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Kinetic calculations of the NTM polarisation current: reduction for small island widths and sign reversal near the diamagnetic frequency

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Motivations and outline

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- Reliable description of NTMs necessary in order to determine onset conditions and stabilisation requirements ($\rightarrow$ ITER)

- Problem at the meeting point of MHD and kinetic theory ($\rightarrow$ required for accurate predictions)

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- Problem at the meeting point of MHD and kinetic theory ($\rightarrow$ required for accurate predictions, e.g. NTM polarisation current)

Outline

- Polarisation current in the presence of a magnetic island

- Solving the drift kinetic equation

- Single-particle motion and full 3D simulations: new conditions for island stability

The Neoclassical Tearing Mode

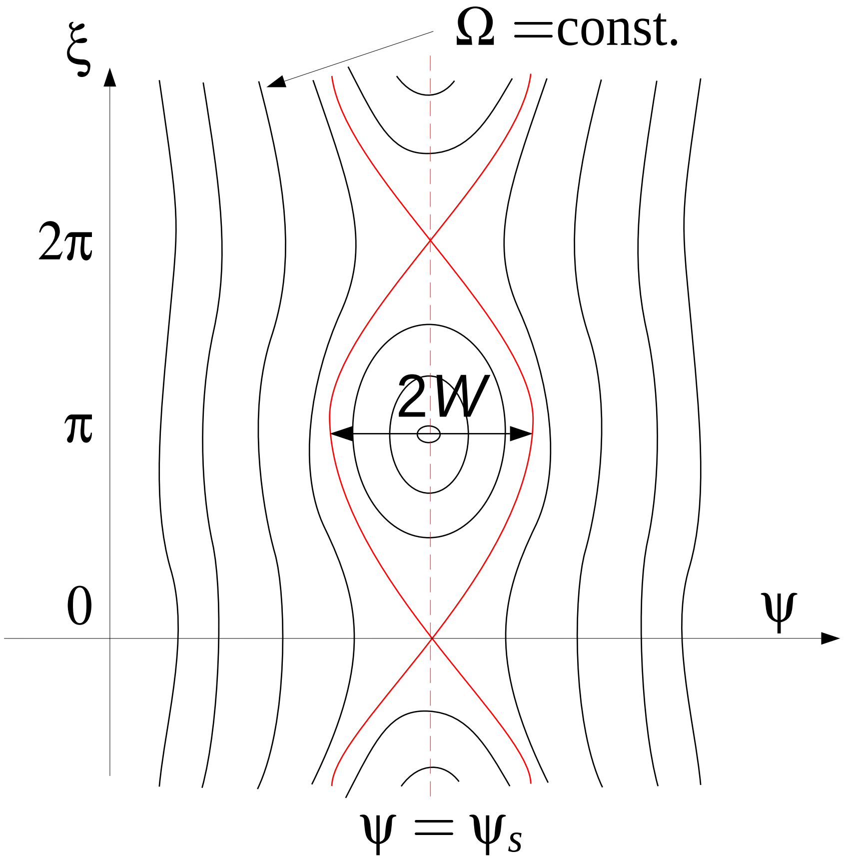
- Island evolution connected with the parallel currents flowing near the resonant surface

$$\frac{\mathrm{d}W}{\mathrm{d}t} = c_1\Delta' + \frac{c_2}{W}\int_{-1}^{\infty}\mathrm{d}\Omega \oint \frac{\mathrm{d}\xi\cos\xi}{\sqrt{\cos\xi+\Omega}}\, j_{\|}^{n.i.}$$

New flux coordinates:

helical flux $\Omega \equiv 2(\psi - \psi_s)^2/W_\psi^2 - \cos\xi$

helical angle $\xi \equiv m\theta - n\zeta$



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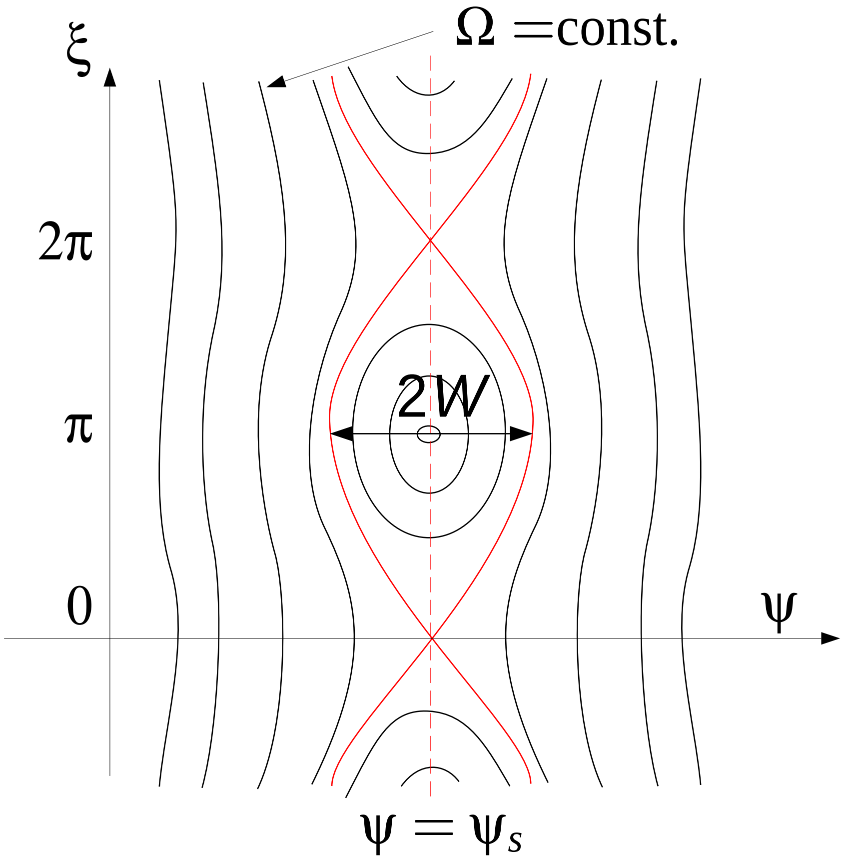

- Destabilising term:

Bootstrap current loss [Qu and Callen, UWPR1985; Carrera et al., PoF 1986]

- Stabilising terms:

(large *W*)

$\rightarrow$ Δ' (current profile, $m \geq 2$) [Rutherford, PoF 1973]

(small *W*)

$\rightarrow$ ...

$\rightarrow$ Polarisation current (?) [Smolyakov et al., PoP 1995; Wilson et al., PoP 1996]

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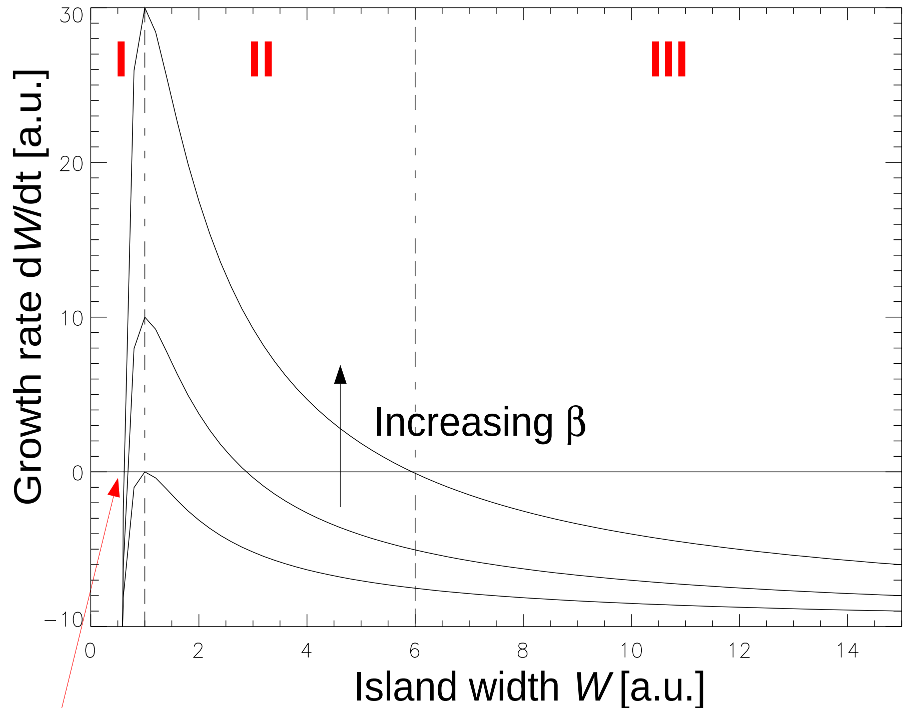

Threshold!

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I: Stabilising mechanism important
II: Bootstrap current important
III: Δ' important

Present understanding of the Neoclassical Tearing Mode

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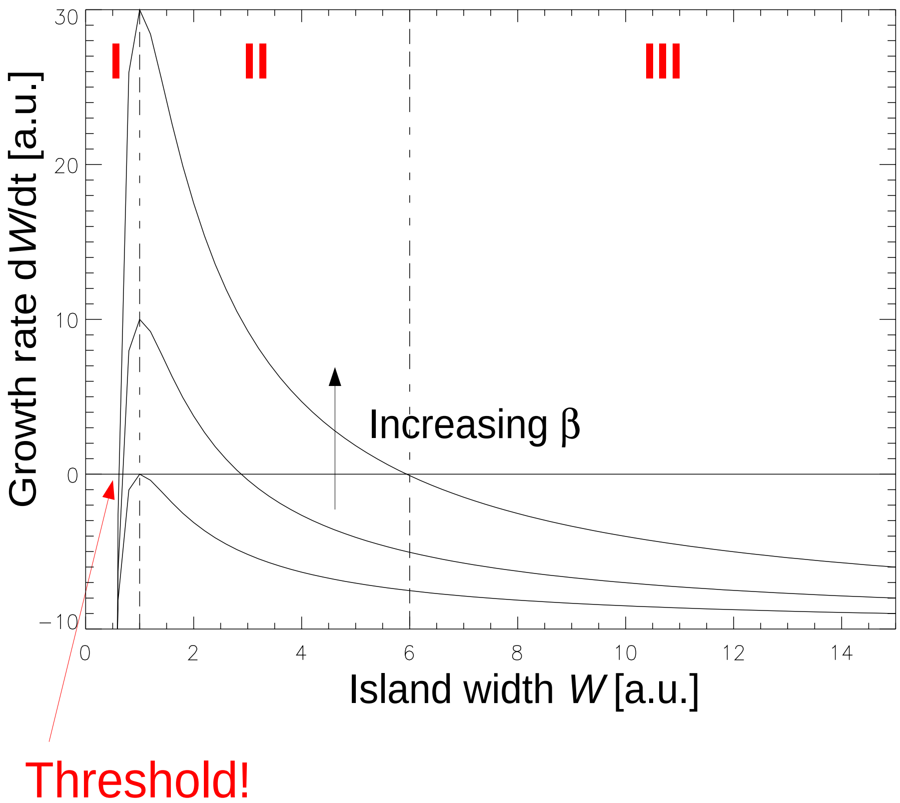

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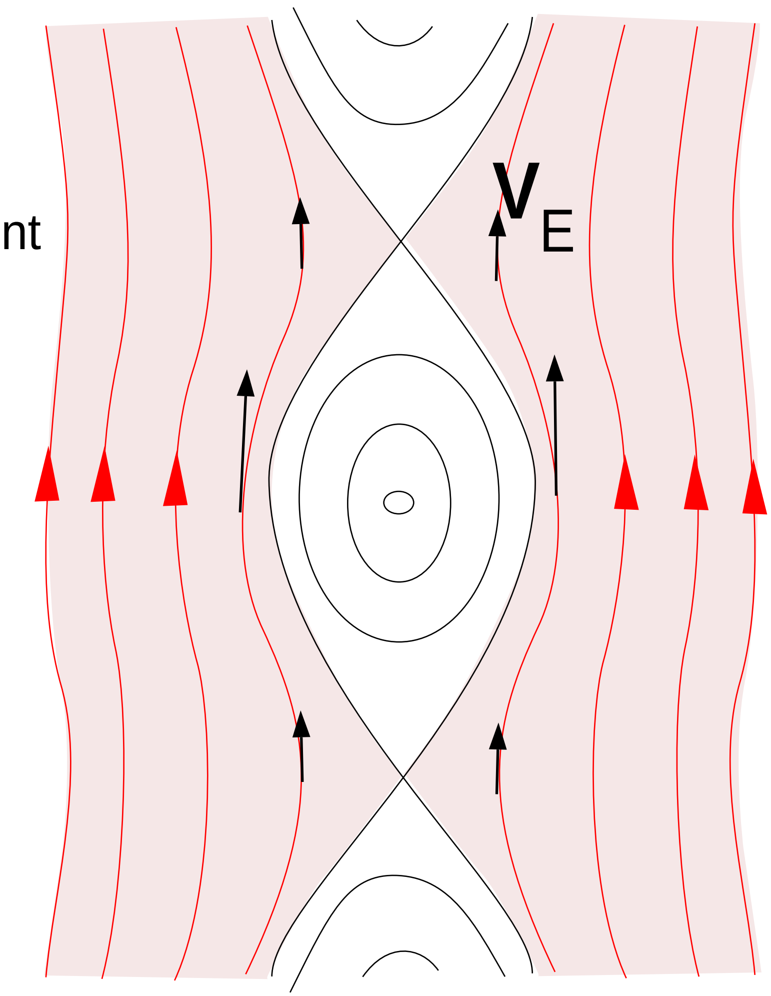
In this talk: **focus on the polarisation current; no mode evolution!**

The island polarisation current

- Island motion with respect to the plasma
$\Rightarrow$ electric field induced (Faraday)

- $E \times B$ motion in the island rest frame: plasma acceleration and deceleration around the *O*-point



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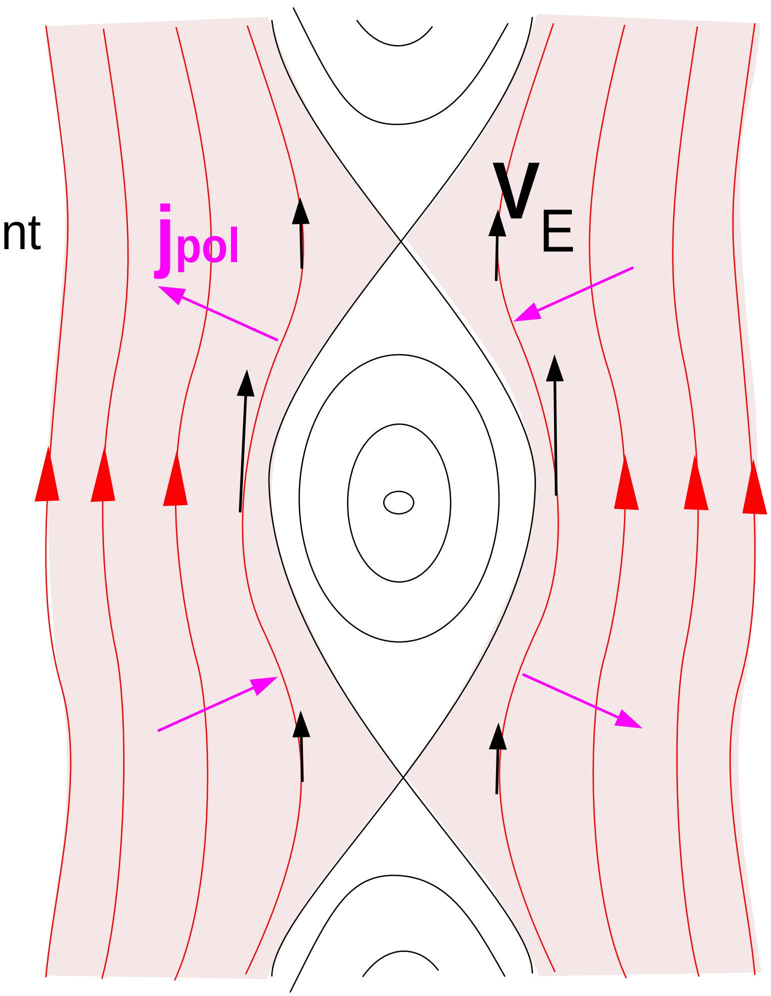
- Variation of plasma inertia balanced by a Lorentz force provided by

$$\boxed{j_{\rm pol}^{\rm class} = \frac{en}{\omega_c}\frac{{\rm d}v_E}{{\rm d}t}}$$

Polarisation current

(⇒ mainly carried by the ions)
[Smolyakov, PPCF 1993]

- Current continuity ($\nabla\cdot\mathbf{J}=0$) ensured by an electron parallel current contributing to the Rutherford equation



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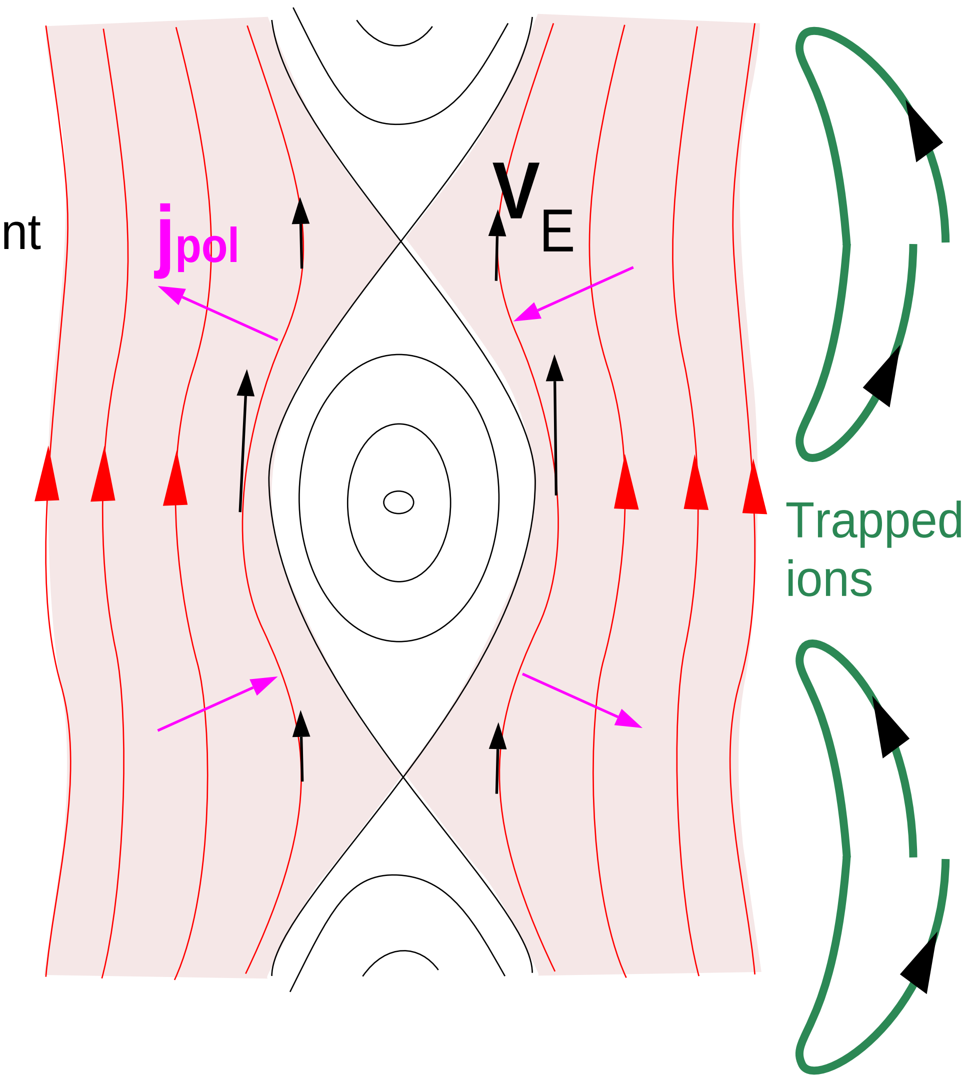


Solving the drift kinetic equation

- Analytical determination of $j_{\parallel}^{n.i.}$ from the drift kinetic equation possible employing the expansion parameters $W/r,\ w_b/W \ll 1$ (and further simplifications...)

- Drift kinetic equation in toroidal geometry with an island structure to be solved

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \left(v_{\parallel}\hat{\mathbf{b}} + \mathbf{v}_d + \mathbf{v}_E\right) \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{e}{m}\frac{\mathbf{v}_d \cdot \nabla\Phi}{v}\frac{\partial f}{\partial v} = C(f)$$

parallel motion — magnetic & electric drift — electric field — collisions

- Representation of the distribution function: $f = f_0 + \delta f = f_M(\psi, \mathcal{E}) + \delta f$

if $\delta f \ll f_0$: reduction of the numerical noise

- The equation for δf is $$\frac{\mathrm{d}(\delta f)}{\mathrm{d}t} = C(\delta f) - \mathbf{v}_d \cdot \nabla f_M - \frac{e f_M}{T}\mathbf{v}_d \cdot \nabla\Phi$$

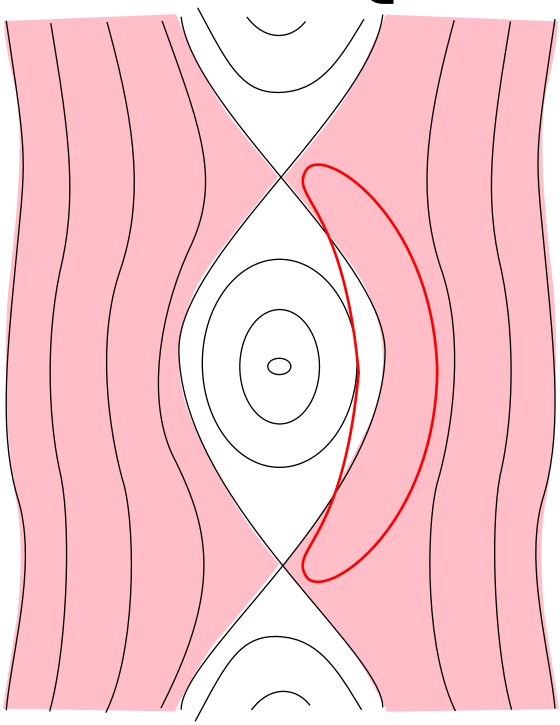
Solution:
- $\delta f \rightarrow$ markers (ions) $\rightarrow$ Hamiltonian equations of motion in Boozer coordinates ($\rightarrow$ HAGIS) [Pinches et al., CPC 1998]
- Collisions: Monte Carlo procedure [Bergmann et al., PoP 2001]

Polarisation current vs. island width

- Simulations performed for the (3,2) mode and large-machine parameters:
$R = 8$ m, $B_0 = 8$ T, $n_i = 10^{20}$ m^{-3}, $T_i = 5$ keV , flat temperature and density profiles

- Local effects ‘‘smeared out’’ by trapped particles overlapping the island

- ASDEX Upgrade: $\begin{cases} W_{\text{seed}} \approx 1 \div 5 \text{ cm} \\ w_b \approx 0.7 \div 3 \text{ cm} \end{cases}$



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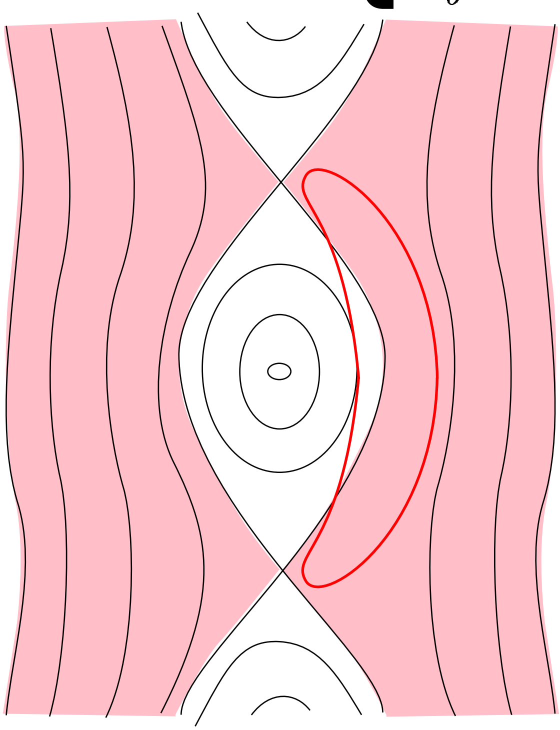
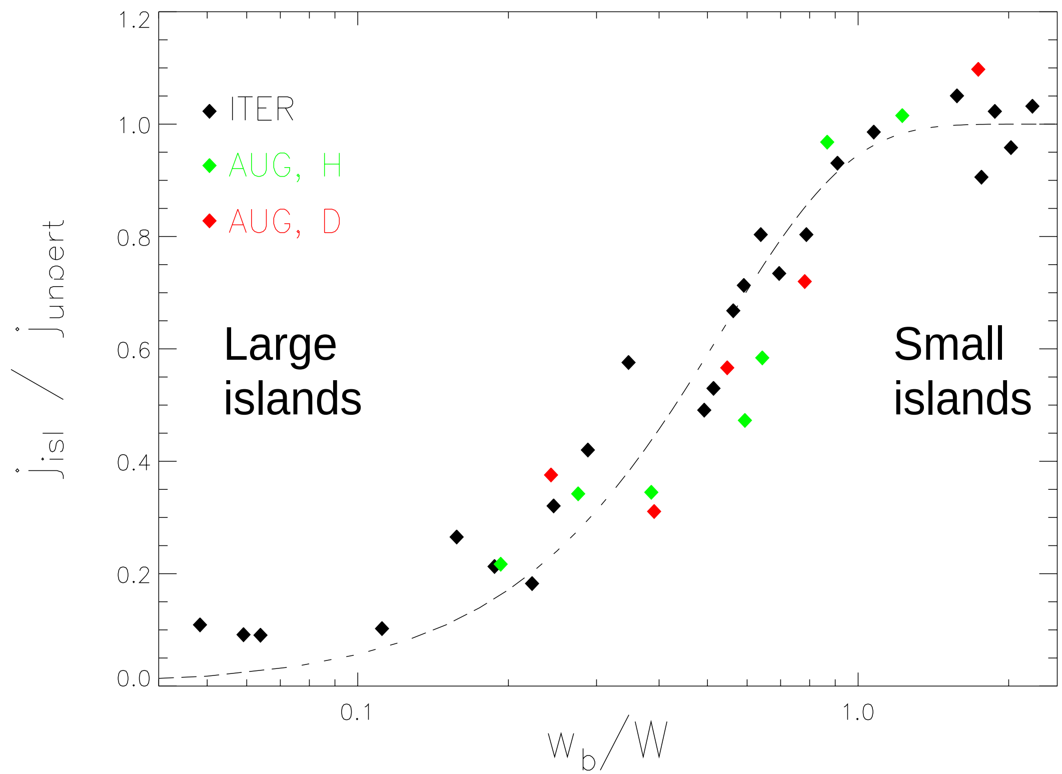


Persistence of the bootstrap current inside a small island



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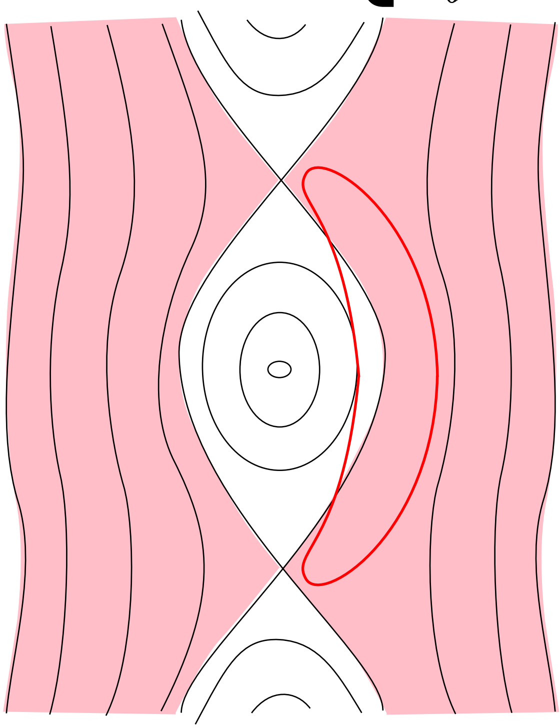

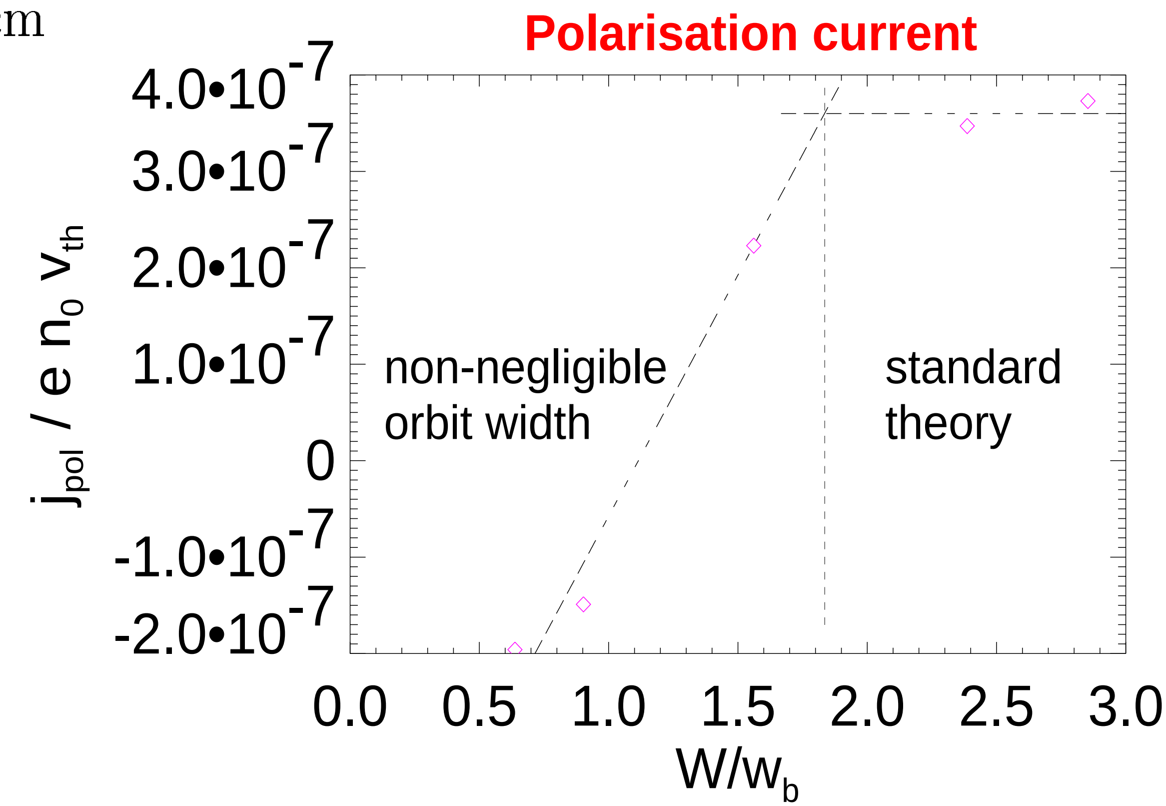

Perpendicular current vs. island rotation frequency

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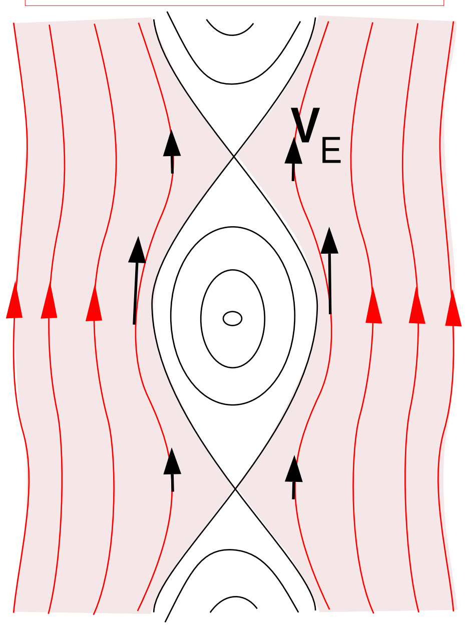
- Behaviour of $j_\perp$ vs. ω puzzling (quadratic scaling with ω expected from fluid picture)

$$\boxed{j_{\rm pol} \propto {\rm d}v_E/{\rm d}t \propto \omega^2}$$



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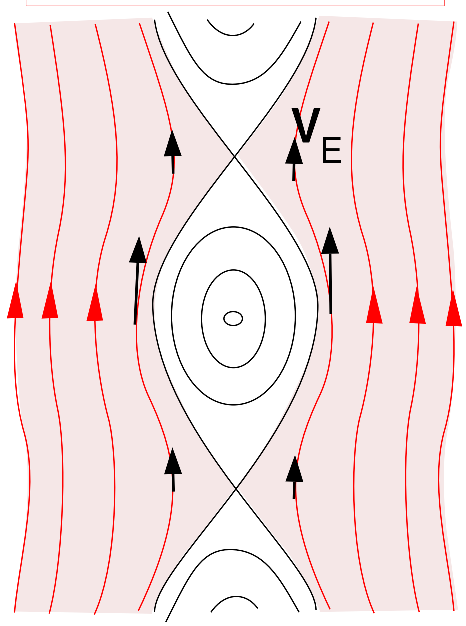
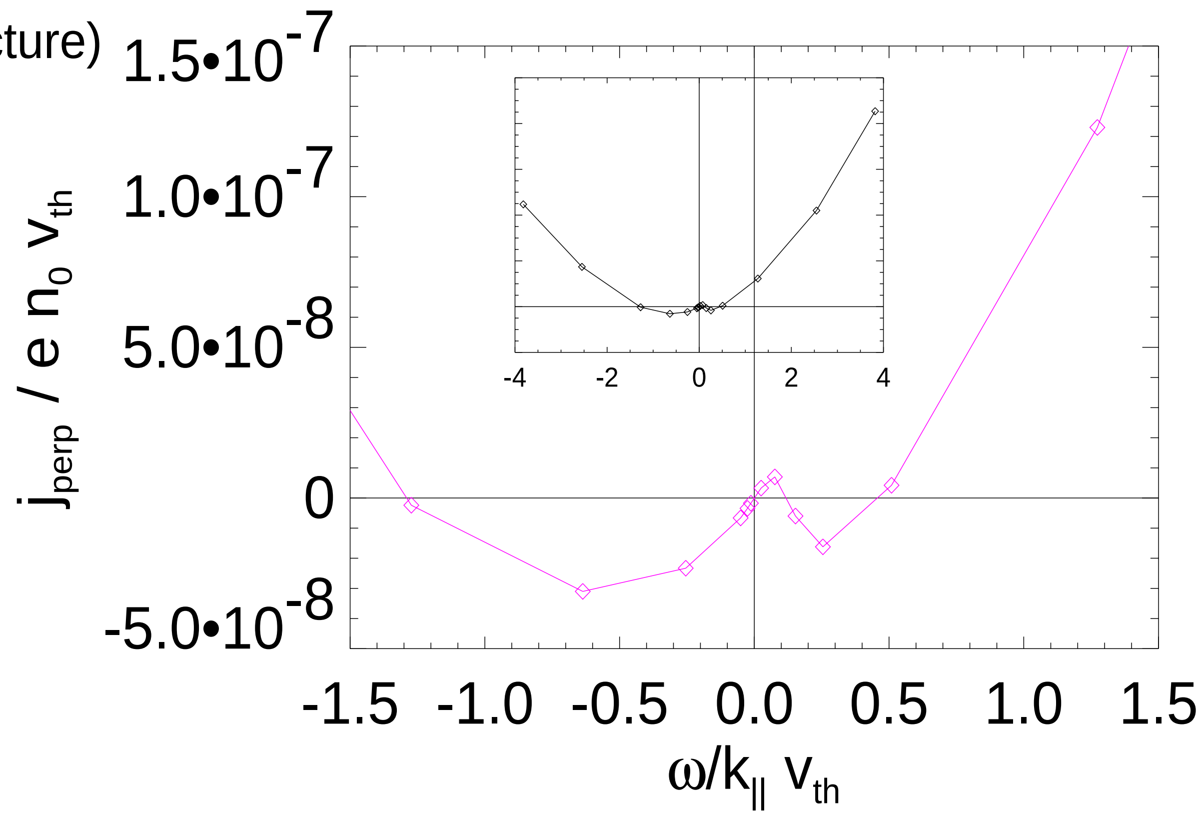
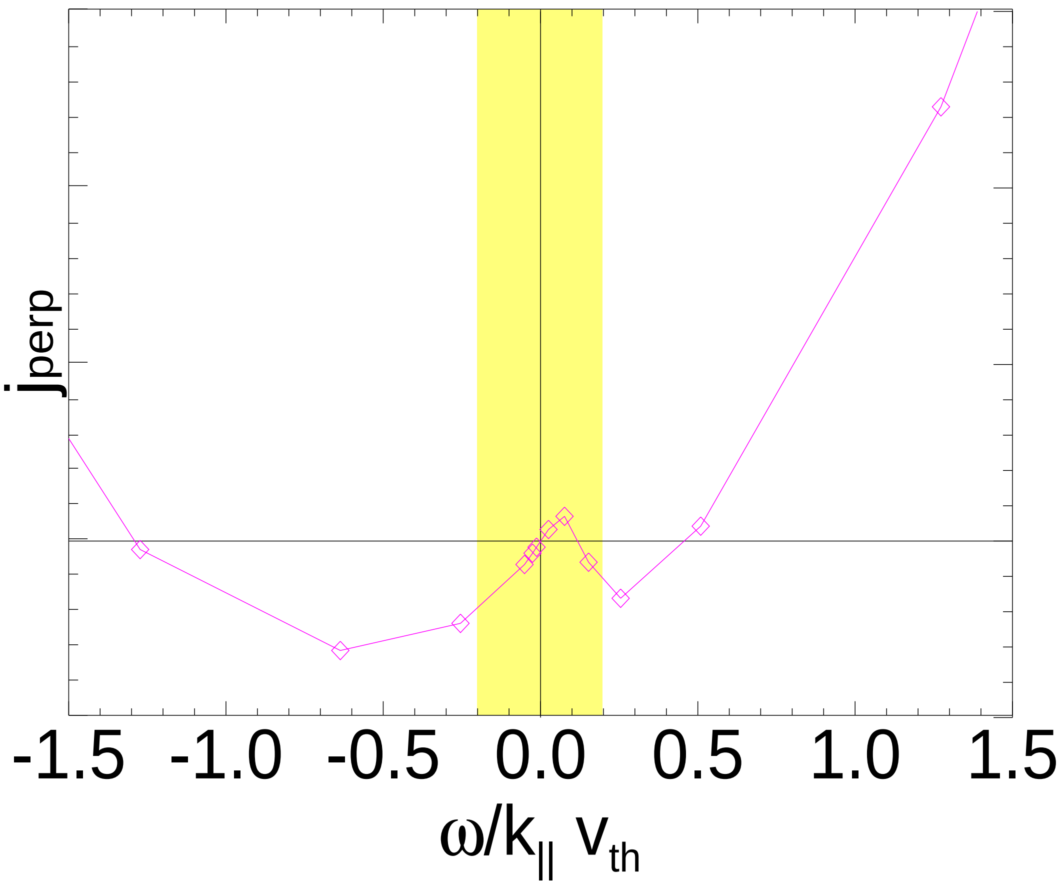
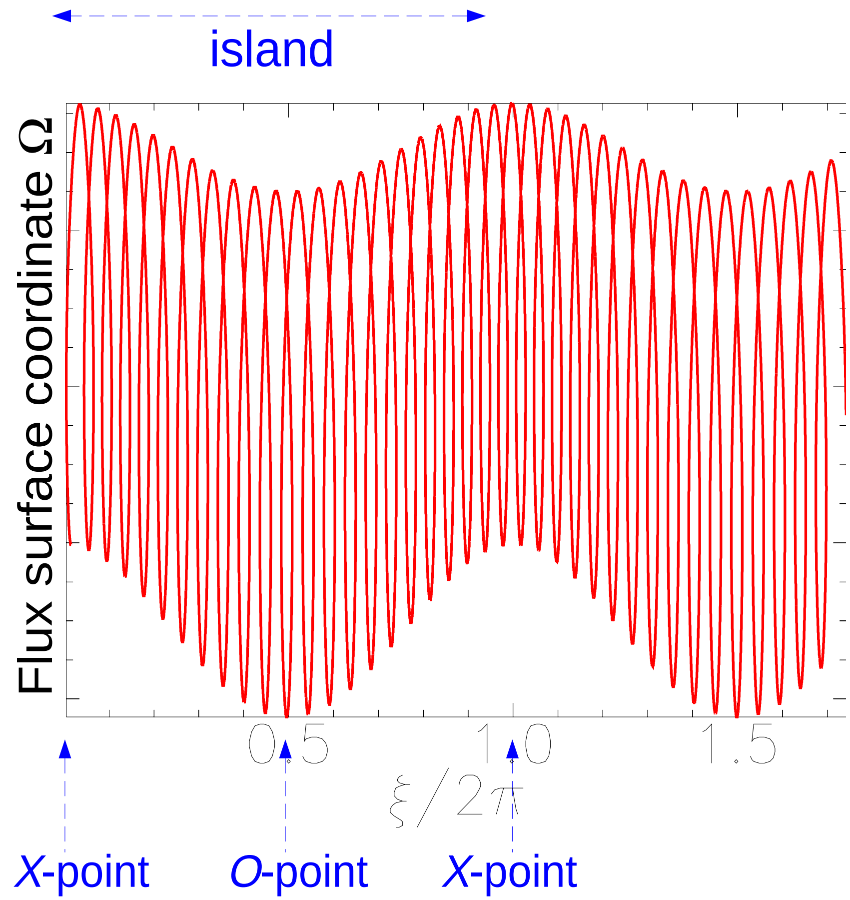




Perpendicular current vs. ω: low frequencies

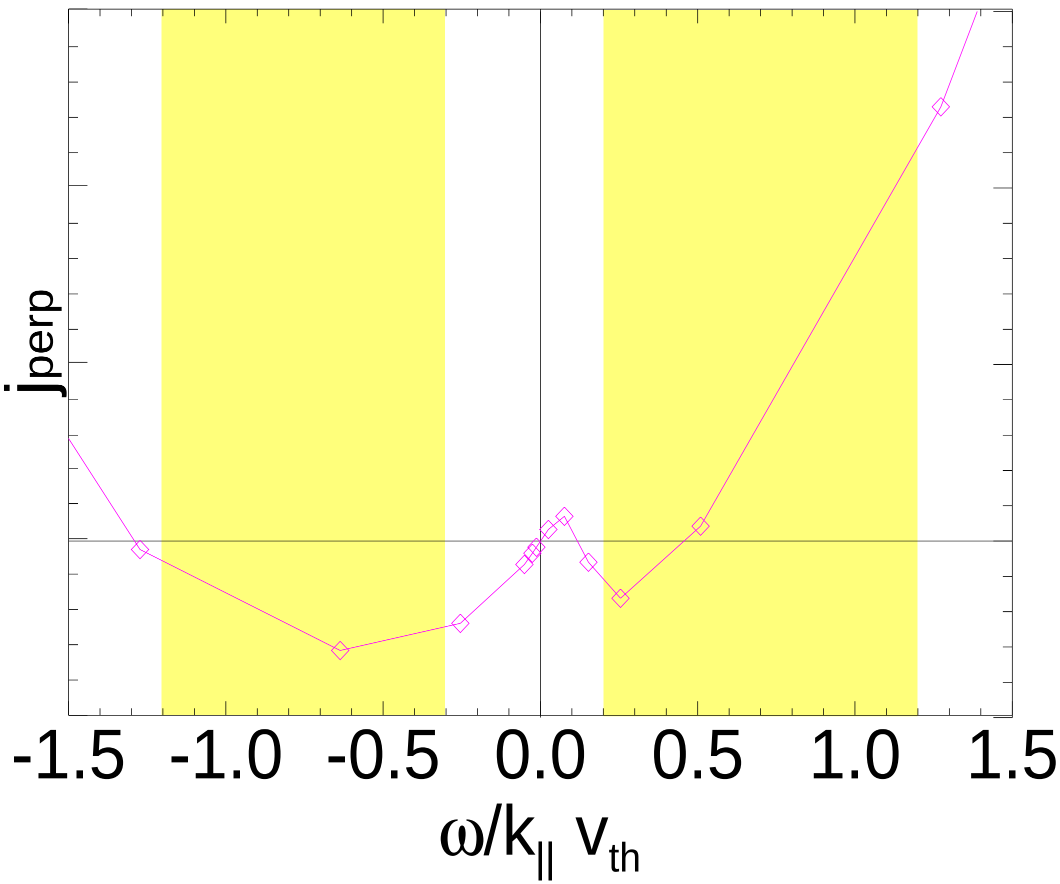


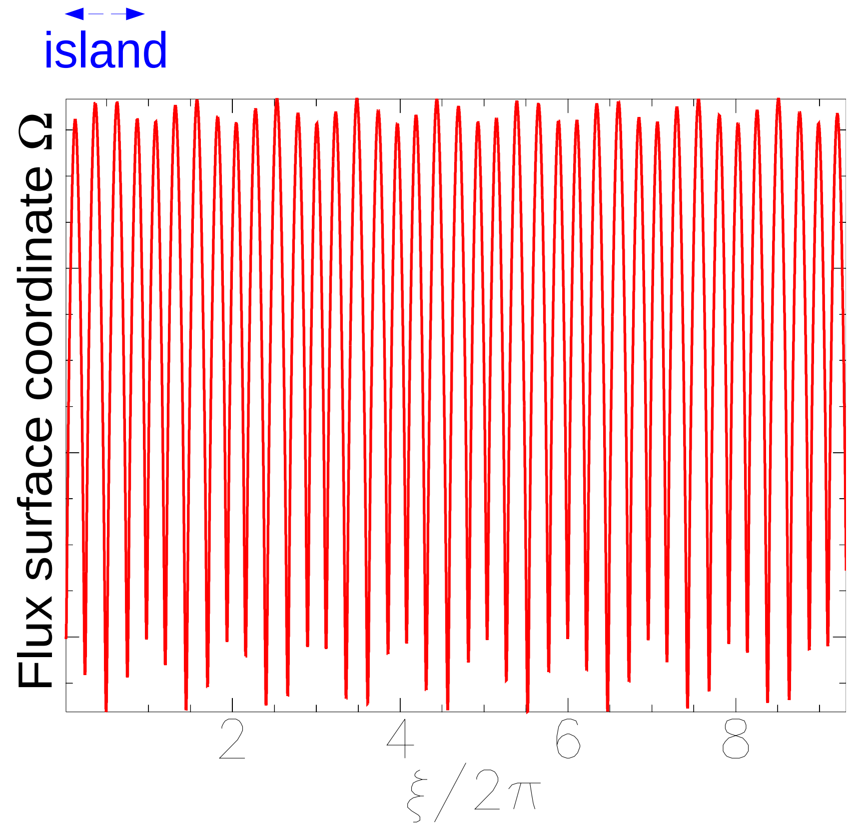
- Toroidal precession compensated by the $E \times B$ drift (island frame) when $2\omega_{tp} \approx \omega$



- Deviation from the perturbed magneitc surfaces due to a combination of magnetic and electric drift (dominates over the polarisation drift)

Perpendicular current vs. ω: transition to higher frequencies

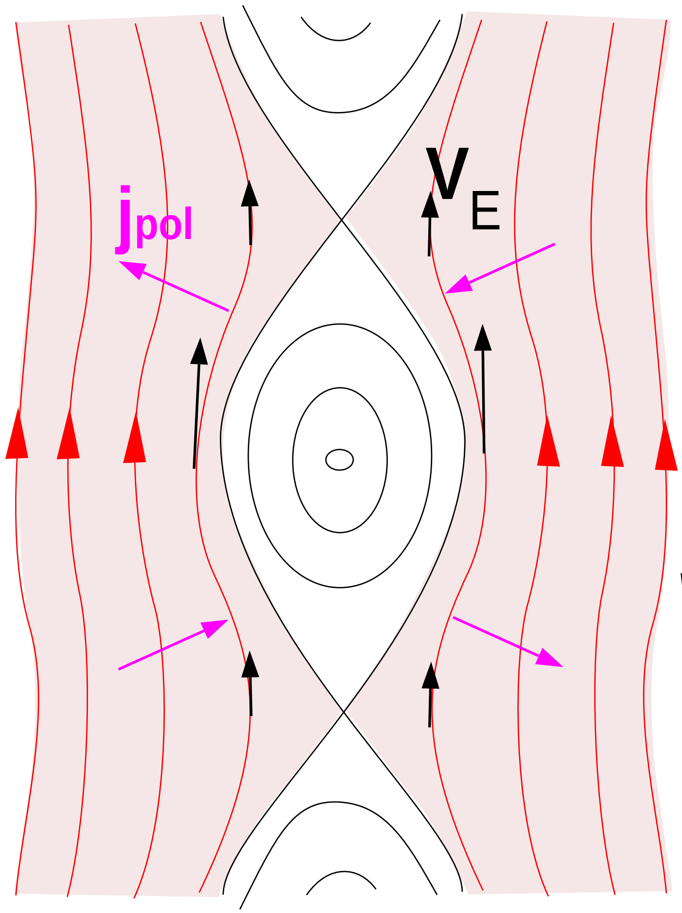


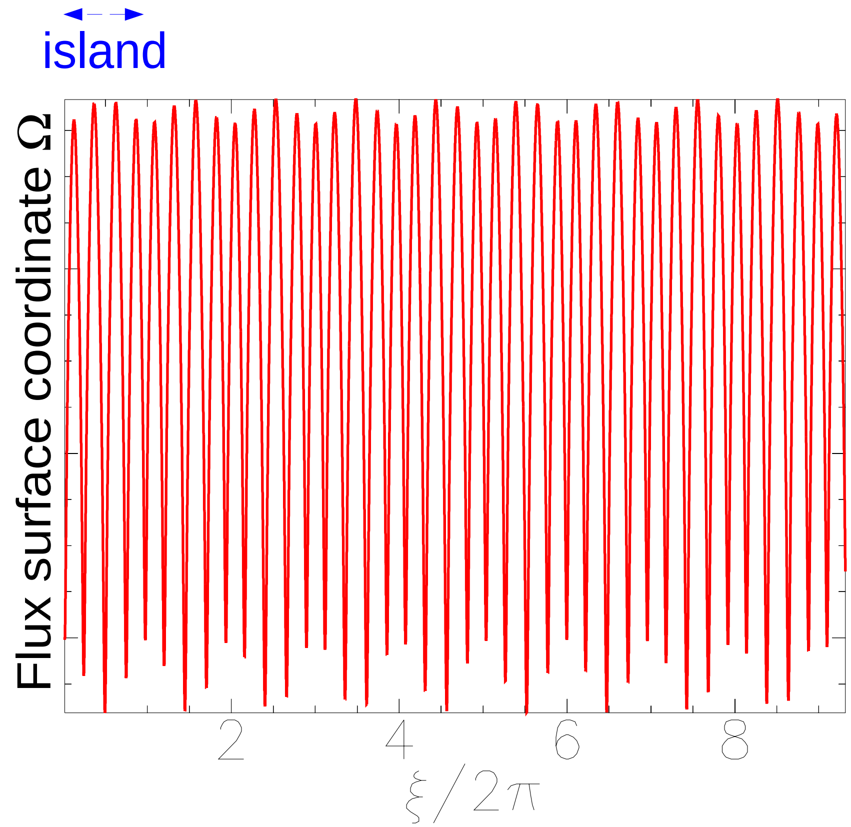


- Transition to higher frequencies
$\rightarrow$ toroidal precession less and less important

- Bounce motion along the perturbed surfaces $\rightarrow$ polarisation current sets on

Perpendicular current vs. ω: transition to higher frequencies





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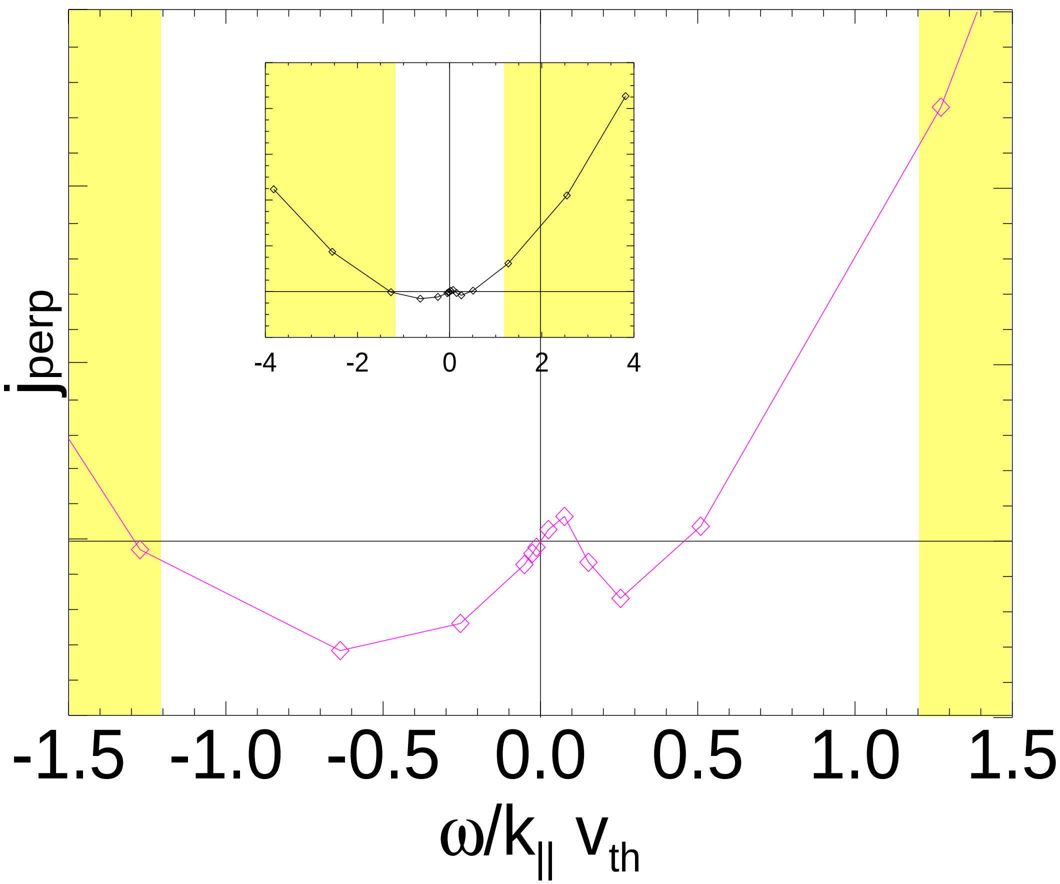
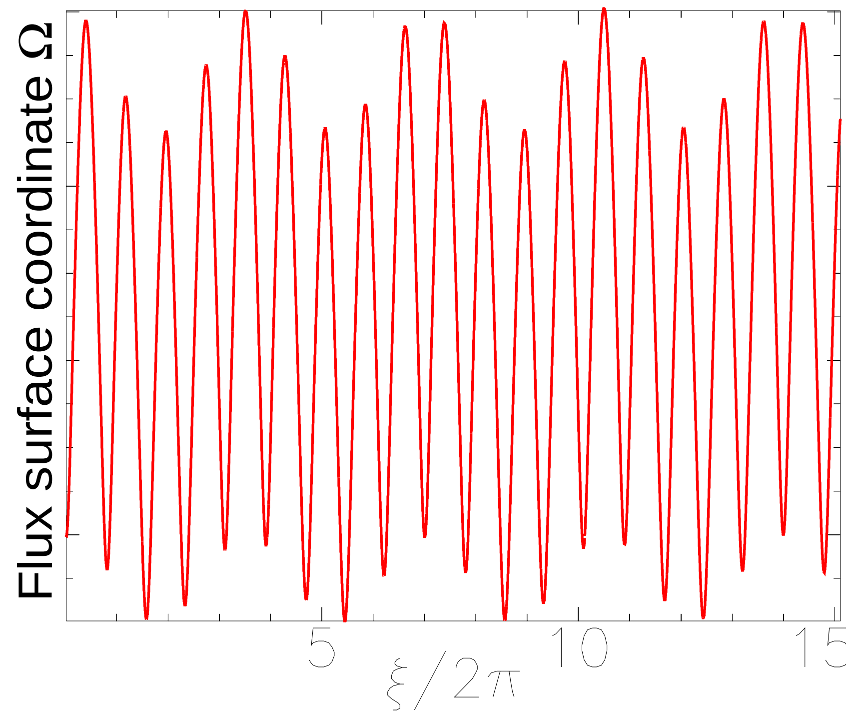
- Bounce motion along the perturbed surfaces $\rightarrow$ polarisation current sets on (cf. fluid picture)

Perpendicular current vs. ω: the "standard" polarisation current





- High frequencies: polarisation current close to "fluid" behaviour $\rightarrow$ quadratic dependence on ω found

- Superposition of island motion and bounce motion $\rightarrow$ current reduction due to slower particles

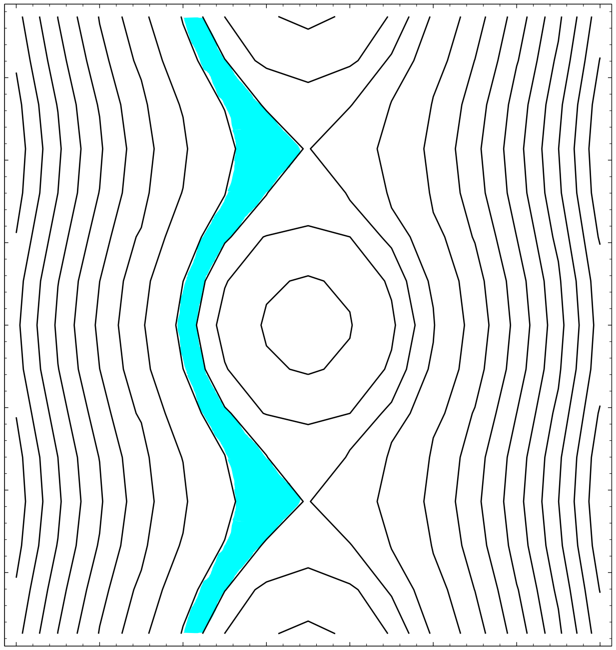
Conclusions

- Complete kinetic description of the ion motion necessary in order to obtain a reliable calculation of the island polarisation current (of the bootstrap current as well)

- Polarisation-current sign influenced by competition between electric and magnetic drift

- Polarisation current strongly reduced for small island widths (comparable to banana width)

Current profiles from the drift kinetic equation



- Macroscopic quantities as moments of the distribution function

- Flux surface averages → cells

$$\langle A \rangle = \lim_{\delta\Omega \to 0} \frac{\int A \mathrm{d}^3\mathbf{r}}{\int \mathrm{d}^3\mathbf{r}} \Rightarrow \frac{1}{n} \left\langle \int A \delta f \, \mathrm{d}^3\mathbf{v} \right\rangle \simeq \frac{\int_{\Omega-\delta\Omega}^{\Omega+\delta\Omega} A \delta f \, \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{v}}{\int_{\Omega-\delta\Omega}^{\Omega+\delta\Omega} f_0 \, \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{v}}$$

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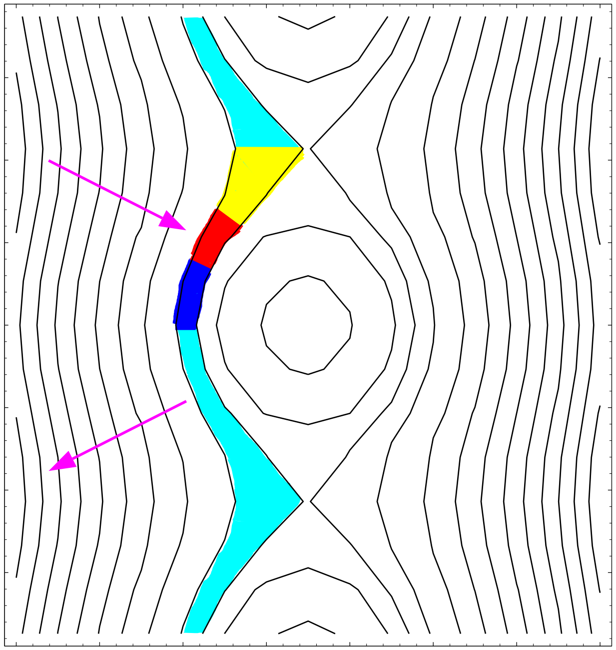

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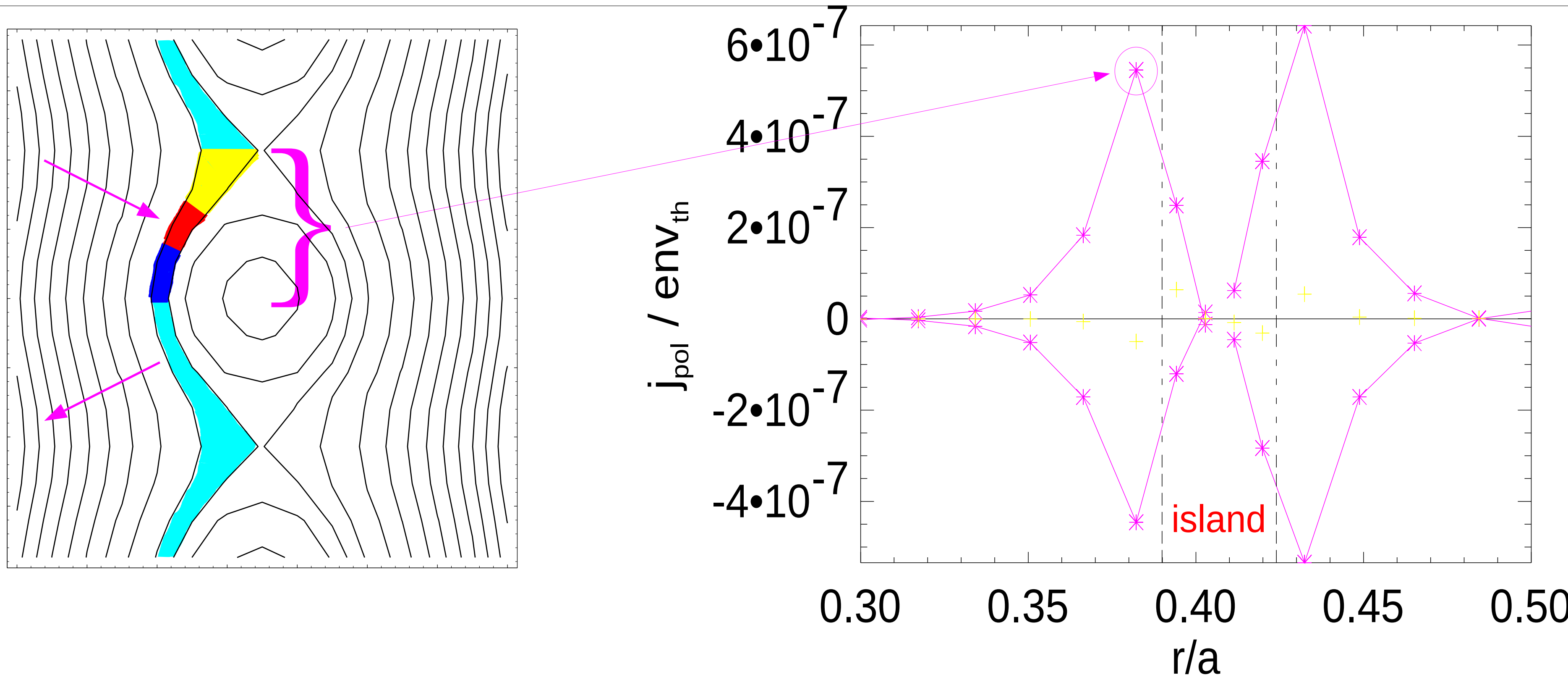

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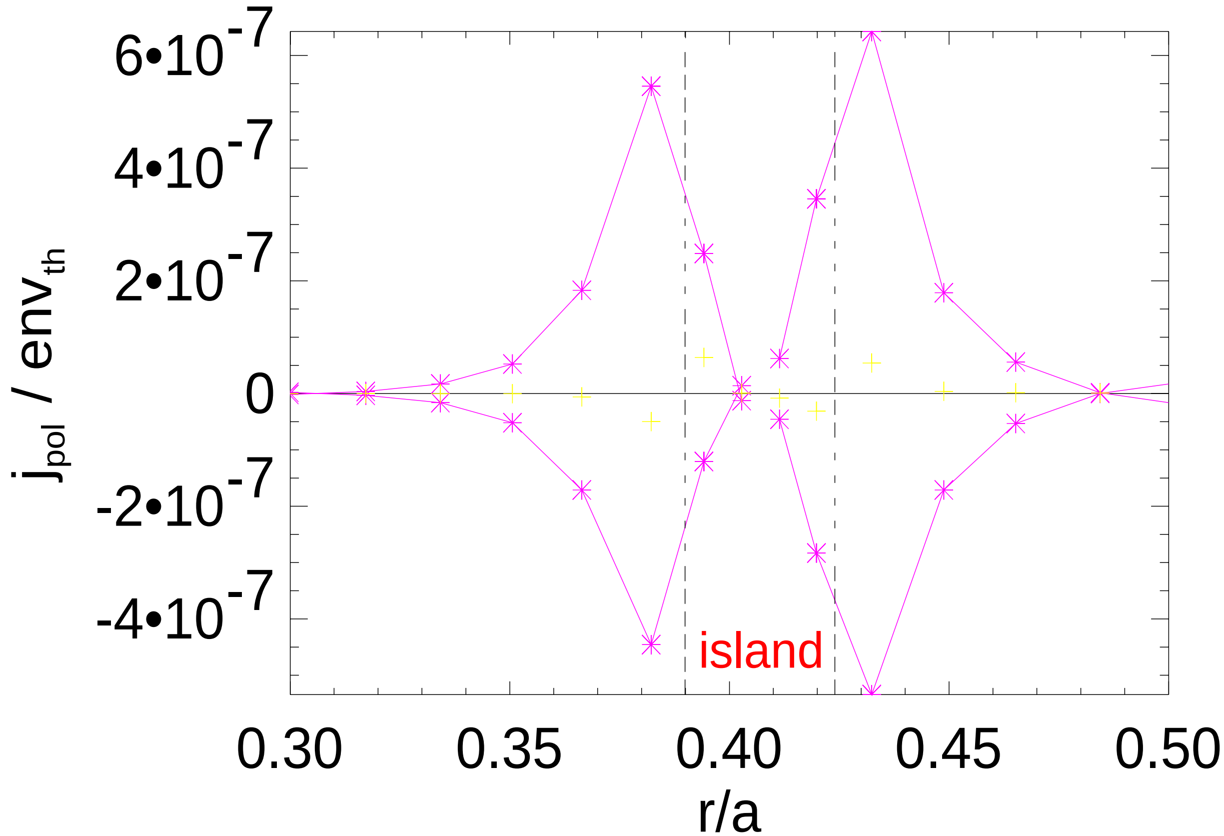
- "Radial" profiles of the polarisation current available!
- Binning in velocity space also possible

Stabilising or destabilising?

- Contribution of the polarisation current to Rutherford equation

$$\Delta'_{\mathrm{pol}} \propto \int_{-1}^{\infty} \mathrm{d}\Omega \oint \frac{\mathrm{d}\xi \cos\xi}{\sqrt{\cos\xi + \Omega}}\, j_{\|}$$

- Parallel current obtained from integration of $\nabla_{\|} j_{\|} = -\nabla_{\perp} \cdot j_{\perp}$ ($j_{\perp}$-profile numerically available)



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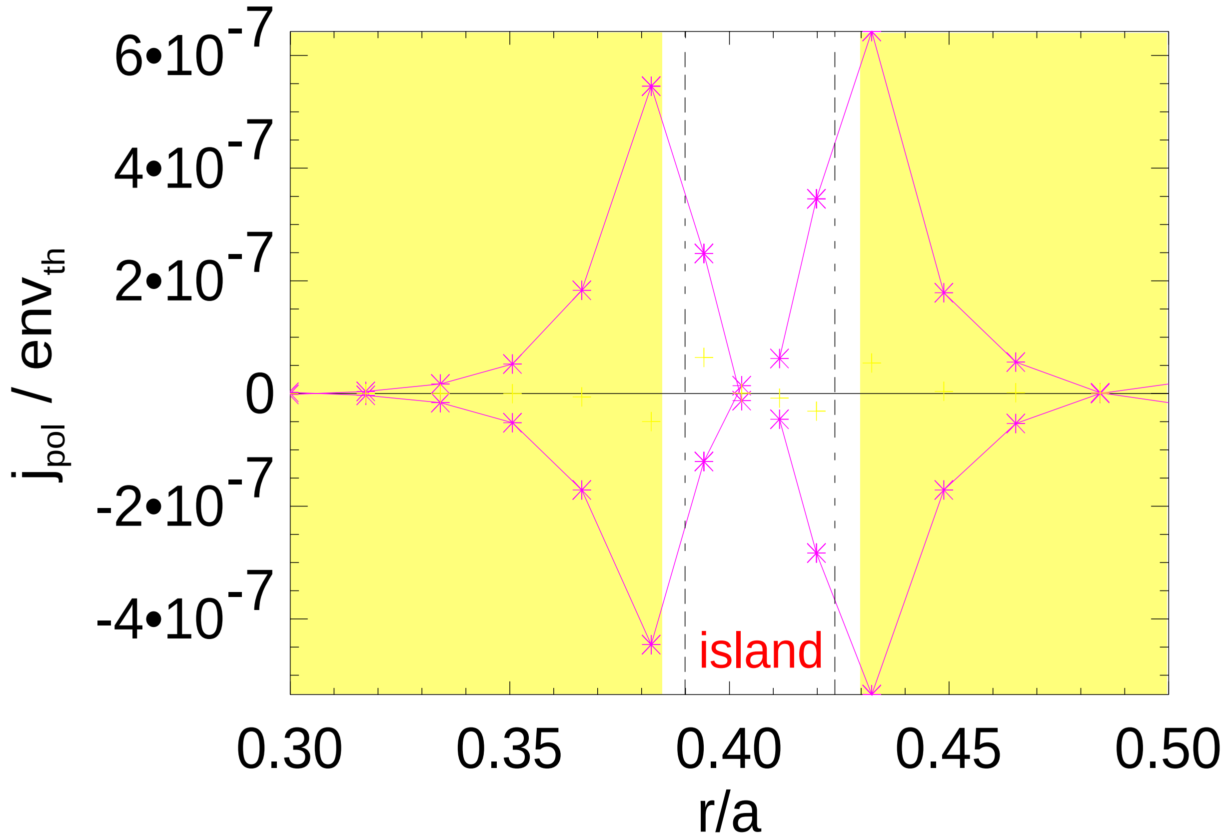

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⇨ **stabilising** if the separatrix is excluded from the radial integration

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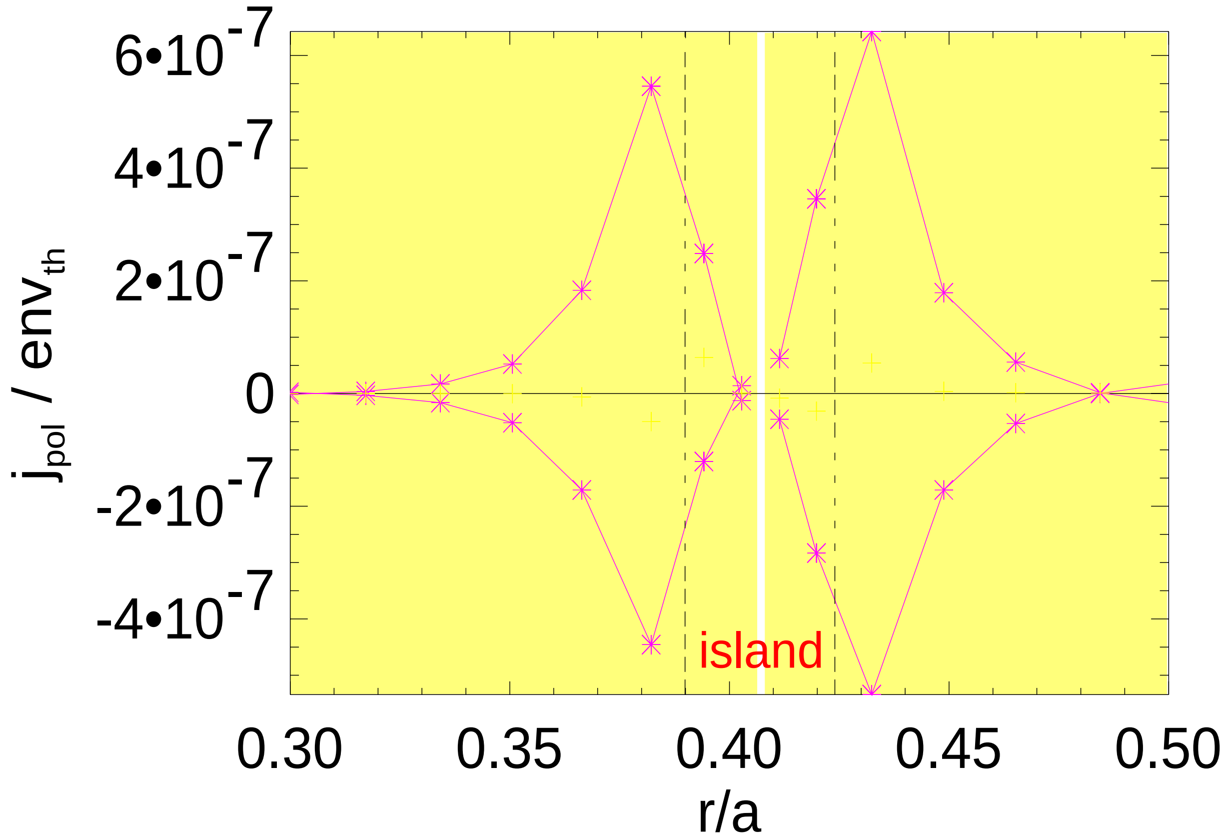

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⇨ **destabilising** if it is included in the radial integration

[Waelbroeck and Fitzpatrick, PRL 1997]