Effect of sheared flows on neoclassical tearing modes

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Motivation

- β limits in advanced tokamaks may be set well below the ideal MHD limit by nonlinear instabilities associated with **neoclassical tearing modes (NTM)**

- Like the classical TMs, NTMs are current driven but the source of free energy is the **perturbed bootstrap current** – a `neoclassical' (toroidal geometry driven) term. NTMs can be **unstable** even when $\Delta' < 0$

- Their interaction and nonlinear evolution in the presence of **equilibrium sheared flows** is not yet fully understood and is the subject of major numerical initiatives e.g. on NIMROD

- Flows are ubiquitous in most tokamaks e.g. from NBI heating, RF heating or self-consistent turbulence.

Role of Flows

- Flows can influence both outer layer and inner layer dynamics for resistive modes.

- They can also bring about changes in linear coupling mechanisms such as toroidal coupling between harmonics.

- Past nonlinear studies – mainly numerical – and often limited to simple situations (e.g. poloidal flows, non-self consistent) reveal interesting effects like oscillating islands, distortion in eigenfunctions etc.

- Also some recent analytic work on the the effect of flow on the threshold and dynamical properties of magnetic islands which are relevant to NTMs

Refs: Chen &Morrison, ’92, 94; Bondeson & Persson, ’86,’88,’89; M.Chu,’98
Dewar & Persson, ’93; Pletzer & Dewar, ’90,’91,’94;

Aim of the present work

- Investigate the nonlinear evolution of NTMs in the presence of **sheared equilibrium flows**

- **Our primary approach is numerical** – solve a set of model reduced MHD equations that contain viscous forces based on neoclassical closures and that permit inclusion of equilibrium flows in a consistent and convenient manner

- Also look at nonlinear evolution of **classical tearing modes** for a comparative study and to obtain a better understanding of the role of flows

- Develop a **generalized Rutherford type model** to seek qualitative understanding of the nonlinear numerical results

Model Equations

- **Generalized reduced MHD equations**

  (Kruger, Hegna and Callen, Phys. Plasmas **5** *(1998) 4169.)*

- *Applicable to any toroidal configuration – no constraint on aspect ratio – exploits smallness of* $\frac{\lambda_\perp}{\lambda_\parallel}$ *and* $\frac{\lambda_\perp}{a}$

- Clear separation of time scales – MHD equilibrium, perp. wave motion and parallel wave motion

- *Final equations evolve scalar quantities on shear Alfven time scales*

- Energy conservation, divergence free magnetic field to all orders

- Include neoclassical closures, equilibrium flows

Model Equations (GRMHD)

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1) \cdot \nabla \phi_1 - \boldsymbol{b}_1 \cdot \nabla \phi_0 = \eta \tilde{J}_{||} - \frac{1}{ne} \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}_e$$

bootstrap current

$$\nabla \cdot \left(\frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (\boldsymbol{V}_1 \cdot \nabla) \left(\nabla \cdot \left(\frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) = (\boldsymbol{B}_0 \cdot \nabla) \frac{\tilde{J}_{||}}{B_0} + (\boldsymbol{B}_1 \cdot \nabla) \frac{J_{T||}}{B_0}$$

$$+ \nabla \cdot \frac{\boldsymbol{B}_0 \times \nabla p_1}{B_0^2} \qquad + \nabla \cdot \frac{\boldsymbol{B}_0}{B_0^2} \times \nabla \cdot \boldsymbol{\Pi}$$

GGJ

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot \boldsymbol{V}_1 = (\Gamma - 1) \left[\eta J_{T||}^2 - \boldsymbol{\Pi} : \nabla \boldsymbol{V} + \boldsymbol{\Pi}_{\boldsymbol{e}} : \nabla \frac{\boldsymbol{J}}{ne} - \nabla \cdot \boldsymbol{q} \right]$$

heat flow

$$\rho \frac{d\widetilde{V}_{||}}{dt} + (\boldsymbol{V}_1 \cdot \nabla) V_{||_0} = -\boldsymbol{b}_0 \cdot \nabla p_1 - \boldsymbol{b}_1 \cdot \nabla p_T - \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla$$

$$\boldsymbol{V} = \Omega(\psi) R^2 \boldsymbol{\nabla}\zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0 \times \nabla\Phi_0}{B_0^2} + V_{0||}\boldsymbol{b}_0 + \frac{\boldsymbol{B}_0 \times \nabla\Phi_1}{B_0^2} + \tilde{V}_{||}\boldsymbol{b}_T$$

Equilibrium flow

- **Neoclassical closure**

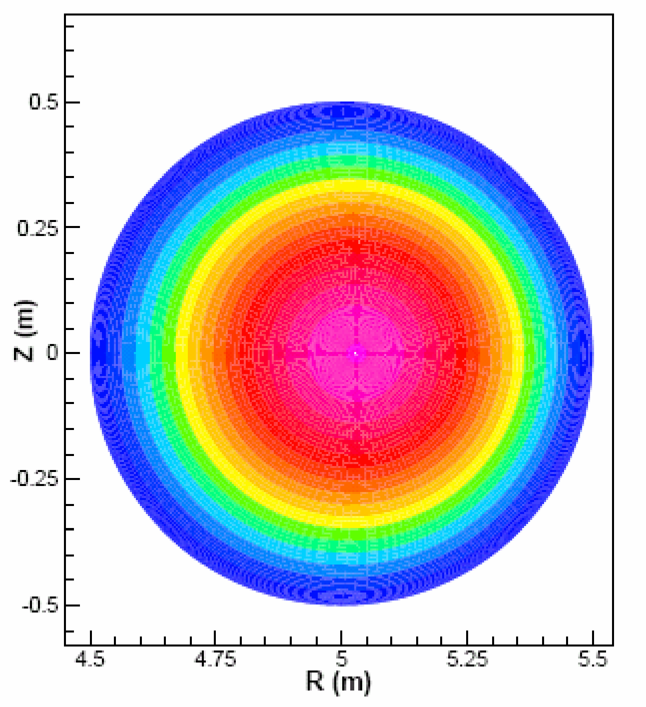
$$\boxed{\vec{\nabla} \cdot \Pi_s = \rho_s \mu_s \left\langle B^2 \right\rangle \frac{\vec{V}_s \cdot \vec{\nabla}\Theta}{\left(\vec{B} \cdot \vec{\nabla}\Theta\right)^2} \vec{\nabla}\Theta,}$$

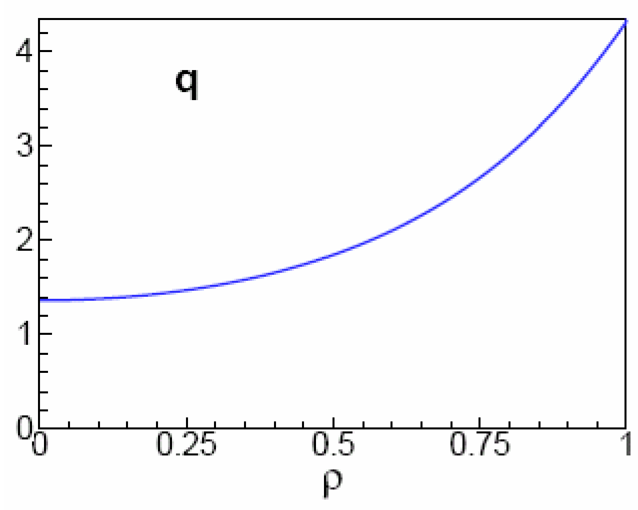
- appropriate for long mean free path limit
- reproduces poloidal flow damping
- gives appropriate perturbed bootstrap current

Numerical simulation

- **GRMHD** eqns solved using code *NEAR* – toroidal initial value code – Fourier decomposition in the poloidal and toroidal directions and central finite differencing in the flux coordinate direction.
- Equilibrium generated from another independent code **TOQ**
- Typical runs are made at **$S \sim 10^5$, low β, sub-Alfvenic flows**
- Linear benchmarking done for classical resistive modes
- For NTMs threshold, island saturation etc. benchmarked in the absence of flows.
- Present study restricted to **sheared toroidal flows**

Equilibrium without flow

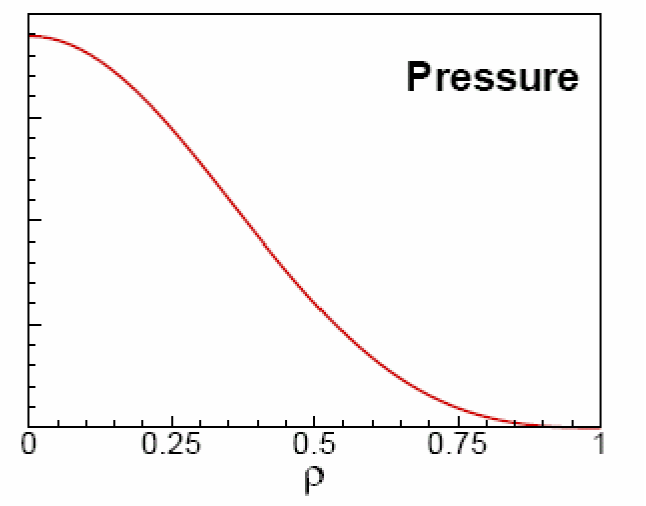




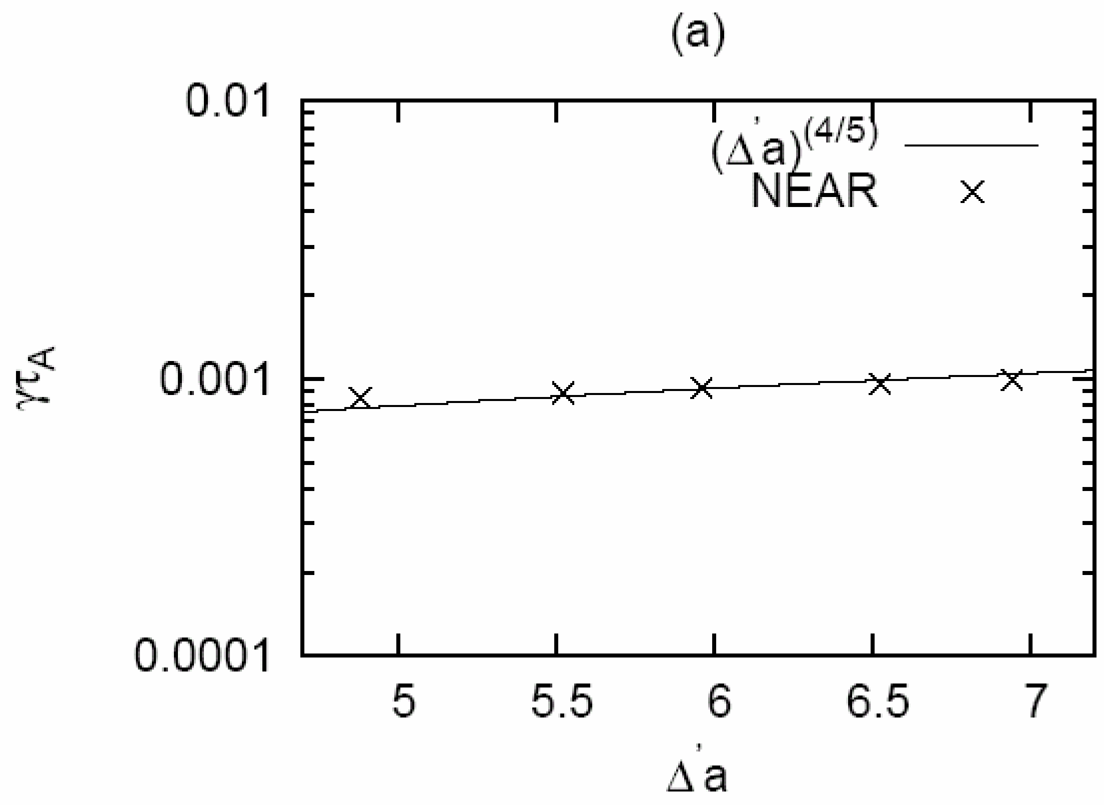
B_o = 4.80 T R_o = 5.00 m

I_P = 0.28 MA   a = 0.50 m

β = 0.26%



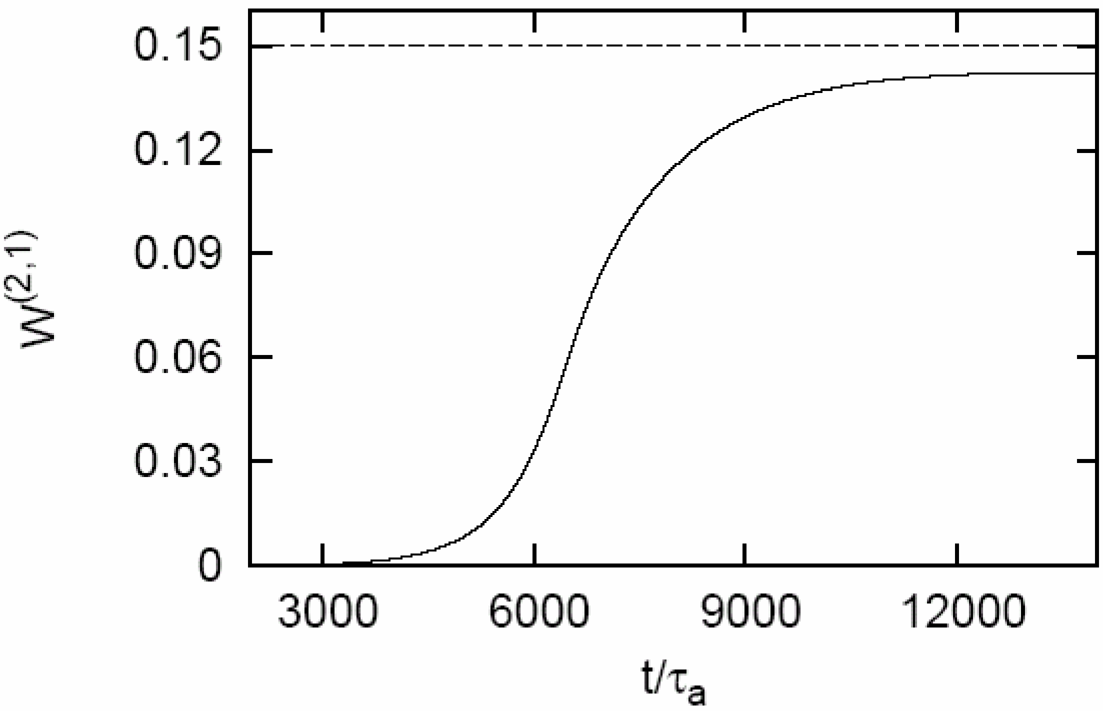
Linear Benchmarking of (2,1) resistive TM



- **Also established the $\gamma \propto S^{-3/5}$ scaling**

Nonlinear saturation of (2,1) resistive TM

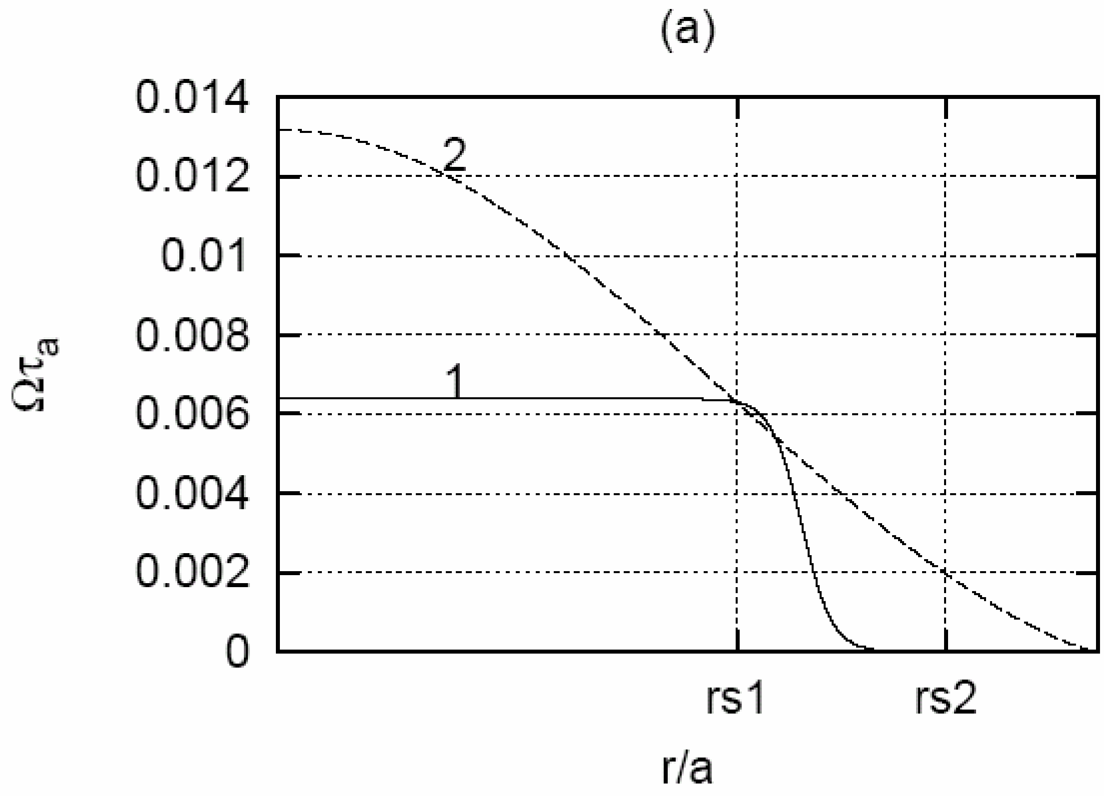
(b)



$$\boxed{\Delta' \rightarrow \Delta'(1 - W/W_{sat})}$$

Hegna, C.C., Callen, J.D., Phys. Plasmas **1** (1994) 2308.

Toroidal flow profiles

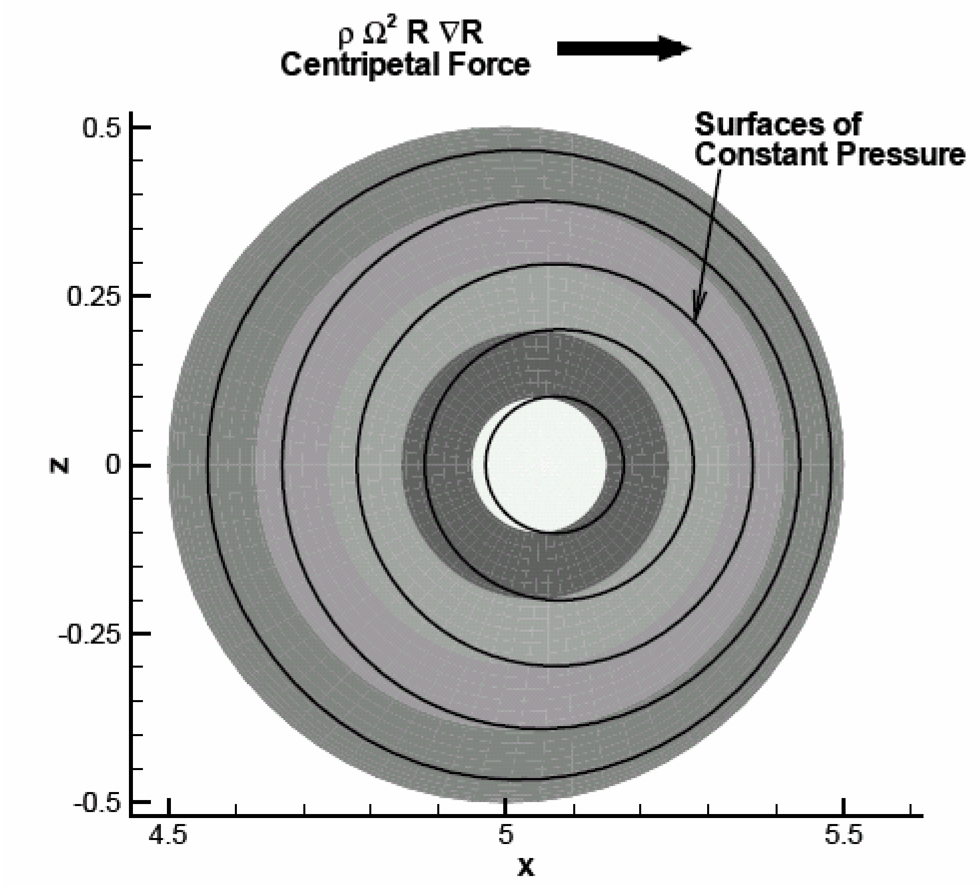


1- differential flow
2- sheared flow

Main points of investigation

- Effects arising from equilibrium modifications
- Influence on toroidal coupling
- Influence on inner layer physics
- Changes in outer layer dynamics
- Nonlinear changes – saturation levels etc.

Equilibrium with toroidal flow

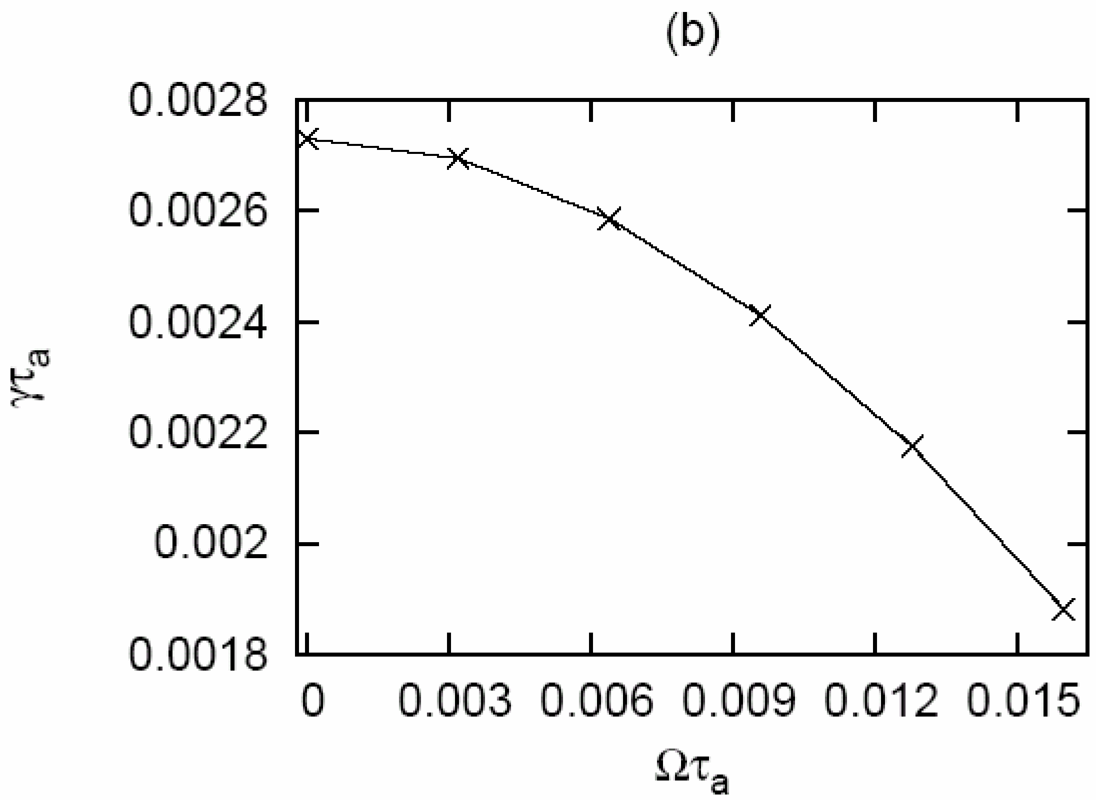


Constant pressure Surfaces shifted from Constant flux surfaces

$$\boxed{p_0 = p_{nf}(\psi_0)\exp\left(\frac{\Gamma}{2}M_s^2(\psi_0)(\hat{R}^2 - \hat{R}_{axis}^2)\right)}$$

Maschke & Perrin, Plasma Phys. 22 (1980) 579

Reduction of (2,1) resistive TM growth with differential flow



- **stabilizing effect due to equilibrium changes e.g. enhancement of pressure-curvature contribution**
- **stabilizing effect due to flow induced de-coupling of rational surfaces**

- **Slab or cylinder**

$$\Delta'\Psi_s = -i\,(\omega - \Omega_s)\,\tau_L \Psi_s \;; \quad \Omega_s = \vec{k}\cdot\vec{V}_0$$

$$\boxed{\gamma = \frac{\Delta'}{\tau_L}} \qquad \boxed{\Omega = \Omega_s}$$

- **Toroidal geometry**

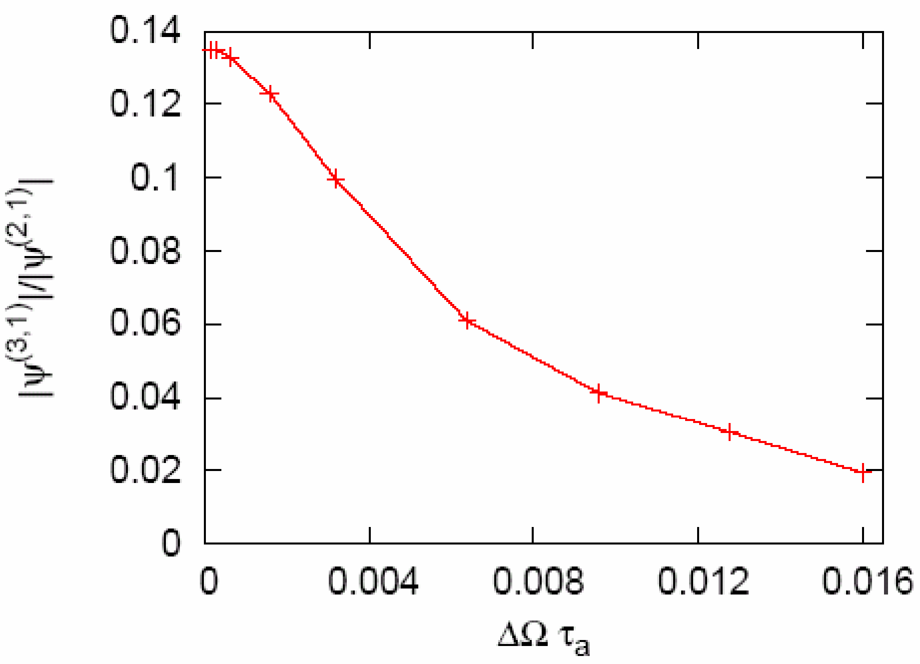
$$\Psi_{\mathbf{large}} = \Delta'\Psi$$ **outer response - Δ' matrix**

$$\Delta(\omega) = -i\,(\omega - \Omega_j)\,\tau_{Lj}\delta_{ij}$$ **inner response**

$$\det\begin{bmatrix} \Delta'_{11} - \Delta_{11}(\omega) & \Delta'_{12} \\ \Delta'_{21} & \Delta'_{22} - \Delta_{22}(\omega) \end{bmatrix} = 0.$$

Quadratic equation

Reduced reconection at the (3,1) surface



- **In the presence of finite flow shear the stabilization effect is smaller**

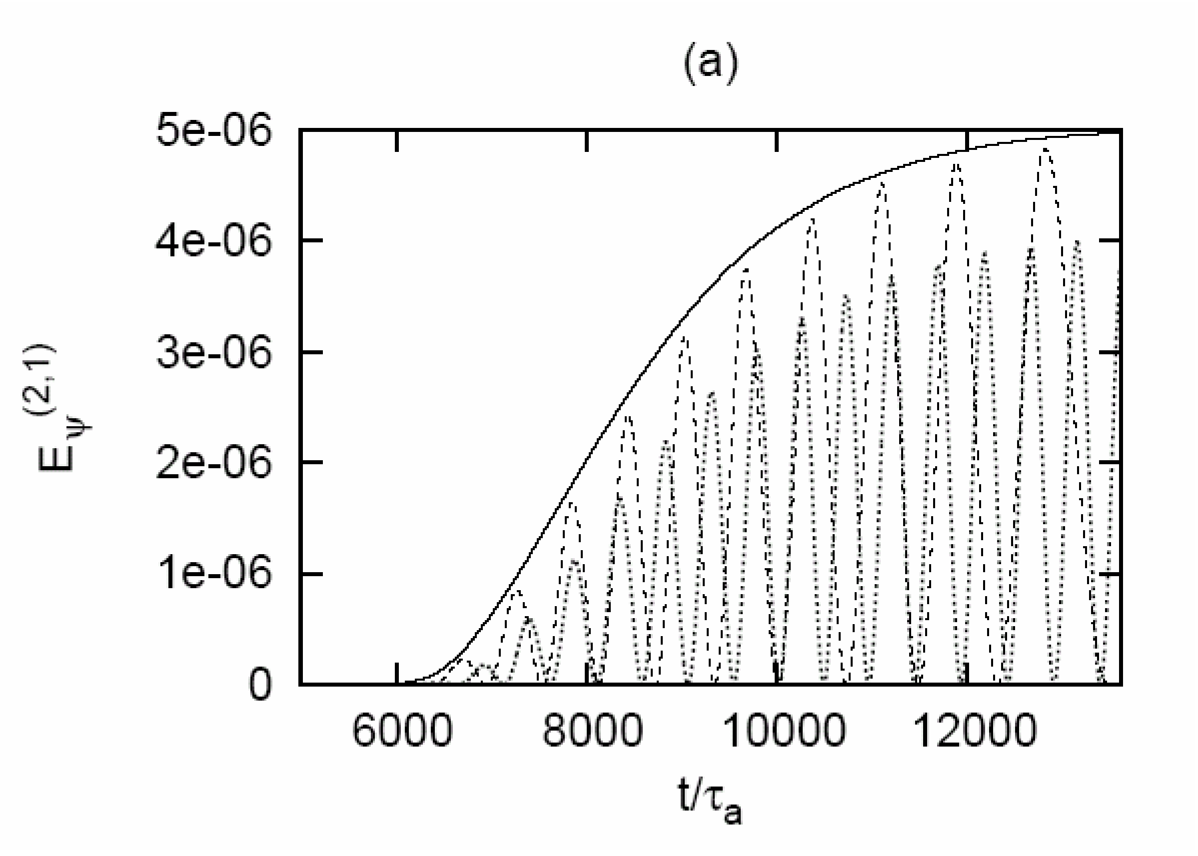
- This can be understood and explained quantitatively on the basis of linear slab theory analysis **(Chen & Morrison, PF B 2 (1990) 495)**

$$\gamma \sim \alpha^{2/5} \Delta'^{4/5} S^{-3/5} \hat{\gamma}$$

$$\hat{\gamma} = \text{flow correction} \geq 1$$

Small flow shear destabilizes the resistive mode through changes in the inner layer dynamics

Nonlinear evolution of (2,1) resistive mode

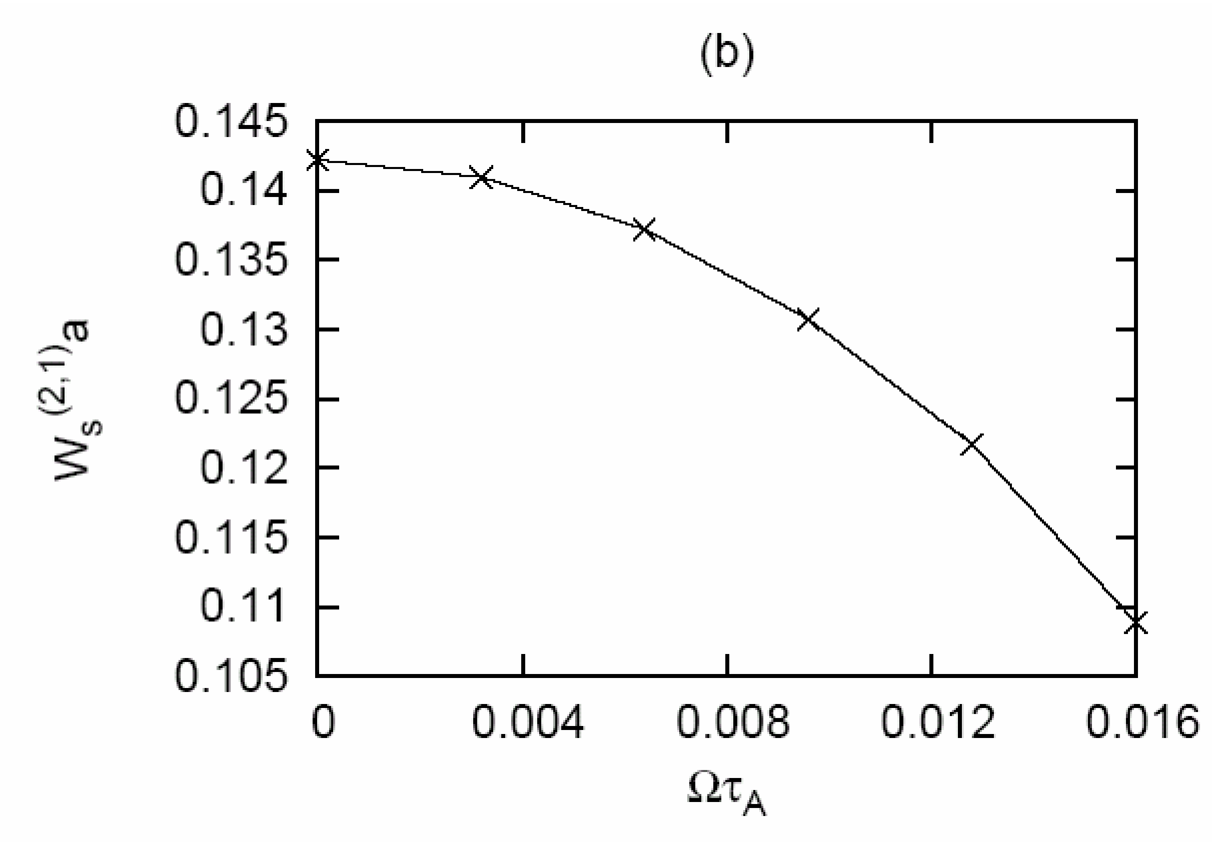


No flow

Sheared flow

Differential flow

Saturated island width decreases with differential flow



Summary of numerical results for classical TMs

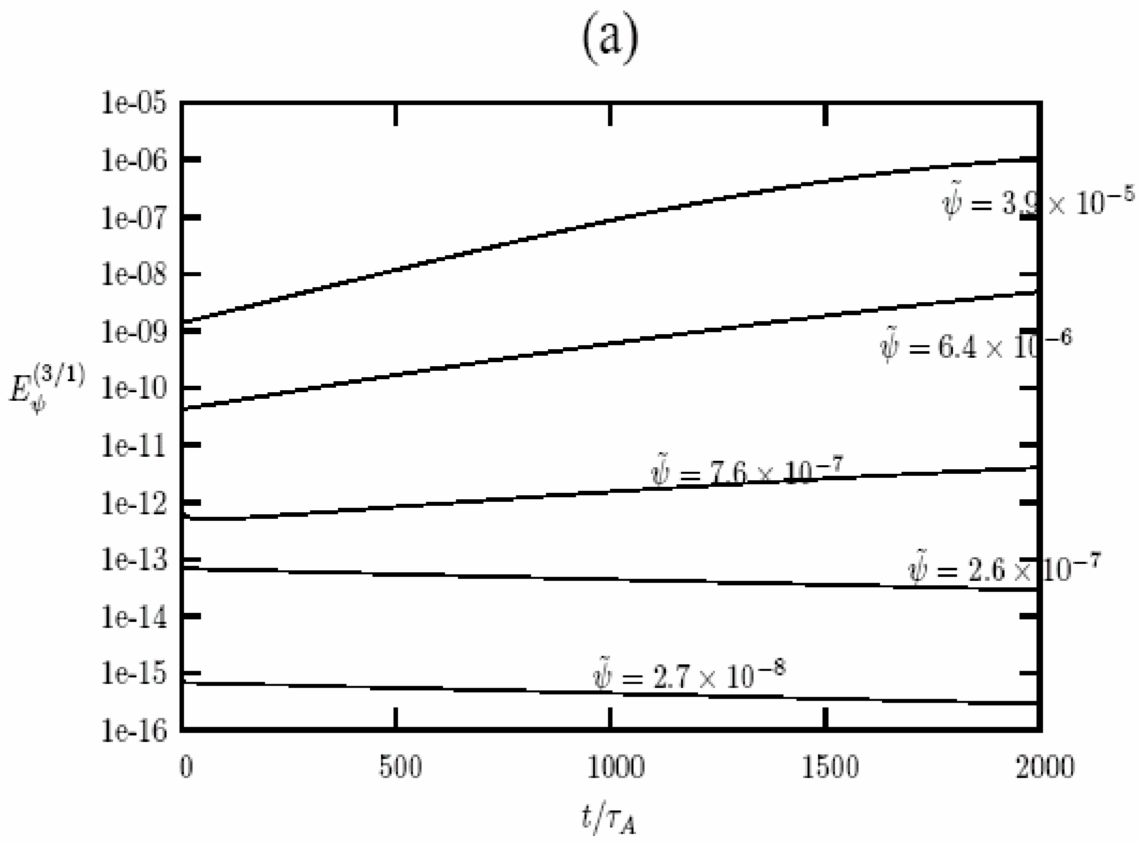
- In the **linear** regime:
  - flow induces mode rotation
  - differential flow : stabilizing influence
    - modification in Mercier criterion
    - decoupling of rational surfaces
  - flow shear: destabilizing influence – consistent with inner layer dynamical theories
- **Nonlinear** regime
  - Above trend continues for differential and sheared flows
  - Mode acquires real frequency which asymptotes to flow frequency
  - Flow reduces saturated island width

Neoclassical Tearing Modes

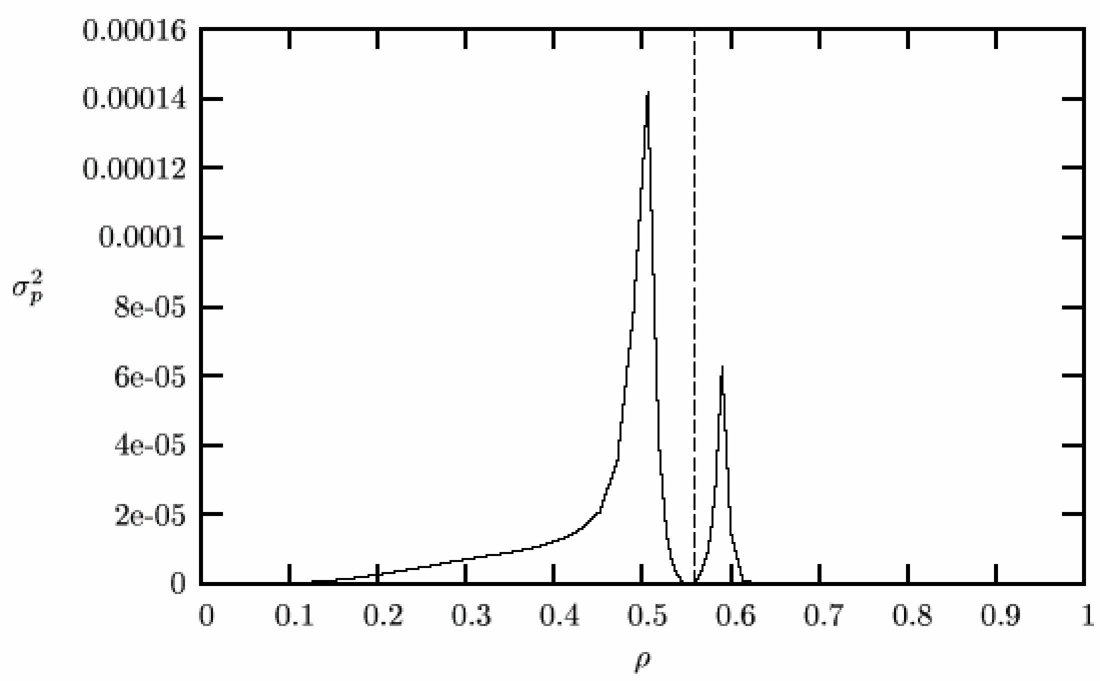
Benchmarking tests in absence of flow:

- **threshold amplitude for instability**
- **nonlinear behavior – island saturation**
- **pressure equilibration**

Existence of threshold amplitude for (3,1) NTM

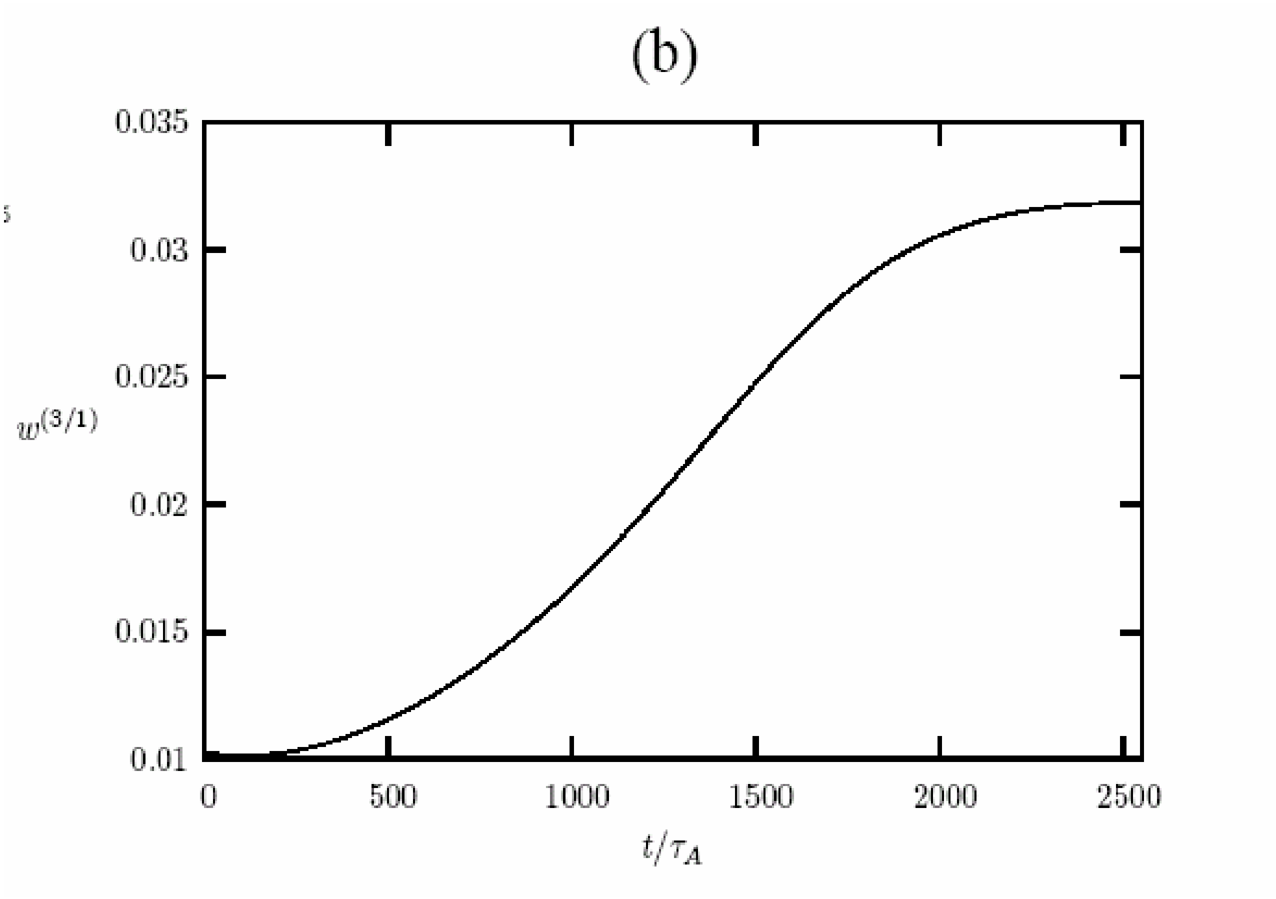


Pressure equilibration

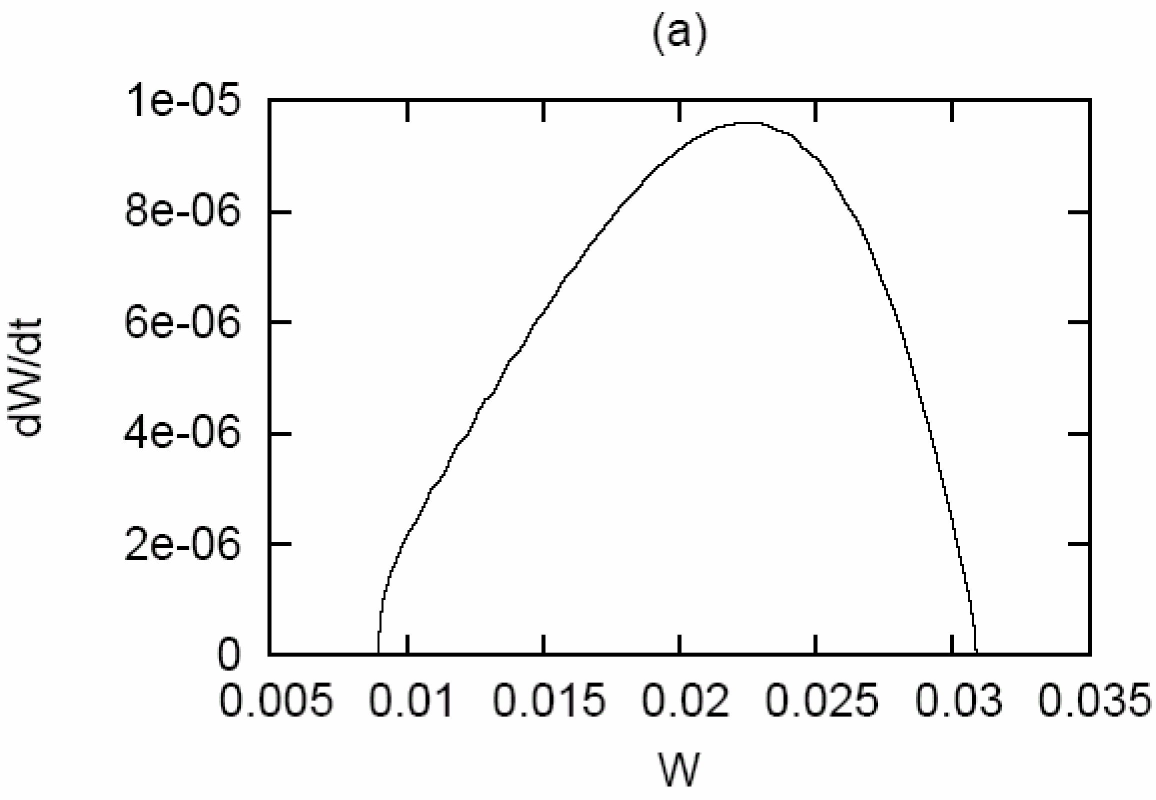


- pressure variance across the island
- pressure flattening ensured by suitable choice of ratio of perpendicular and parallel thermal conductivities - typically $\chi_{||} / \chi_{\perp} \sim 10^6$.

Nonlinear evolution of (3,1) NTM island width



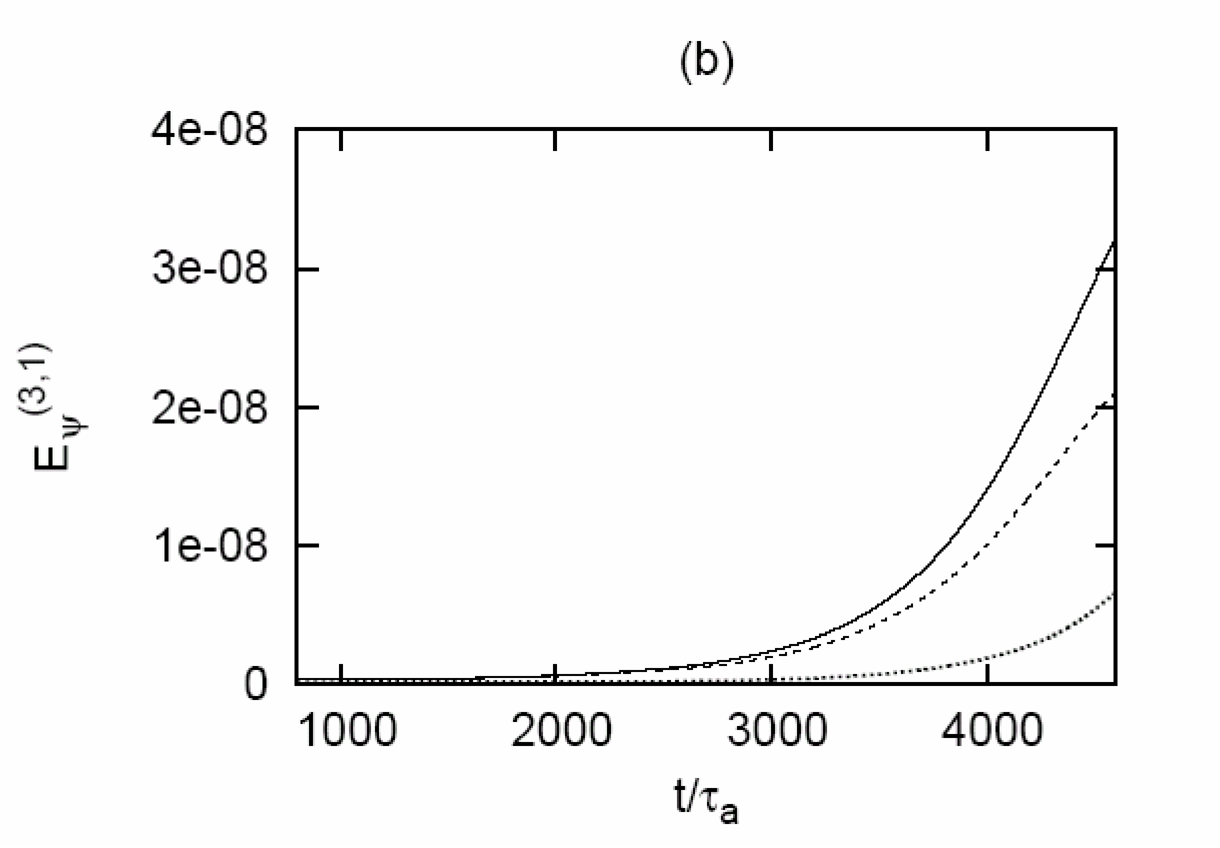
``Phase diagram'' of (3,1) NTM



NTM with flows

- self-consistent equilibria generated by TOQ
- two types of flow profiles – differential flow, sheared flow
- attention paid to pressure equilibration

Nonlinear evolution of (3,1) NTM



No flow

Sheared flow

Differential flow

Summary of numerical results for NTMs

- In the quasi- linear regime:
  - differential flow : stabilizing influence
  - flow shear: destabilizing influence – consistent with quasi-linear theory

- Fully nonlinear regime – we experience numerical instabilities in the presence of flows – possible cause - inadequate pressure equilibration – this is being investigated.

Analytic Model

- **Single helicity calculation**
- **Flow effects incorporated in polarization current term**
- **two fluid model**
- **neoclassical effects in Ohm's law**
- **simple pressure evolution equation & neglect parallel dynamics**
- **phenomenological model for GGJ effect**
- **use both matching conditions to get island evolution equation as well as temporal evolution of real frequency**

Island equation with sheared flow

Pressure/curvature

Neoclassical current

$$0.4\frac{\partial W}{\partial t} = D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.6\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right.$$

$$\left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + \frac{{\omega_E'}^2}{W}\right)\right]$$

differential flow

polarization current

flow shear

$$0.82\frac{\partial}{\partial t}\left[W(\omega-\omega_E) + \frac{\omega_E'}{2}W^2\right] = -12.6\frac{\mu_e}{W}(\omega-\omega_E)$$

$$-\frac{1}{4\sqrt{2}}\left(\frac{nsV_A}{R^2 q}\right)^2 W^4\Delta_s'$$

Summary and Conclusions

- **Presented numerical simulation results, using a model set of GRMHD equns. with neoclassical viscous terms and toroidal flow, for nonlinear evolution of resistive TMs and NTMs**

- Differential flow has a stabilizing influence – can be understood intuitively in the linear regime as occurring from decoupling of rational surfaces - the decoupled surface appears as a conducting surface and exerts a stabilizing influence. Same trend continues in the nonlinear regime – no analytic theory exists in the nonlinear regime

Summary and Conclusions (contd)

- Flow shear has a destabilizing influence in the small shear limit examined by us. In the linear regime it is consistent with past analytic work (**e.g. Chen, Morrison, PFB(1990)495**). In the quasi-linear regime our single mode Rutherford type calculation shows similar trend.

- Flow induces a real frequency in the mode which can be understood from our Rutherford type model equation – the shift in the frequency is proportional to the amount of shear.

- Final saturated island widths for NTMs in the presence of flow has not yet been resolved numerically and is presently under investigation