
Advances in Comprehensive Gyrokinetic Simulations of Transport in Tokamaks

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GYRO is the most physically comprehensive global nonlinear gyrokinetic code

- GYRO is an Eulerian (continuum) [not PIC (Lagrangian)] 5D gyrokinetic code. Development began in 1999 and design milestones completed in early 2003.
- GYRO operates **both** as a **cyclic flux tube code** (flat profiles at vanishing Δ^*) or near full radius slice **0 boundary condition global code** (profile variation with finite Δ^*).
- GYRO is very versatile with all the **physics needed for physically realistic simulations** of all transport channels in a tokamak **core** plasma:
 - toroidal ITG mode physics
 - real tokamak geometry
 - trapped and passing electrons
 - finite beta
 - e-i pitch angle collisions
 - equilibrium sheared ExB and toroidal rotation profiles
 - inputs real experimental profiles**== “full physics”**

Some limitations and fundamentals about GYRO, global gyrokinetic codes, and broken gyroBohm scaling at finite Δ_*

- Standard gyrokinetic & Poisson-Ampere equations **assume leading order in small Δ_* only.**
Typically Δ_* and $\tilde{n}/n_0 < O(1\%)$ [see Frieman & Chen 1982; Antonsen & Lane 1980]
- GYRO consistently retains only this **leading order in Δ_*** : Thus in GYRO,
all finite Δ_* effect breaking gyroBohm scaling result from profile variation alone.
- GYRO will likely be inaccurate in a steep gradient pedestal where $\tilde{n}/n_0 > O(10\%)$.
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- GYRO will likely be inaccurate in a steep gradient pedestal where $\tilde{n}/n_0 > O(10\%)$.
Consistently relaxing these **small Δ_*** approximations likely require 6D Vlasov equations.
- **For flat profiles and all Δ_*** , global GYRO with 0 BC gives the same gyroBohm scaled transport as the flux-tube GYRO with cyclic BC, i.e. the boundary conditions are “benign”.
- **With profile variation**, going from **small to very small Δ_*** , the local diffusivity from a global code approaches the the gyroBohm scaled diffusivity from a local flux tube code.

The mystery since the first 1990 IAEA DIII-D experiments:
tokamak transport is always gyroBohm sized, but
Why are L-modes Bohm scaled at such small experimental values of β* ?
_Why are H-modes (and Ohmic) discharges typically gyroBohm scaled ?

Outline:

- Realistic simulation of Bohm scaled DIII-D L-mode β_* pair
- First steps to a steady state core gyrokinetic transport code
- Mechanisms breaking gyroBohm scaling: local versus non-local

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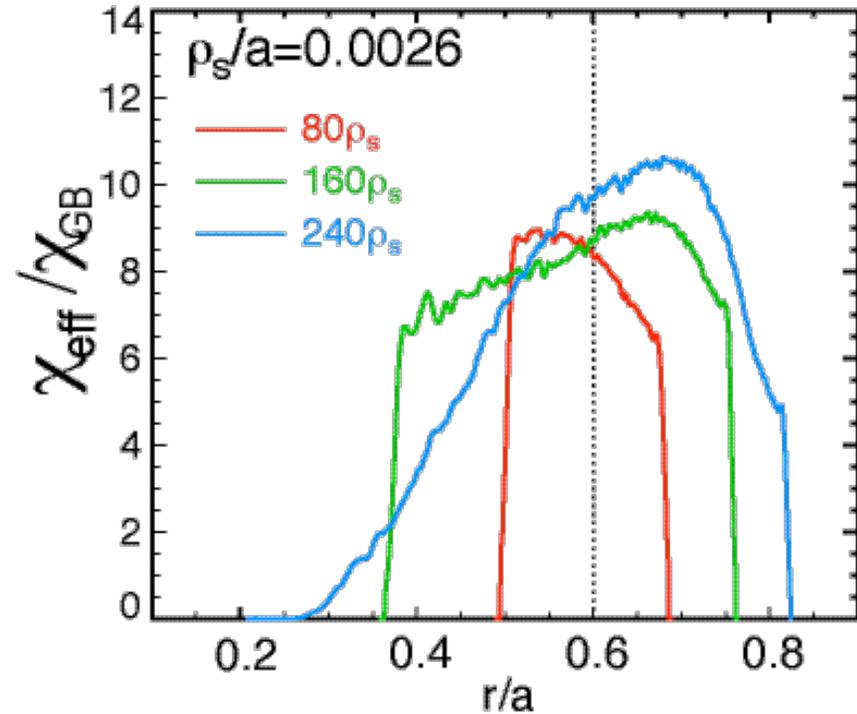
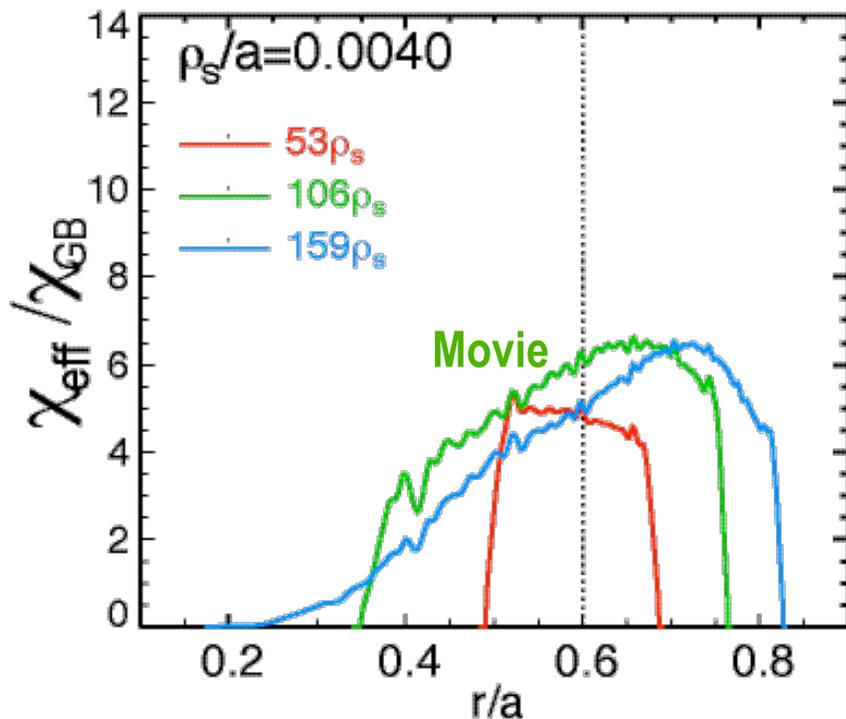
- Realistic simulation of Bohm scaled DIII-D L-mode β_* pair
- First steps to a steady state core gyrokinetic transport code
- Mechanisms breaking gyroBohm scaling: local versus non-local
- Heuristic theory/model for non-local transport
- Corrugated flux surface “equilibrium” profiles and local dynamos
- Neoclassical flows embedded in turbulence
- ExB shear quenching of electron driven turbulence
- Plasma and impurity flow pinches

Realistic simulation of Bohm scaled DIII-D L-mode α_* pair

Transport at slice center nearly independent of slice size suggests non-local effects are small and not responsible for Bohm scaling here.

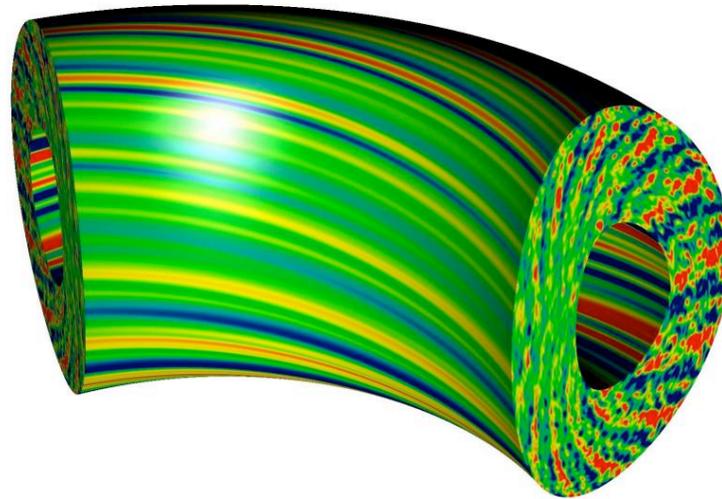
$$\rho_{eff} = (\rho_i + \rho_e)/2 \quad \rho_{GB} = (c_s/a)\rho_s^2 \quad \alpha_* \equiv \rho_s/a \quad \rho_s \equiv c_s/\rho_i$$

- Bohm scaling is $\rho/\rho_{GB} \quad 1/\alpha_* \quad \alpha_*(1.05T)/\alpha_*(2.10T) = 0.0040/0.0026 \approx 8/5$



- “Full physics” except $\sqrt{m_i/m_e} = 20$ not 60 which is difficult for large slice sizes here.
- Transport 4x experimental levels for 20 but 2x experimental for physical $\sqrt{m_i/m_e} = 60$.

$B=1.05T$ $\beta_* = 0.004$ medium slice simulation movie



- Movie is DIII-D L-mode discharge #101381 with “full physics” except $\sqrt{m_i/m_e} = 20$
- Halo is separatrix
- Stagnation point at $r/a=0.6$ corresponds to movie reference frame with $E_r=0$
- GYRO movie downloads at <http://Fusion.gat.com/comp/parallel/>

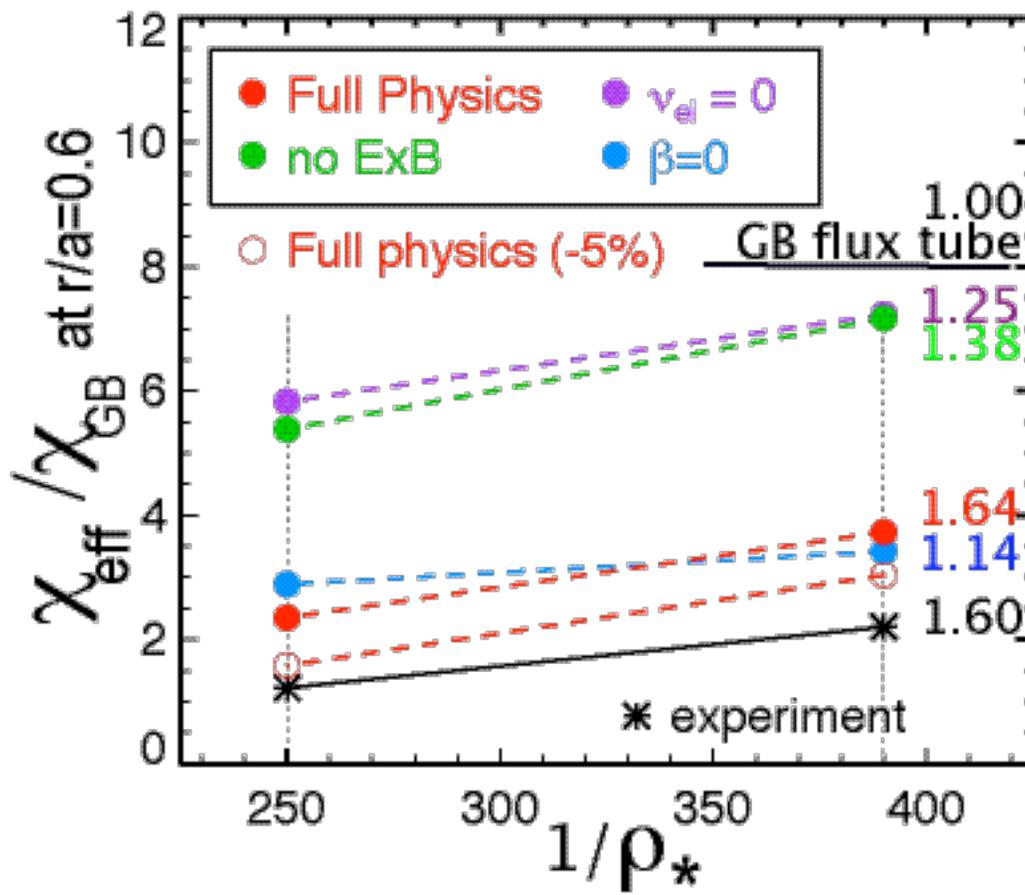
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Code: GYRO

Authors: Jeff Candy and Ron Waltz

Get within 2x of experimental diffusivity with Bohm scaling only if retaining “Full Physics” (and real electron mass).

$$\sqrt{m_i/m_e} = 20 \square 60 \quad \chi_{eff} / \chi_{GB} \text{ ratio: } 8/5 \square 3.6/2.2 = 1.64$$



$$\chi_{eff} / \chi_{GB} (\text{flux tube no ExB}) = 8$$

- Turning off **ExB**, or **e-i collisions**, or **beta** drops from **Bohm ratio 1.64** toward gyroBohm scaling ratio 1.00

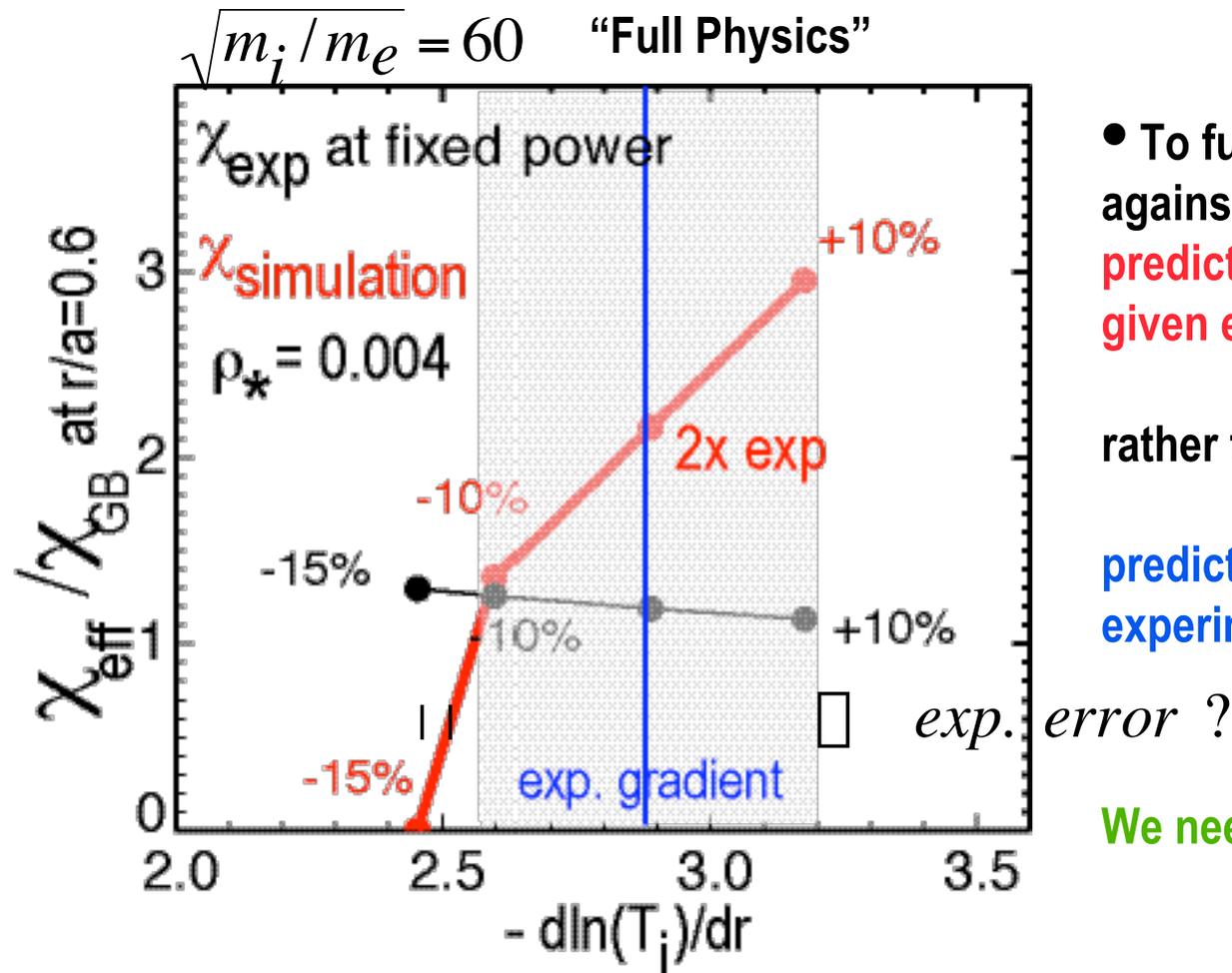
All reduce $\chi_{net} = \chi_{max} \square \square_E$

- Decreasing the ion temperature gradient by **5%** gets to 1.5X exp.

The DIII-D core is very stiff:

Transport power flow is very sensitive to temperature gradient.

A small 10% reduction brings simulation into agreement with experiment



- To fully test a stiff transport model against experiment, one must predict the temperature profile from given experimental power flows;

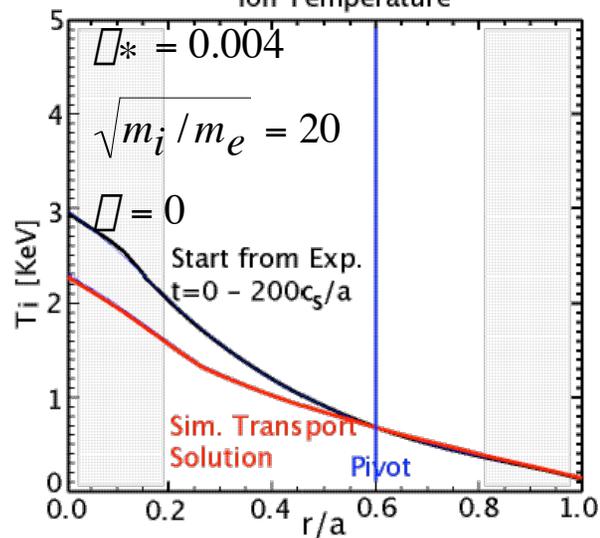
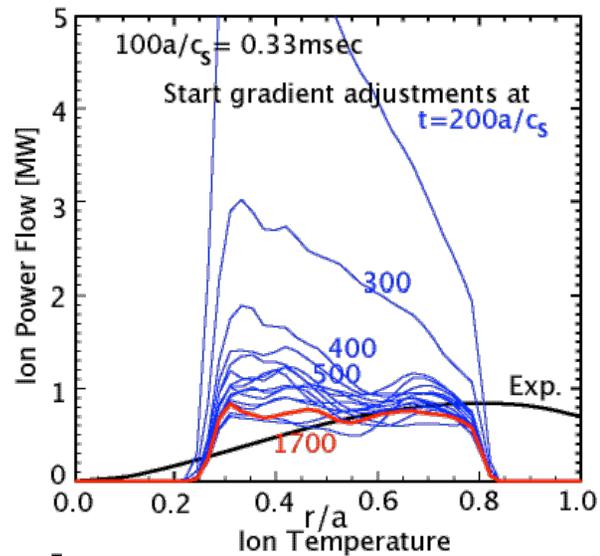
rather than

- predict the power flow from the given experimental temperature profile.

We need a gyrokinetic transport code !

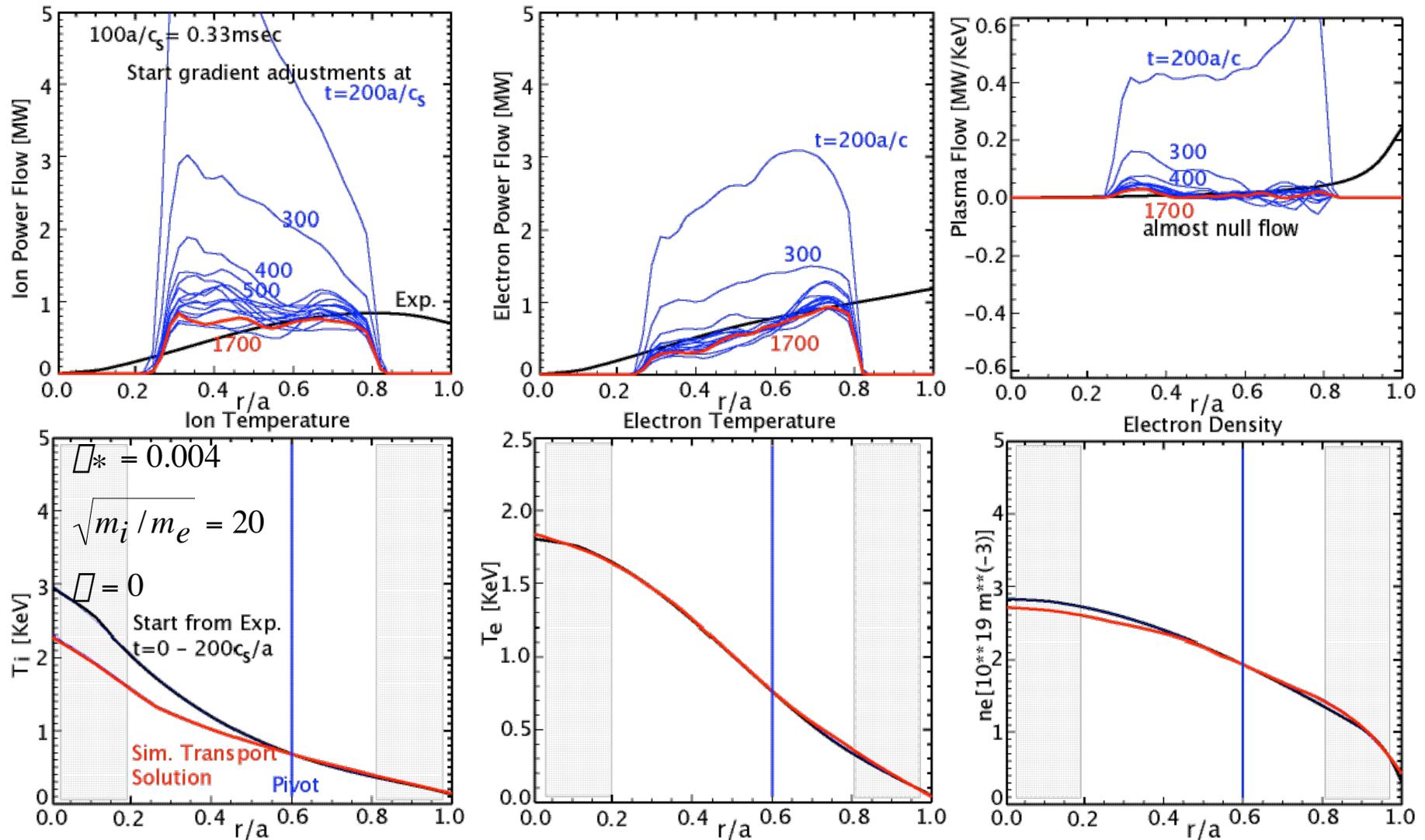
First steps to a steady state core gyrokinetic transport code

At every radius, use a negative feedback adjustment in the gradients to drive the simulation flows to the experimental flows: $\Delta grad_{sim} / grad_{exp} = \Delta \Delta_{FB} [flow_{sim} - flow_{exp}] / flow_{exp}$
 Integrate up and down the simulation gradient from a fixed pivot to get the simulation profile.



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- Within the next two years, we hope to have a gyrokinetic steady state core transport simulation with self-consistent fusion power for an ITER scale burning plasma **given an H-mode pedestal boundary condition.**

Local mechanisms breaking gyroBohm scaling

- The profile of maximum growth rate γ_{\max} and ExB shear rate ω_E are exquisitely matched, so Bohm scaling is not due to experimental dissimilarity.

- Mode phase velocity shear rates $\omega_s = |\omega_* \pm \omega_E|$ can have a ω_* dependence.

When $\omega_s / \gamma_{\max} \sim \omega_* / \omega \sim O(1)$ there can be a significant stabilizing effect breaking gyroBohm scaling: $\gamma \sim \gamma_{GB} (1 - \omega_s / \gamma_{\max}) \sim \gamma_{GB} (1 - \omega_* / \omega) \sim \gamma_{GB} (\omega / \omega_*)$

Contrast with empirical Mixed Bohm/gyroBohm Model: $\gamma \sim \gamma_{Bohm} \omega (1 + \omega_* / \omega)$

- This local shearing mechanism was previously discussed [Waltz et al IAEA 1996] and illustrated with ITG-ae simulations [Waltz, Candy & Rosenbluth APS 2001 and IAEA 2002]

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However for the DIII-D L-mode ω_* pair realistic “full physics” simulation

- The ω_* dependence of the velocity shearing rate ω_s seems too weak, to quantitatively account for the Bohm scaling.

Non-local mechanisms breaking gyroBohm scaling

- Linear toroidal coupling between singular surfaces gives a non-local mechanism which **drains** turbulence from **unstable** regions and **spreads** turbulence into **stable** (or less unstable regions).

This non-local mechanism **breaks gyroBohm scaling toward Bohm in the unstable** regions:

$$\chi \sim \chi_{GB} (1 - \chi_* / \chi)$$

and **toward super-gyroBohm in the stable (or less unstable)** regions:

$$\chi \sim \chi_{GB} (1 + \chi_* / \chi)$$

- This non-local mechanism was found to give a small breaking effect in ITG-ae simulation **using realistic profiles with a central stable core** but could account for transport in linearly stable central core plasmas or ITBs [Waltz, Candy & Rosenbluth APS 2001 and IAEA 2002]
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However the DIII-D L-mode χ_* pair realistic “full physics” simulation had no stable edge and the stable core was not covered by the smaller simulation slices...yet Bohm persisted.

- Thus it seems unlikely to us that this non-local mechanism gives a quantitative explanation.
- Nevertheless **non-local effects** can be important, and we have developed a heuristic model for including them into gyroBohm local transport models like GLF23 [Waltz et al IAEA 1996]

Summary and Conclusions

- **GYRO is the most physically comprehensive gyrokinetic code with the “full physics” presently thought required to realistically simulate core tokamak transport in all channels.**
- **Bohm scaled transport in DIII-D L-modes in matched $\bar{\omega}_*$ -pairs has been obtained.**
- **ExB shear appears to be important in obtaining Bohm scaling in DIII-D L-mode.**
GYRO simulations track the gyroBohm scaled DIII-D H-modes which have larger $\bar{\omega}_*$ but much lower ExB shear rates.
- **Core transport is stiff. Simulated core power and plasma flows can be matched with to experimental flows with small (10%) adjustments in the ion temperature gradients.**
- **We demonstrated the first steps to a core gyrokinetic transport code.**
- **We have investigated both local velocity shear and non-local drainage mechanisms for breaking gyroBohm scaling.**
.....But neither appear to give quantitatively accurate accounts of the Bohm scaling in the realistic DIII-D L-mode simulations.

Other recent conclusions (discussed in paper & poster)

- Simulations demonstrated a heuristic theory (model) for incorporating non-local transport in local gyroBohm models (like GLF23). The theory is based on linear toroidal coupling of singular surfaces and the partial formation of global modes broken up by zonal flows.

- The model locally averages local growth rates over a length L

$$L/a \approx 1/[T_{glob} \chi_E] \approx (\chi_*/\hat{s})(a/R)/[\chi_{loc}^{net}/(c_s/a)]^{1/2}$$

- GYRO has shown that the equilibrium flux-surface averaged radial gradient (and divergence) profiles are highly corrugated on the scale of a few ion gyroradii and tied to singular surfaces.

“Dynamo” current density corrugations may affect tearing stability but do not produce much net current beyond the neoclassical current voltage relation

- Ion-ion collisions and the neoclassical curvature drift drive was added to GYRO.

Even with large orbit effects (finite χ_*), the “cross-talk” between turbulent and neoclassical flows appears to be weak.

- The EXB shear quench rule: $\chi_E/\chi_{max} = 2 \pm 0.5$ (s- χ geometry) has been extended from ITG to trapped electron mode (TEM) turbulence.

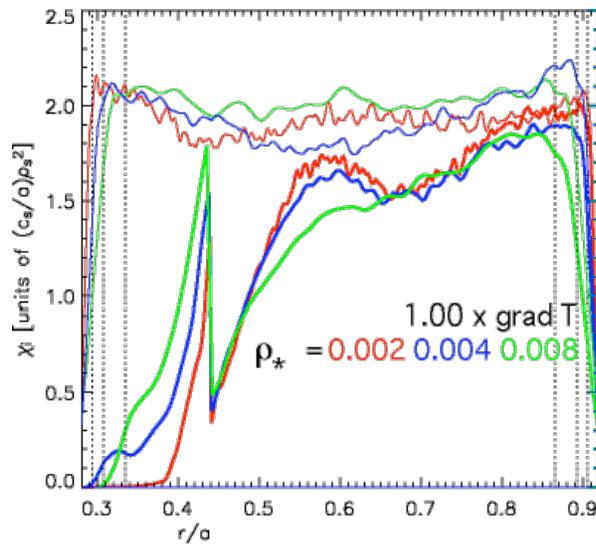
For purely toroidal rotation, a large parallel shear rate $\chi_p = (Rq/r)\chi_E$ can drive up $\chi_{max}(\chi_p)$ faster than χ_E and no quench may result.

- Multi-species have been added to GYRO. Thermally pinched plasma flows and impurity pinches are found to be in good agreement with the 1996 GLF23 model.

Simulations with non-local transport

- Simplified ITG-ae simulations with flat and piecewise flat profiles.

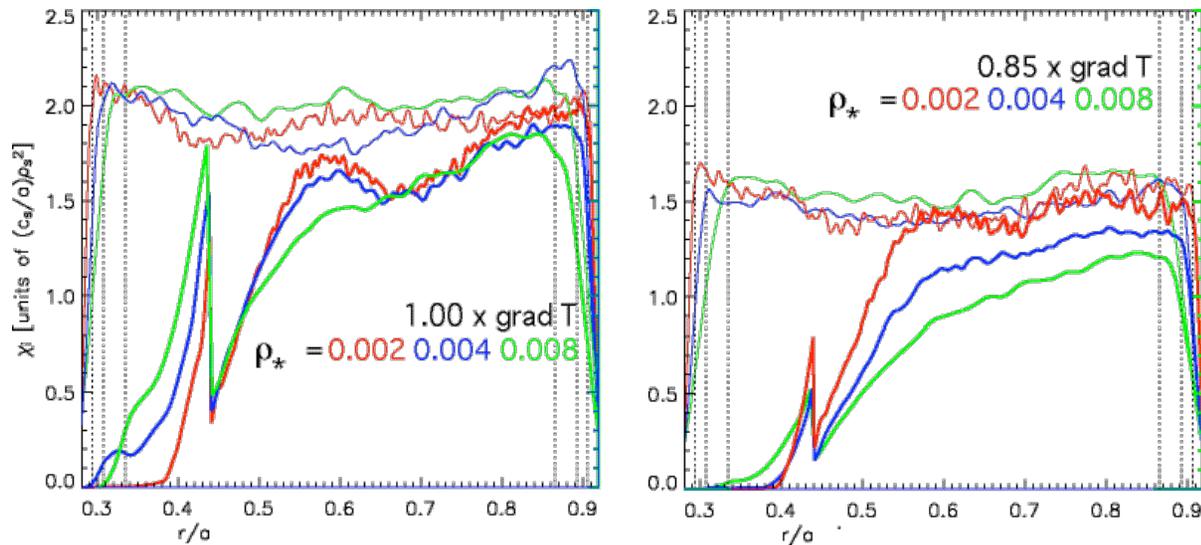
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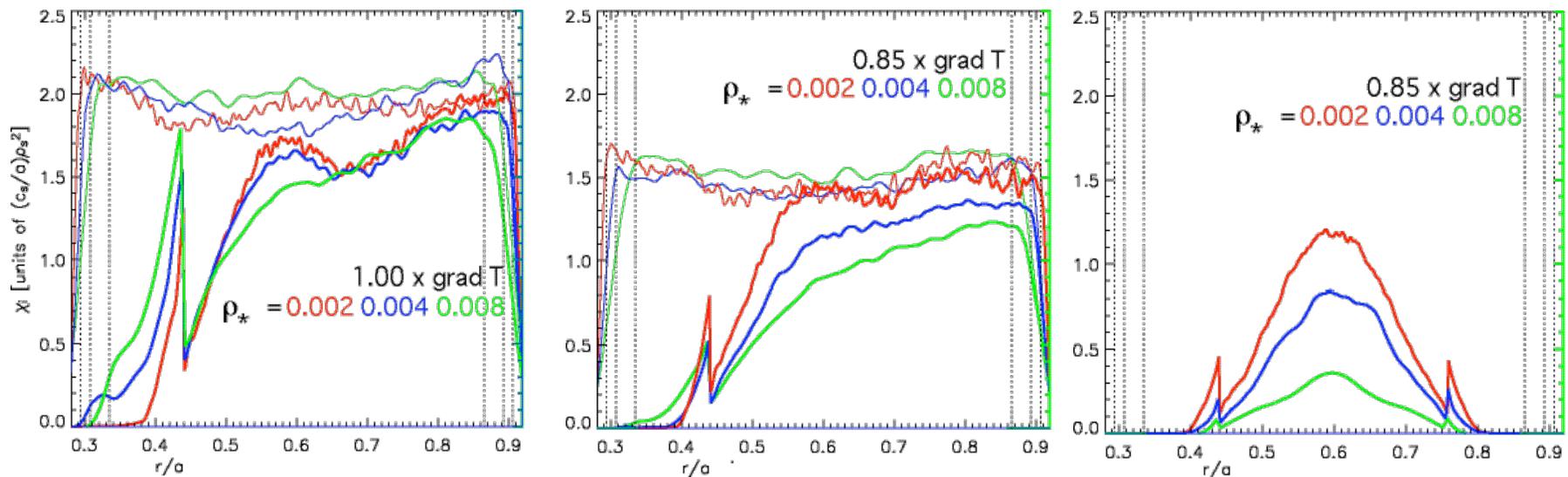


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.....easier to get Bohm near threshold

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- Lower driving rate increases the “non-local connection length” L
.....easier to get Bohm scaling nearer to threshold
- Adding an **unphysical stable edge** region to the right, doubles the drainage from the unstable region and makes Bohm scaling easier to get

Heuristic theory (model) for non-local transport

- Local transport models (like GLF23) use quasilinear theory with the gyroBohm spectral weight $I_k(x) = [(e |\tilde{\chi}_k| / T_e) / \kappa_*]^2 \kappa_{k_loc}^{net} / [c_s / a]$
- We propose to replace the **local (net) growth rate** $\kappa_{k_loc}^{net}$ by a **non-local growth rate** from radially averaging over the whole plasma (with a localized weight) :

$$\kappa_k^{net}(x) = \int_0^a dx' / [2L(x')] \kappa_{k_loc}^{net}(x') \exp[-\kappa |x - x'| / L(x')]$$

- BC:** $\kappa_{k_loc}^{net}(x' < 0) = \kappa_{k_loc}^{net}(0)$ and $\kappa_{k_loc}^{net}(x' > a) = \kappa_{k_loc}^{net}(a)$
- If $L/a \ll 1$ then $\kappa_k(x) \approx \kappa_{k_glob}^{net}(x)$ the **global eigenmode growth rate**. Both $\kappa_{k_loc}^{net}$ and $\kappa_{k_glob}^{net}$ are independent of κ_* .

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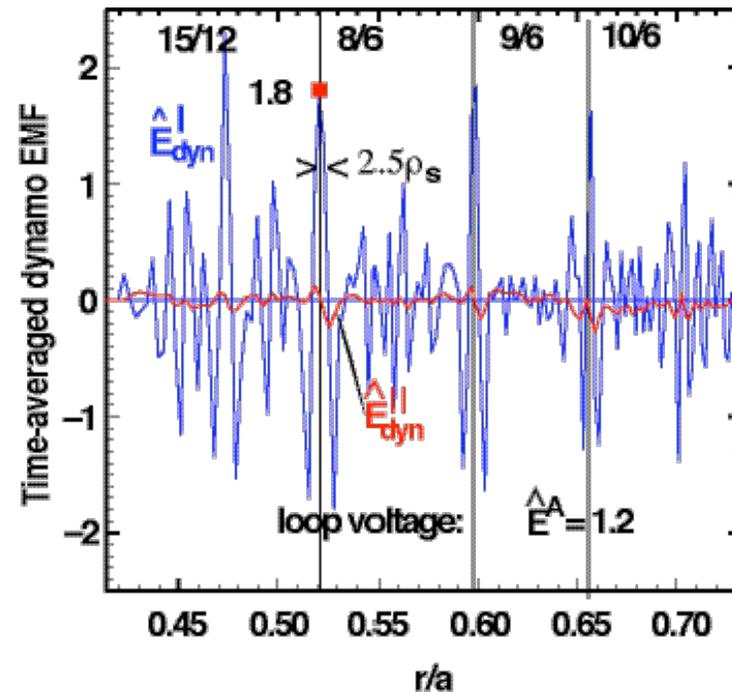
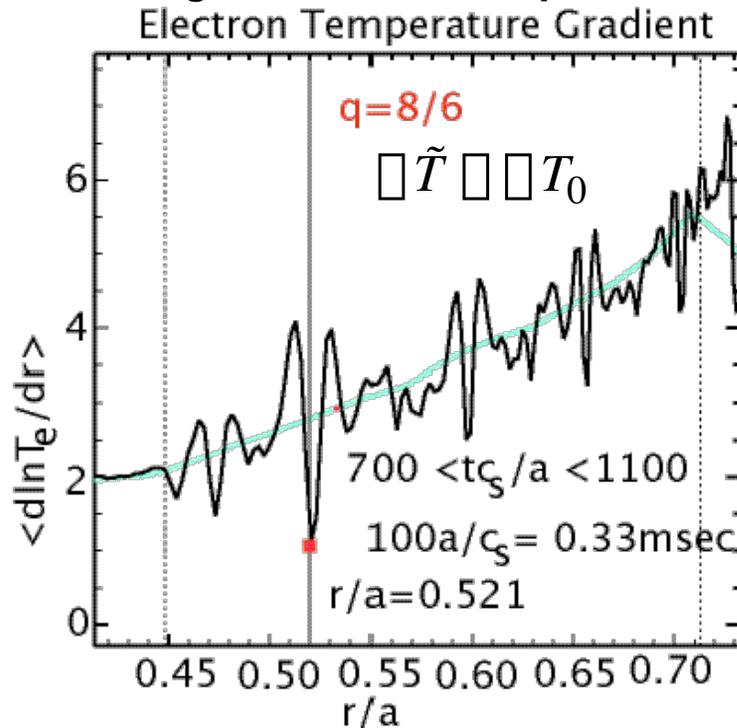
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- **But** for small Δ_* plasmas, **global eigenmodes take a long time to form** T_{glob} , and if the turbulence is more strongly driven, the **zonal flows** have larger ExB shearing rates $\tilde{\omega}_E$ and more **quickly break up the global modes before they fully form**.
- Our heuristic theory argues $L/a \approx 1 / [T_{glob} \tilde{\omega}_E] \approx (\Delta_* / \hat{s})(a/R) / [\Delta_{loc}^{net} / (c_s / a)]^{1/2}$ which explains the piecewise flat profile simulations.

Profile Corrugations and dynamos

- “Equilibrium” flux-surface averaged radial profiles of gradients (and divergences) are highly corrugated on the scale of a few ion gyroradii.
- The corrugations are “components of zonal flows” tied to low order singular surfaces

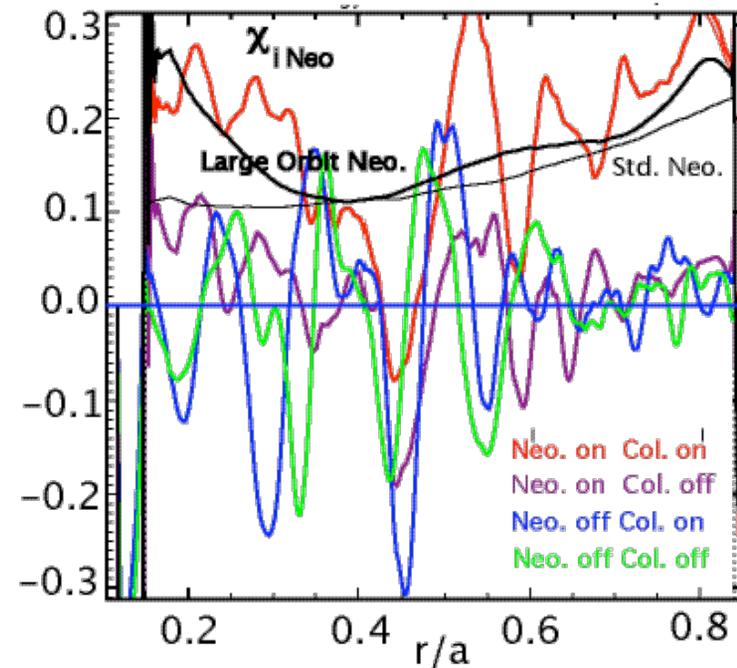
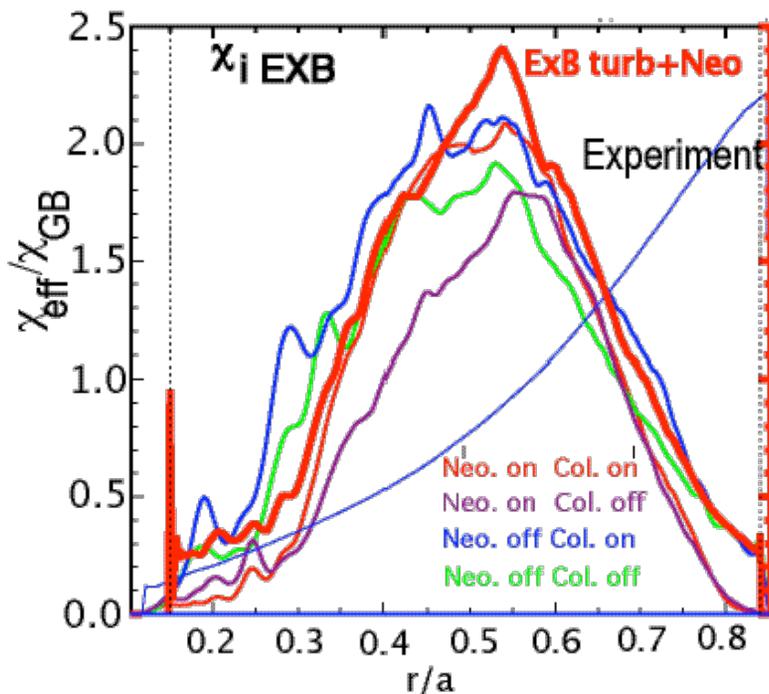


- There are current density corrugations $j_{dyn} / j_0 = E_{dyn} / E^A$ responding to dynamo EMF's which drive little net current but may effect tearing mode stability.
- EMF(I) is from magnetic flutter (like the MHD \square - dynamo) and EMF(II) is electrostatic.

“Cross-talk” between neoclassical and turbulent flows

- GYRO now has a conserving Krook ion-ion collisions and a neoclassical driver term to $n=0$ equation giving the $O(\epsilon_*)$ poloidal deviation to the equilibrium. Neoclassical flows result.
- At $\epsilon_* \rightarrow 0$ there is no “cross talk” between turbulent and neoclassical flows....they are additive.

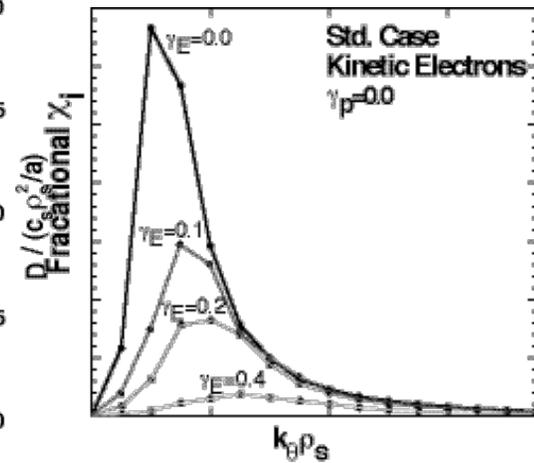
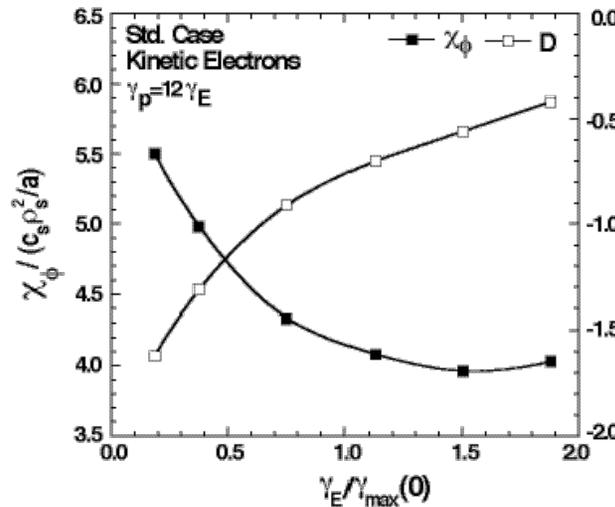
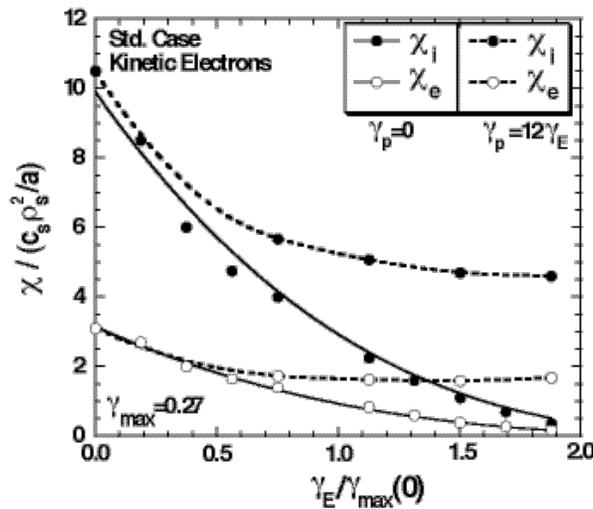
For finite - ϵ_* the cross talk appears to be small if any.



- The “turbulent” neoclassical flows $\langle (\tilde{f} + ze\tilde{n}/T)_{n=0} \rangle_{Dx}$ are radial averages of the large orbit neoclassical flows (which deviate somewhat from the standard $\epsilon_* \rightarrow 0$ flows).
- Plasma flow (not shown) is ambipolar (with conserving Krook and non-adiabatic part only)

EXB shear quenching and trapped electron driven modes

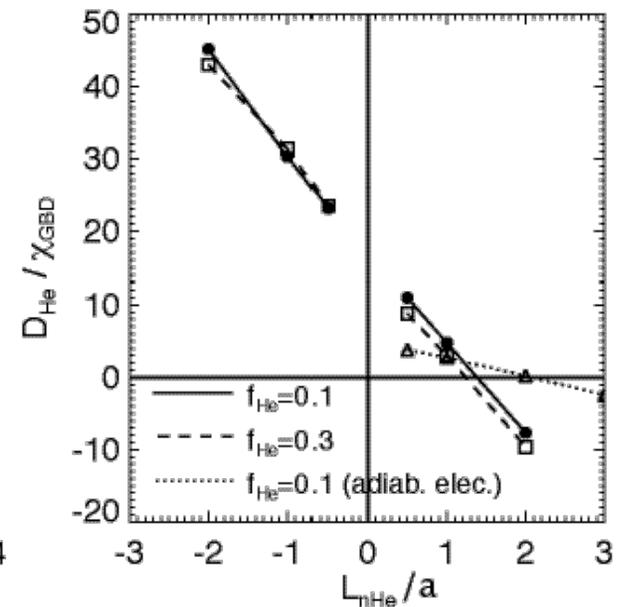
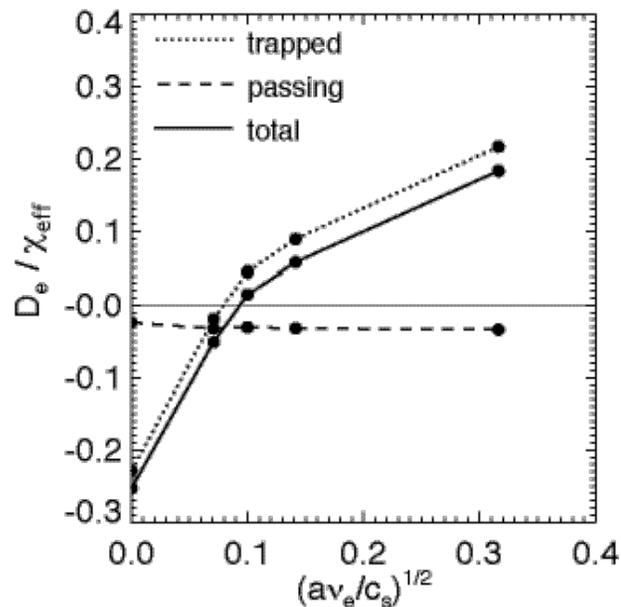
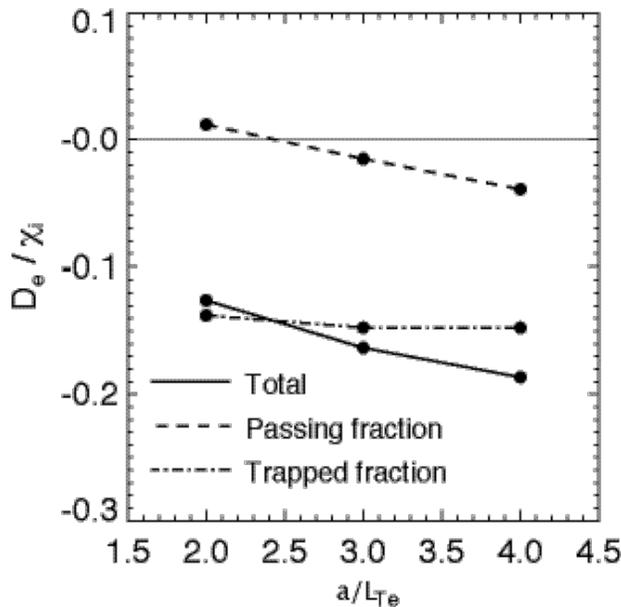
- The EXB shear quench rule: $\bar{\nu}_E / \bar{\nu}_{\max} = 2 \pm 0.5$ (s- \square geometry) has been extended from ITG to trapped electron mode (TEM) turbulence: GA-Std case $a/L_T = 3, a/L_n = 1, R/a = 3, q = 2, s = 1$
- For purely toroidal rotation, a large parallel shear rate $\bar{\nu}_p = (Rq/r)\bar{\nu}_E$ can drive up $\bar{\nu}_{\max}(\bar{\nu}_p)$ faster than $\bar{\nu}_E$ and no quench may result.



$\bar{\nu}_p = \bar{\nu}_E = 0$	L_n / L_T	$[\bar{\nu}_{\max}, \bar{\nu}]$	$\bar{\nu}_i / \bar{\nu}_{GB}$	$\bar{\nu}_e / \bar{\nu}_{GB}$	$D / \bar{\nu}_{GB}$
ITG-ae	3/1	0.13, -0.31	3.54 ± 0.37		
ITG/(TEM)	3/1	0.27, -0.33	10.7 ± 2.6	3.2 ± 0.6	-1.9 ± 0.5
ITG/TEM	2/2	0.28, -0.014	11.0 ± 2.2	11.3 ± 2.2	4.0 ± 0.8
TEM	2/3	0.43, +0.029	23.9 ± 3.7	27.7 ± 4.5	5.6 ± 0.9

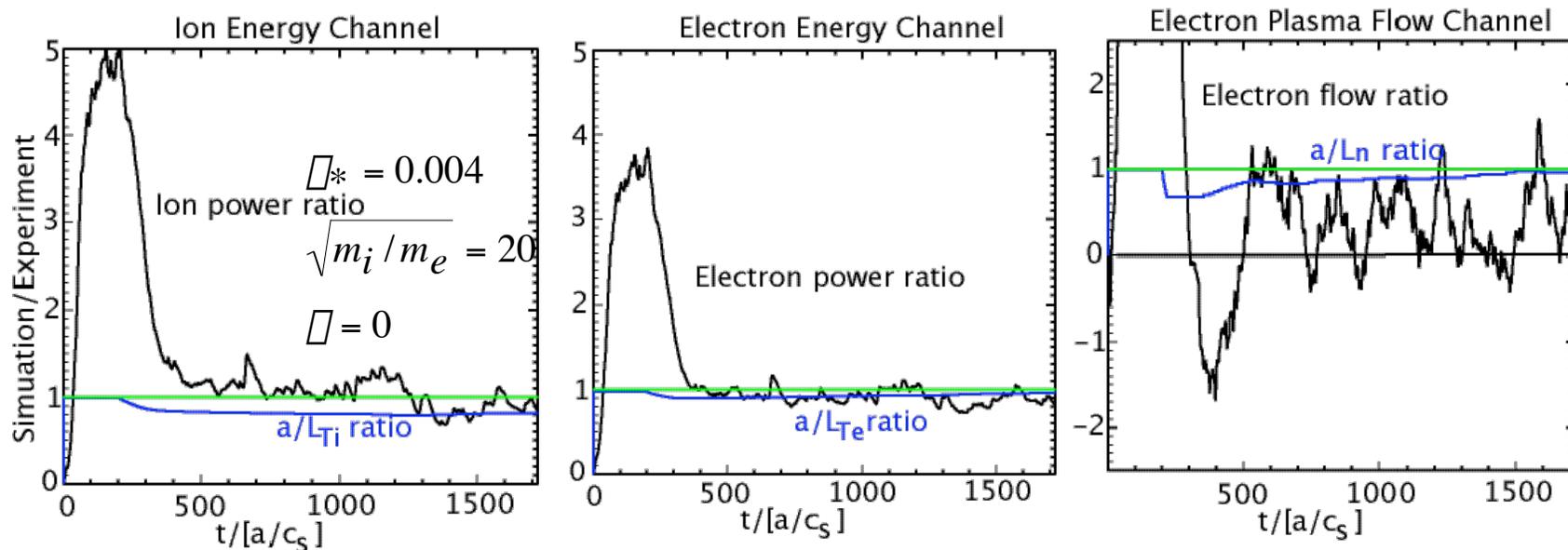
Plasma and impurity flow pinches

- Multi-species have been added to GYRO. Thermally pinched plasma flows and impurity pinches are found to be in good agreement with the 1996 GLF23 model.
- Pure plasma thermal pinch driven by trapped electrons.
- Adding e-i collisions move null flow point to higher $\square_e = L_n / L_{Te}$
The thermal pinch allows some peaking of null flow cores.
- Helium transport described by D-V paradigm: $\square_{He} = D_{He} n_{He} / L_{He} = n_{He} [D^d_{He} / L_{He} \square V_{He}]$



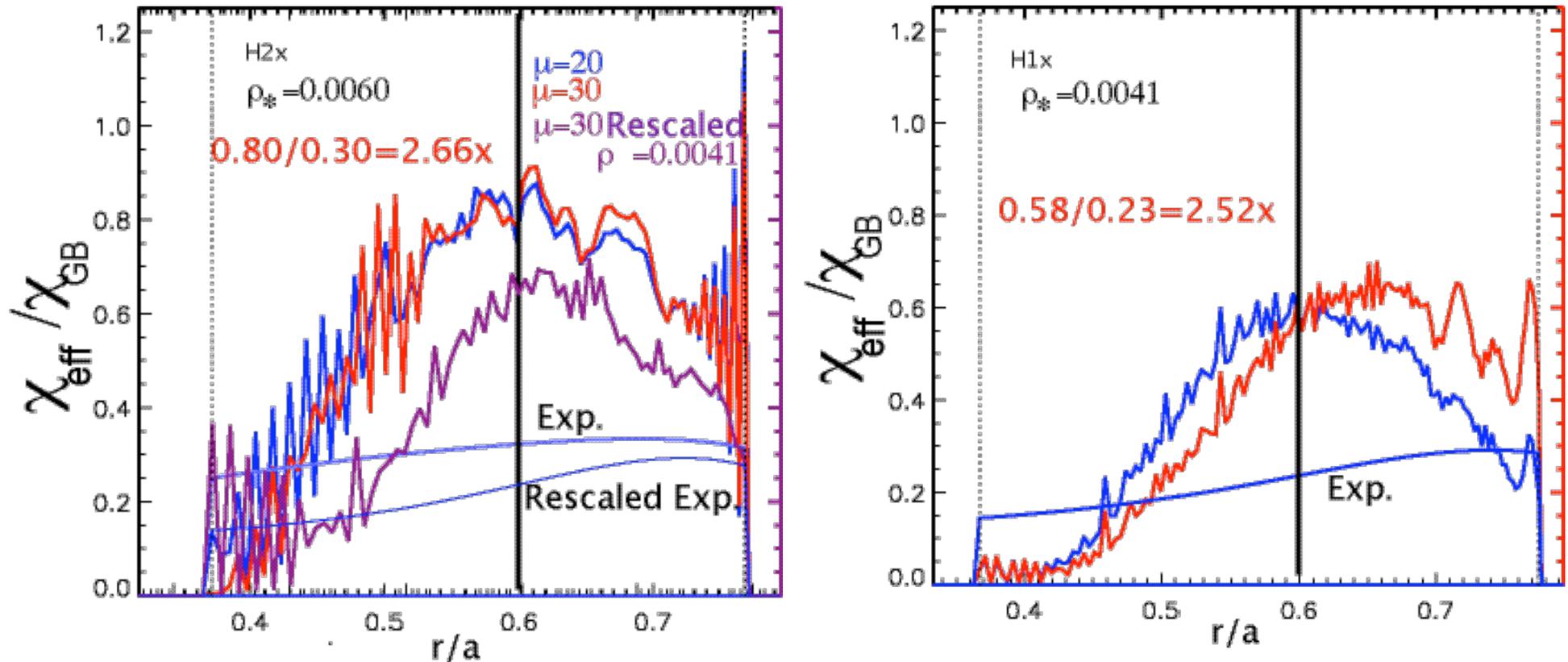
GA Std. Case: $a/L_T = 3, a/L_n = 1, R/a = 3, q = 2, s = 1$

- Because of core stiffness [a/L_{Ti}] and near null plasma flow from a thermal pinch [a/L_n vs a/L_{Te}], adjustments to match experimental flows are small.....10-20% here.



- Within the next two years, we hope to have a gyrokinetic steady state core transport simulation with self-consistent fusion power for an ITER scale burning plasma **given an H-mode pedestal boundary condition.**
- This will operate as a master prototypical transport code driving many parallel slave gyrokinetic gyroBohm flux-tube simulations at each radius.

Preliminary gyroBohm DIIID H-mode simulations



- The similarity condition on the best DIIID H-mode are rather poor.
- The “gyroBohm” scaling actually tends to super-gyroBohm $r/a < 0.7$
- Nevertheless the GYRO simulation tracks the experiment
- Rescaling from $\rho_* = 0.0041$ data to $\rho_* = 0.0060$ we get exactly gyroBohm.