NUMERICAL STUDY OF MAGNETIZED-SHEATH FORMATION USING AN EULERIAN VLASOV CODE*

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1. Introduction

Many problems of plasma-wall interaction require an understanding of the boundary layer of a magnetized plasma in contact with a wall. In tokamak physics, for instance, configurations with shallow angles of the magnetic field with the wall are necessary for divertor operation.

There is an abundant literature which studies this problem with the assumption that the electron density can be calculated from a Boltzmann relation [1],

$$n_e = n_0 \exp\left(e\Phi/kT_e\right) \tag{1}$$

or using particle-in-cell (PIC) codes [2]. The validity of the Boltzmann approximation relies on the assumption that the electron velocity distribution remains Maxwellian, which is in principle verified only in the presence of collisions and in the absence of a strong electric field. In a collisionless plasma, and close to the wall in the presence of a strong electric field, electrons should also be described by a kinetic equation. On the other hand, PIC codes are noisy, so that it is difficult to obtain accurate distribution functions and the related velocity moments from them.

In our ongoing work we use two versions of a spatially one-dimensional Eulerian Vlasov code [3] to study the problem of sheath formation at a magnetized plasma-wall transition. In both versions, the ion motion is fully resolved with three velocity dimensions. The difference between the two code versions lies in the treatment of the magnetized electrons: In the first version ("SHEATH1"), their distribution function is obtained from a collisionless kinetic equation assuming that their motion is restricted along the magnetic field. The results do confirm that the distribution of the electrons is not Maxwellian. The second version of the code ("SHEATH2") uses a Boltzmann relation for the electron density. Our near-term goal is a detailed comparison of the results of the two code versions for the same set of system parameters. In the present paper we present, as a first step, some new results obtained with the kinetic-electron code SHEATH1.

2. Model and basic equations

We use a cartesian coordinate system x, y, z in which the wall is represented by the (y, z) plane $(x = 0)$ and the bulk-plasma boundary is located at $x = L$. The magnetic field, constant in space and time, lies in the (x, y) plane and makes an angle α with the y axis. We assume that all quantities are independent of y and z. In what follows, all quantities are normalized: time t to $\omega_{pi}^{-1} = \sqrt{\varepsilon_0 m_i / (n_0 e^2)}$(inverse ion plasma frequency), velocity v to $c_s = \sqrt{kT_e/m_i}$ (ion sound speed), position x to $\lambda_{De} = c_s/\omega_{pi}$ (electron Debye length), densities n_e, n_i to n_0 (electron density at the bulk-side boundary), potential Φ to kT_e/e, electric field E_x to $kT_e/(e\lambda_{De}) = \sqrt{n_0 kT_e/\varepsilon_0}$, the ion velocity distribution function f_i to n_0/c_s^3, and the electron velocity distribution function f_e to n_0/c_s.

The electrostatic field is calculated from the usual relation $E_x = -\partial\Phi/\partial x$, where the electrostatic potential must be determined by solving Poisson's equation

$$\frac{\partial^2 \Phi}{\partial x^2} = -(n_i - n_e). \tag{2}$$

The electron and ion densities are related to the respective velocity distribution functions by the velocity integrals

$$n_i(x,t) = \int d^3v\, f_i(x, v_x, v_y, v_z, t), \qquad n_e(x,t) = \int dv_\parallel f_e(x, v_\parallel, t) \tag{3}$$

The ion velocity distribution function is calculated from the ion Vlasov equation

$$\frac{\partial f_i}{\partial t} + v_x \frac{\partial f_i}{\partial x} + (E_x - v_z \omega_{ci} \cos\alpha) \frac{\partial f_i}{\partial v_x} + v_z \omega_{ci} \sin\alpha \frac{\partial f_i}{\partial v_y} + (v_x \omega_{ci} \cos\alpha - v_z \omega_{ci} \sin\alpha) \frac{\partial f_i}{\partial v_z} = 0, \tag{4}$$

and in SHEATH1 the electron velocity distribution function is calculated from the electron Vlasov equation

$$\frac{\partial f_e}{\partial t} + v_\parallel \sin\alpha \frac{\partial f_e}{\partial x} - \frac{m_i}{m_e} E_x \sin\alpha \frac{\partial f_e}{\partial v_\parallel} = 0. \tag{5}$$

Alternatively, in SHEATH2 the Boltzmann relation (1) is used for calculating the electron density.

For the distribution functions, the boundary conditions at $x = 0$ are those of a perfectly absorbing wall and zero plasma-bound flux, while at the bulk-plasma side $(x = L)$ the boundary conditions result from the assumption that the derivative of any quantity is zero. For the potential, we in addition require, without loss of generality, that $\Phi(x = L) = 0$. The simulation is initialized with a spatially uniform state consistent with the boundary conditions and characterized by cut-off Maxwellian ion and (in SHEATH1) electron distribution functions.

3. Results for "kinetic" electrons

As a reference case, we choose the parameters $\alpha = 6^\circ$, $m_i/m_e = 3672$, $T_e = T_i$, and $\rho_i/\lambda_{De} = 14.142$ (with ρ_i the ion gyroradius). the final states we obtain turn out to be oscillating. The oscillations can be seen to some extent in the electron phase-space contour plots of Fig. 1, in the spatial profiles of the ion and electron current of Fig. 2 (where they are seen most clearly), and in the electron kinetic-energy-density profiles shown in Fig. 3. Other quantities show very small oscillations. In addition, we clearly see that the electron distribution functions are not Maxwellian.

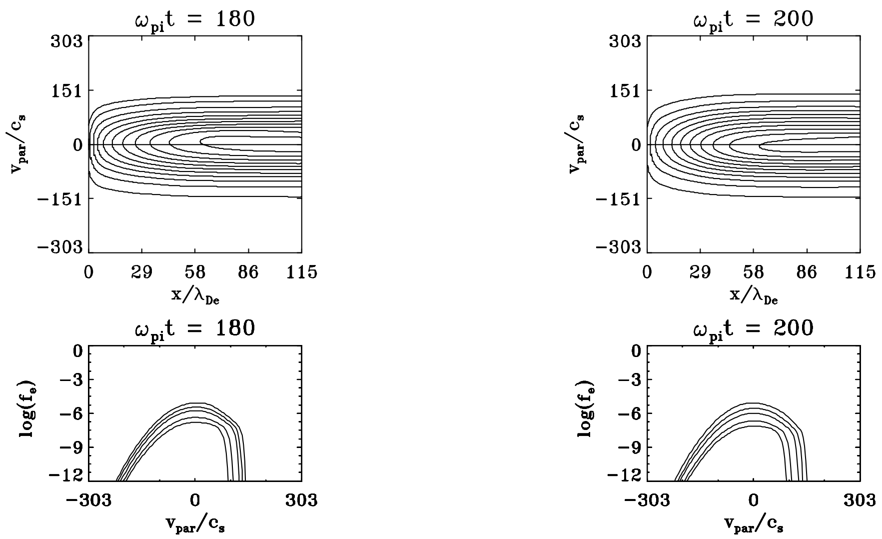

*Fig.*1. Electron distribution function as contour plots in $(x, v_{\|})$ phase space (top) and as $\log f_e(v_{\|})$ for various x values (bottom) at $t = 180$ (left) and $t = 200$ (right).

4. Discussion and conclusions

One of the most striking features of the present work is that we see strong oscillations for practically all parameters considered. This is in obvious contrast to the work of Gerhauser and Claaßen [1], where no significant oscillations were observed. As the primary cause for this difference we have identified with high probability the different boundary conditions applied in the two approaches at the bulk-side boundary: While Gerhauser and Claaßen prescribed a fixed input distribution for the ions, our boundary conditions consist in forcing zero spatial gradients for all quantities involved. This explanation was confirmed by a test run (with Boltzmann-distributed electrons, not shown here) in which we too prescribed fixed boundary conditions, resulting in a non-fluctuating final state.

Obviously, a major effort is still needed to understand the influence of system parameters (such as length or magnetic field) on the oscillations observed. Since the system simulated is bounded by two planes and hence formally represents a "plasma diode," some clues to this problem can possibly be found from previous work on other types of plasma diodes [4], where it was shown that the final states of such systems may crucially depend on the system parameters and boundary conditions.

Another major conclusion to be drawn from this work is the observation that sheath regions are likely to exhibit time-dependent behaviour, which is in striking contrast with the fact that most sheath models existing in the literature are time-independent. Hence, the time-dependency aspect of plasma sheaths should receive much more attention than has been the case to date.

Of course, the "definitive" solution to the sheath problem would require self-consistent consideration of the entire bounded plasma system including the bulk plasma, but since such solutions are not likely to be available very soon, the next realistic step required is to find out which bulk-side boundary conditions come closest to this real situation.

*Work performed within the Association EURATOM-ÖAW.

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[4] S. Kuhn, Contrib. Plasma Phys. 34 (4), 495 (1994).

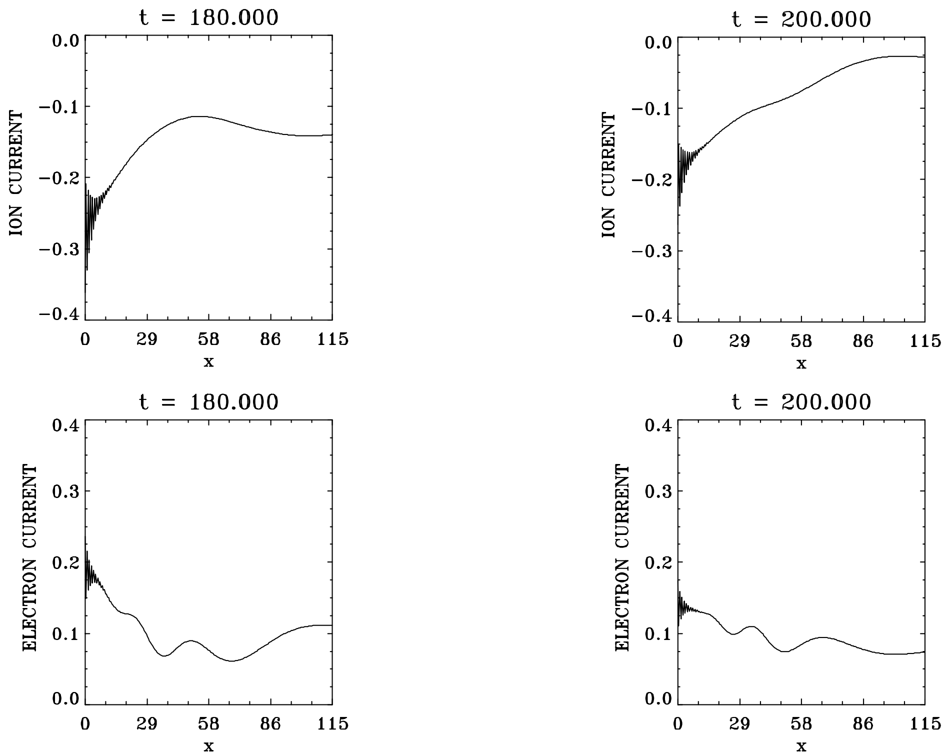

Fig.2. Spatial profile of the ion current density normal to the plate for $t = 180$ (left) and $t = 200$ (right).

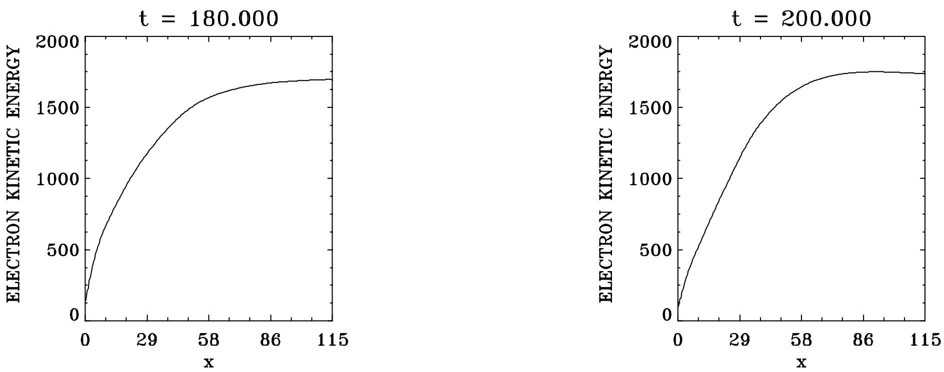

Fig.3. Electron kinetic-energy density profile for $t = 180$ (left) and $t = 200$ (right).