L_T)^{1/2}$,

  – toroidal branch

  – find different poloidal mode structures and local eigenvalues for $s > 0$ and $s < 0$, eg either side of q_{min}

  – involve even, odd and mixed parity (in poloidal angle, θ) modes

- Coupling coefficient $C^T \sim \exp(-1/\varepsilon)$, $\varepsilon \sim b_T^{1/3} s^{1/2}$, for most strongly coupled mode (mixed parity)

  – weak coupling at low s or b_s,

  – but, nevertheless, requires very long wavelength to have negligible coupling near q_{min}! - say n ~ 6 for $\rho_* = 1/256$

- There is also a slab branch: $\varepsilon \sim b_T^{1/2} s^{3/4}$

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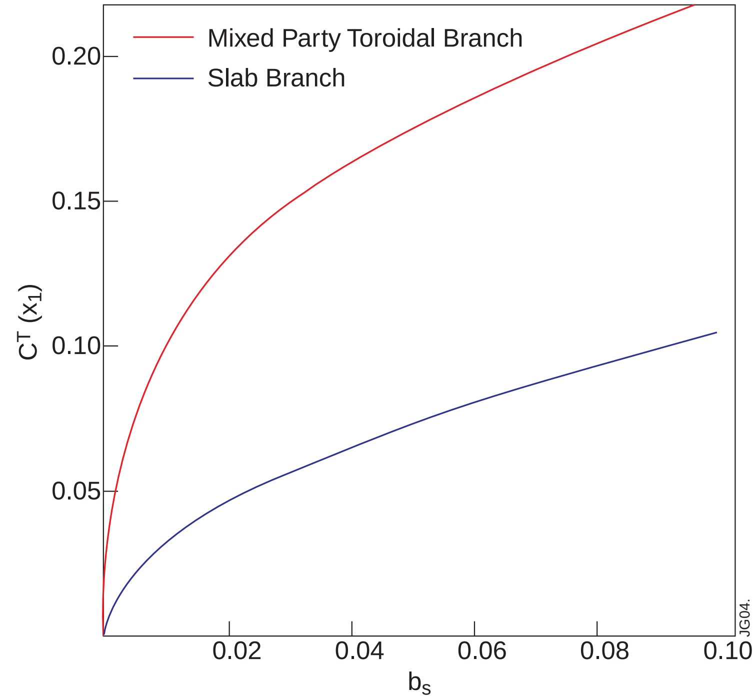
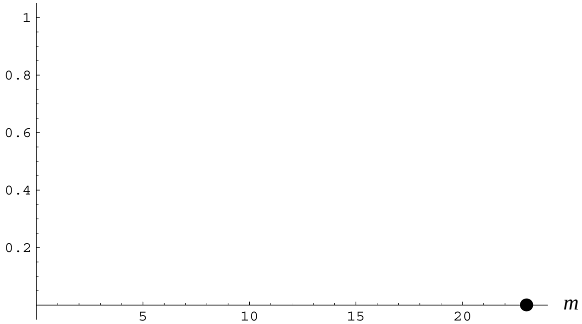
Toroidal coupling coefficient, $C^T(x_1)$ evaluated at x_1, the nearest resonant surface to q_{min}, for ITG modes as a function of $b_s = (k_\perp \rho_s)^2$



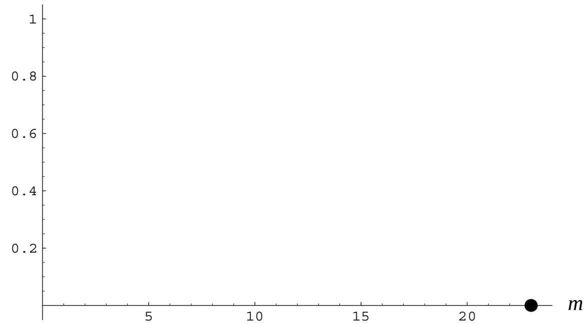
- Different eigenmodes for a_m spectrum

a_m



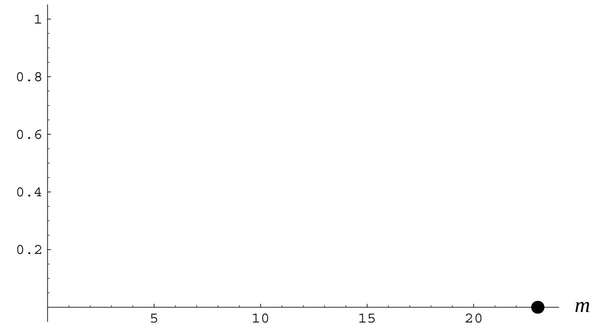
q_{min} $m\text{-}nq_{min}$

a_m



q_{min} $m\text{-}nq_{min}$

a_m



q_{min} $m\text{-}nq_{min}$

- Narrowing of spectrum of a_m as mode centre nears q_{min} (quadratic variation of $1/L_T$)

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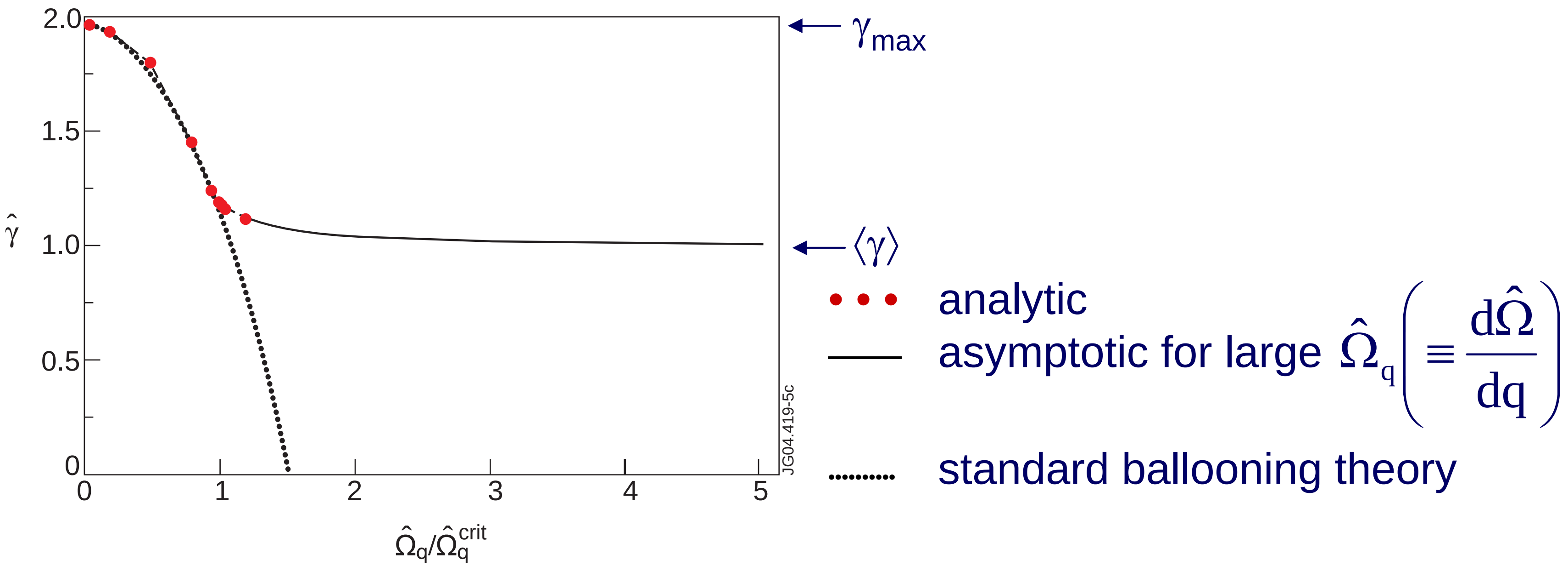
TOROIDAL ROTATION SHEAR: MHD BALLOONING

- Analytic solution of Doppler-shifted ($\gamma \rightarrow \gamma$ - inΩ'x) recurrence relation for small (α, s) and quadratic radial variation of α



- Continuous evolution from stationary plasma result: $\gamma = \gamma_{max}$ (k) to sheared rotation result: $\gamma = \langle\, \gamma$ (k)$\rangle$ *(Waelbroeck & Chen, Phys Fluids B* ***3*** *(1991) 601)*

- Transition at $\hat{\Omega}_q \sim \hat{\Omega}_q^{crit} \sim 0$ (1/n)
  - transition sharper as Λ^2 = nqsr/L_* increases
  - stationary result rapidly becomes invalid

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COMPLEMENTARY NUMERICAL 2D SOLUTION

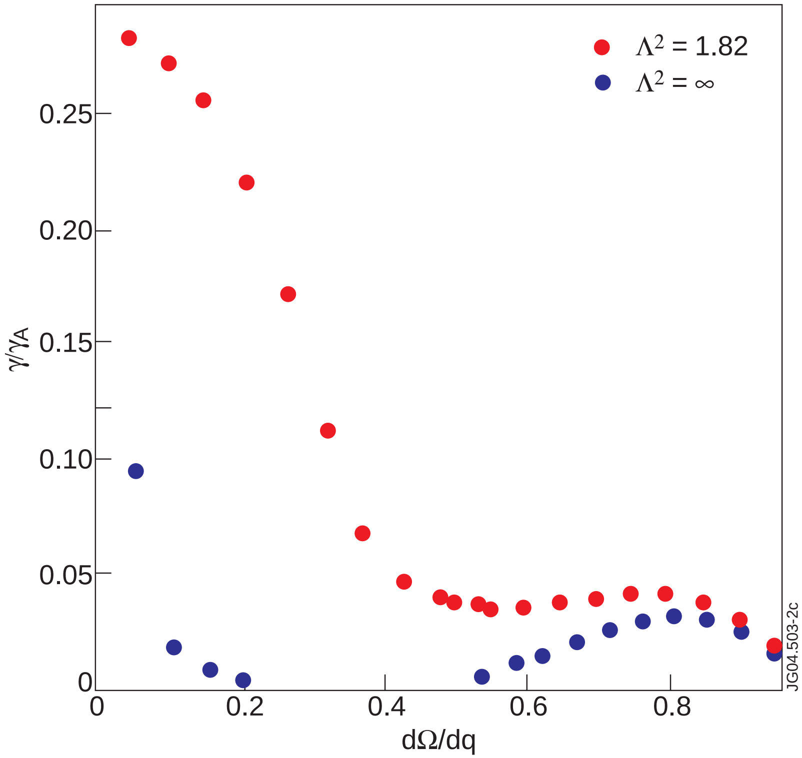
- α ~ s ~ 0(1) with stronger flow shear using numerical 2D eigenmode technique

- For α = 2, s = 1, recover result of time-dependent solution by Miller et al *(Phys Plasmas **2** (1995) 3676)* when n $\rightarrow \infty$

- Example: α = 1, s = 1

  – Finite n: γ drops less rapidly as n decreases

  – Can pass through another unstable region before finally stabilising at $d\hat{\Omega}/dq \cong 1$



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CONCLUSIONS

- New formalism for stability calculations at low s, relevant to ITBs
  - recurrence relation between ‘modelets’
- Ballooning mode radial structures tend to narrow at low s and near q_{min}
- MHD ballooning modes can be stable in ITBs with high pressure if they are sufficiently narrow (due to favourable average curvature)
- ITG modelets can be weakly coupled near q_{min}, but only if very long wavelength
  - different poloidal mode structures and eigenvalues on either side of q_{min} so ITB acts as a ‘barrier’ to mode structures
- Toroidal rotation shear: γ_{MHD} reduces rapidly for $d\hat{\Omega}/dq \sim 0(1/n)$, finally stabilising for $d\hat{\Omega}/dq \cong 1$, ie v ~ sv_A

  $\Rightarrow$ limited validity of standard (stationary) ballooning calculations

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