6. STUDIES ON NON-INDUCTIVE CURRENT DRIVE

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6.1. INTRODUCTION

This project has had essentially two main lines of research:

- Studies on lower-hybrid (LH) current drive (CD)
- Development of kinetic codes to solve the Fokker-Planck (FP) equation.

6.2. STUDIES ON LHCD

6.2.1. Introduction

Within this line of research, the following main activities were carried out in 2000:

- Development of a fully 3-D (toroidicity plus ripple) tokamak ray-tracing code for LH wave propagation;
- Study of diffraction effects on LH wave propagation.

6.2.2. Fully 3-D tokamak equilibrium code

The effects of toroidal and magnetic ripple on the propagation of LH waves for tokamaks with weak toroidicity (e.g., TRIAM-1M) have been addressed, where a large aspect ratio approximation for the toroidal magnetic equilibrium with ripple perturbations based on Shafranov geometry, and consistent up to the first order in ripple and toroidal perturbations, was used. To extend this to tokamaks with high toroidicity, analytical and numerical studies of a more general non-axisymetric toroidal magnetic equilibrium with magnetic ripple, based on Hamada flux coordinates, have been started.

Hamada coordinates form a particular magnetic-flux coordinate system useful for non-axisymetric MHD calculations because they provide important simplifications in several contexts. In particular, each contravariant field component becomes a flux label even when there is no toroidal symmetry, and the current density and magnetic field lines are straight, whereas the Jacobian of the coordinate transformation equals one. This simplifies the analytical equilibrium formulation, although the non-axisymetric equivalent of the Grad-Shafranov equation will still be too complex. This is a nontrivial computational problem and only approximate iteratively numerically solutions can be found. However, by coupling these solutions to a LH ray-tracing code, the effects of magnetic ripple on the propagation of LH waves can be numerically determined for arbitrary toroidal geometries. This enables one to assess if magnetic ripple is sufficient to explain the LH spectral gap problem on several tokamaks, and in future will be used for comparisons with the theoretical analysis on LH wave propagation in toroidal plasmas with magnetic ripple. This is an extension of the work already done in cylindrical geometry.

6.2.3. Diffraction effects on LHCD

As for the analysis of diffraction effects on the propagation of LH waves, the problem of the calculation of the field intensity near caustics in the beam-tracing formalism has started to be addressed, but has not yet been solved. In addition, a more realistic axisymmetric magnetic equilibrium for diffraction and ray-tracing calculations has been developed (including elipticity and triangularity, besides the Shafranov shift), which leads to a method of solution for the Grad-Shafranov equation that is somewhat different from the standard one.

6.3. KINETIC CODES TO SOLVE THE FOKKER-PLANCK EQUATION

6.3.1. Introduction

Within this line of research, the main activities carried out in 2000 regarded the improvement of kinetic codes for RF heating and current drive (H&CD) using both:

- Finite differences;
- Transition probabilities.

6.3.2. Finite-difference codes for FP equations

For a wide class of problems (describing the evolution of systems far from equilibrium), it is possible to reduce its detailed and often cumbersome description down to a much more manageable

formulation, in which FP equations play a fundamental role. However, excluding a few particular situations, the solutions to such FP equations are not available in an analytically closed form, being numerical and approximate solutions the only resource when proper modelling is required. Therefore, the ability to efficiently provide accurate and stable numerical solutions to general FP equations, being them linear or not, establishes itself as a key factor in several areas of knowledge, ranging from Physics and Engineering to Economics and Social sciences. Even restricting the scope of applications to plasma physics, FP equations are fundamental when modelling Coulombian collisional transport, being it applied to astrophysical or thermonuclear (either magnetically or inercially confined) plasmas, and general wave-plasma interactions, resulting either in wave damping and energy absorption or in wave scattering, just to name a few examples.

Since the numerical solution of FP equations is, in most applications, only a part of a more complex modelling scheme, the major goal of any solving method is to reduce as far as possible the cost in computing effort, without compromising, in the process, accuracy or stability. As the computational cost relates directly to the overall size of the mesh used to obtain a discrete version of the FP equation in question, one obvious way to reduce such cost is to employ non-uniform meshes, distributing the mesh points location according to the real needs of the problem being addressed. The guiding for this rearrangement should be established by minimising the errors introduced when turning a continuous FP equation into a series of discrete relations, increasing the accuracy for a given number of mesh points or, conversely, reducing the mesh size whilst maintaining the accuracy level.

Within the most widespread approaches to numerically solve FP equations, some finite-difference schemes may be singled out, which are intrinsically particle conserving and able to preserve the non-negative character of their solutions, as well as exact representations of equilibrium state. When dealing with non-linear problems, this type of scheme may be extended to non-uniform grids, not only by means of proper weighting coefficients, but also by redefining the concept of quasi-equilibrium solutions. Indeed, this is the key factor in order to maintain a rather general approach when solving FP equations, without resorting to particular properties of certain FP operators as, for instance, their ability to be rewritten in certain convenient form.

The above considerations were successfully applied to the solution of two representative non-linear problems in plasma physics, that is, the Coulomb scattering of a like-charged particle population and the Compton scattering of photons, in frequency space, due to the interaction with a electronic population in thermal equilibrium. The weakly non-linear nature of the first is related with the integral form of the friction and diffusion coefficients, which in turn makes the computational effort to scale roughly as $N^{2.4}$, with N being the number of cells in the mesh. The second is strongly non-linear and represents a severe test to the proposed scheme.

Since the Coulomb scattering problem is energy conservative by definition, one suitable process to evaluate the numerical solution quality is to compute the magnitude of energy dissipation effects, introduced by the discretization procedure. It may be shown that such energy dissipation effects are quantified by an integral functional involving the mesh size function $\Delta(x)$ and some of its derivatives, the minimisation of such functions being used as a guide in designing suitable non-uniform grids. In fact, even a rough design consisting in the juxtaposing of two uniform grids of different mesh sizes (a finer one for the low velocity zone and a larger one for the high velocity zone), is able to reduce energy dissipation when compared with uniform grids of the same global size (Fig. 6.1).

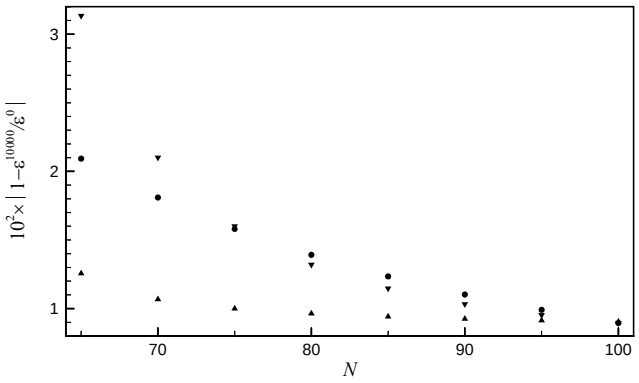

Fig. 6.1 - Relative energy dissipation. The circles stand for uniform grids, the inverted triangles for juxtaposed grids and the regular triangles to variational optimised grids.

However, as the mesh size in the high velocity zone is increased (decreasing the overall number of mesh points *N*), the results of such rough design gets worse and worse until they exceed the dissipation levels of uniform grids. To overcome this process, a more rigorous approach must be used when designing the high-velocity grid, based upon variational principles applied over the energy dissipation integral functional. This procedure gives rise to intrisincally non-uniform grids (Fig. 6.2), whose performance is clearly exhibited in Fig. 6.1. It is interesting to state that this approach is in everything similar to the variational calculus of Classical Mechanics, which has the Lagrangian equations of motions as one of its major outcomes.

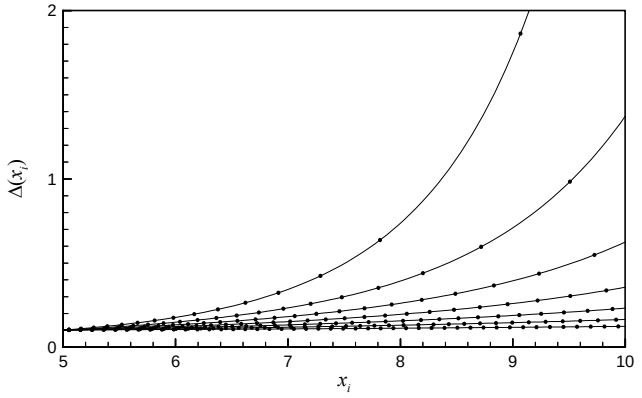

Fig. 6.2 - Several variational optimised grids in the high-velocity domain, for N=65, 70, 75, 80, 85, 90 and 95.

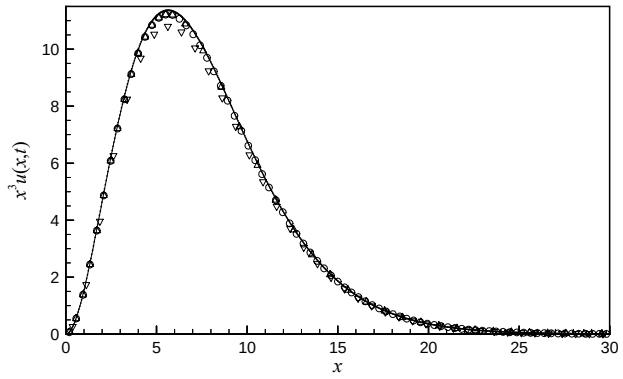

Fig. 6.3 - Photon reduced distribution function, where the solid line stands for the analytical solution. The open circles and the inverted triangles depict the numerical solution achieved with a N*=79 and a* N*=40 uniform grid respectively. The non-uniform grid with* N*=40 is shown with regular triangles.*

A good picture of how the finite difference scheme behaves under strong non-linear conditions may be set up by solving the Compton scattering problem, where the friction contribution depends on the square of the distribution function. Along with such non-linear character, this affecting mainly the quasi-equilibrium computation, the singular behaviour exhibited by the distribution function at low energies co-operates in worsening the accuracy as the number of cells in uniform grids is reduced (Figs. 6.3 and 6.4). However, a simple redistribution of the mesh sizes, while keeping fixed the number of cells, is sufficient to enhance by five times the accuracy in the computed steady-state solutions. When compared against previous results, which reported relative errors of the order 0.005 and lower for the same uniform grid, it may be suggested that the proposed scheme does not impose itself as a profitable alternative to already proposed ones. Nevertheless, it should be kept in mind that such previous schemes rely upon either the knowledge of exact quasi-equilibrium solutions, or the possibility of rewriting the FP equation in a convenient form. Needless to say, neither one of such approaches are readily available for general non-linear problems, thus stressing the relevance of the developed scheme.

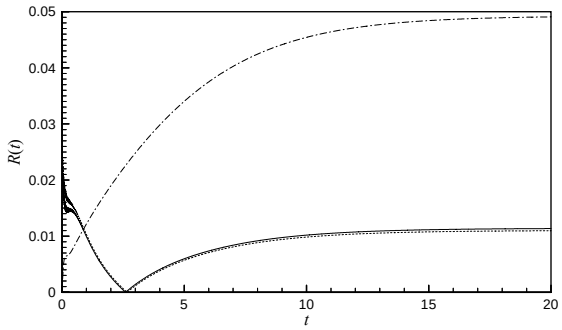

Fig. 6.4 - Relative error, with respect to the analytical solution, versus time. The dash-doted line stands for the N=79 uniform grid results, the dotted line for the N=40 uniform grid and the solid line for the N=40 non-uniform grid.

6.3.3. Transition-probability formulation of the electron kinetics during RF H&CD

A great variety of current scientific studies, from physics to biology to social sciences, involve the FP equation, an example being the problem that motivated the present work, the electron kinetics during RF H&CD of fusion plasmas. Other examples abound in various areas, and are as diverse as the modelling of muscle contraction, the study of neural networks or of interacting social groups, the interstitial atom clustering processes in diatomic materials when defects are produced in a solid and

more can be found for instance in the theory of non-equilibrium thermodynamics, to name but a few illustrative cases. Such a broad application has, therefore, turned this equation into one for which developments at any level (theoretical or numerical) can potentially interest a vast audience. Of the various approaches used to solve the FP equation, path sums and Gaussian short-time propagators are known to offer a very simple and clear view of the kinetics involved and, moreover, to lead to a straightforward numerical implementation. As a consequence, they have been recently gaining some interest as a valuable alternative to Monte Carlo.

In what regards the application of path sums and Gaussian short-time propagators to FP calculations for LH and electron-cyclotron (EC) CD studies, new advancements have been achieved. In particular, novel and improved boundary conditions, internal as well as external, were introduced, the former type to be used exclusively in two-dimensional (2-D) models whereas the latter (needed both at low and high velocities) to be used in 2-D and 1-D alike. Interestingly enough, the low- velocity external boundary condition has a much greater impact on Monte Carlo (than on propagator) calculations, as the tests with the well-known 1-D model of LHCD have shown. This illustrates the advantages of path-sum schemes, where the electron distribution function may be effectively divided into a Maxwellian bulk and a perturbation (which essentially vanishes in the low-velocity region, and is thus virtually independent of the boundary conditions there).

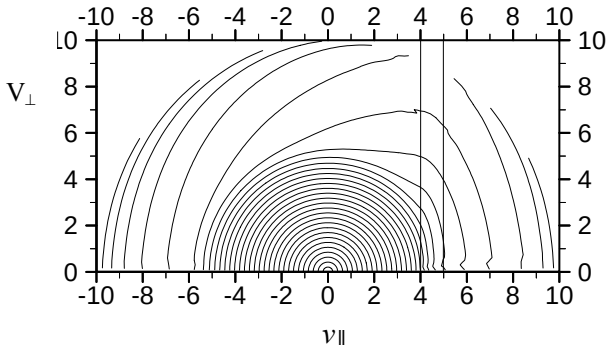

Fig. 6.5 - Electron distribution function for a typical ECCD calculation with D=0.25 between $v_{\parallel}$=4 and $v_{\parallel}$=5

A 2-D code is in progress to treat the electron kinetics during LHCD, as well as, ECCD, solving widely used models. It has been compared to equivalent finite-difference solutions, showing that for moderate values of the wave induced diffusion coefficient (D~0.1) good results are achievable (Fig. 6.5). Unfortunately, for higher values, owing to time-step restrictions and concomitant constraints on grid spacing, the handling of very large propagator matrices is required when solving 2-D FP equations, which leads to some difficulties. These are greater for LHCD than for ECCD, due to the strong wave-induced (electron) diffusion in the parallel direction in the former case, as opposed to perpendicular in the latter.