RESISTIVE BALLOONING INSTABILITY IN TOKAMAK EDGE PLASMAS

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Abstract. To investigate the origin of Enhanced D_α (EDA) operation in Alcator C-Mod, analytic dispersion relations for Resistive Ballooning Modes (RBM) in the edge pedestal are developed. Sound wave propagation and coupling to drift waves reduce linear growth rates, but no stability threshold is found. Growth of the drift RBM peaks at relatively low values of toroidal mode number, n.

1. **Introduction**

In EDA operation in Alcator C-Mod [1] a benign instability, the quasi coherent mode, is observed in the edge pedestal where the local pressure gradient is steep, but insufficient to drive Ideal ballooning instability. The quasi coherent mode is favoured by high safety factor, q, and high collisionality, consistent with linear stability predictions for Resistive Ballooning Modes (RBMs) [2]. Several recent studies, both linear [3] and non-linear [4, 5], have been devoted to this topic. In much of this work a complex set of two-fluid Braginskii equations is solved numerically, sometimes in a complicated equilibrium containing, or simulating the proximity of, a magnetic separatrix. Instability is generally found, but there has been little discussion of a stability threshold (as observed in Alcator C-Mod). In this paper we return to the single fluid resistive MHD equations [2] to derive analytic dispersion relations for RBMs adding various physical effects, including (1) sound wave propagation, (2) Δ'_B drive from the Ideal region in ballooning space, with poloidal angle $\theta \sim 1$ and (3) two-fluid diamagnetic effects. In Sec. 2 the analytic theory of single fluid resistive MHD ballooning modes is extended by employing an ordering in which the growth rate γ is of the order of the sound frequency $\omega_s = C_s/Rq$. The extension to the drift-resistive ballooning dispersion relation is then obtained. This adds coupling to electron and ion drift waves. Solution of the dispersion relation for the $s - \alpha$ model equilibrium is presented and the results are summarized in Sec. 3.

2. **Resistive and Drift-Resistive Eigenmode Equations.**

In the $\gamma \sim \omega_s$ ordering, numerical solution of the 4th order resistive ballooning equations for an $s - \alpha$ model [6] indicated that a stability threshold may occur at higher

ω_s, but stability boundaries were not determined. However, in the $\gamma \sim \omega_s$ ordering, if $\epsilon = (n^2q^2/S)^{1/3} \ll 1$, with the Lundquist number $S = \tau_\eta/\tau_A$, a two length scale analysis of the 4th order equations reduces them to a single second order ODE in the long scale variable $X = \epsilon Z/\sqrt{\hat{\gamma}}$, where $Z = s\theta$, $s = rq'/q$ and $\hat{\gamma} = \gamma\tau_A/\epsilon$:

$$\frac{d}{dX}\frac{X^2}{1+X^2}\frac{dF}{dX} - FQX^2 = 0, \tag{1}$$

$$Q = \frac{\hat{\gamma}^3}{s^2}\left[1 + 2q^2\frac{\omega_s^2}{\gamma^2+\omega_s^2}\right] - \frac{\alpha^2}{2s^2}\left[\frac{\gamma^2}{\gamma^2+\omega_s^2}\right]$$

with $\alpha = -2Rp'q^2/B^2$. Equation (1) can be solved exactly, and the solutions matched to the ideal region ($X \to 0$) solution, where $F \propto 1 + \Delta'_B/Z$, for $Z \gg 1$. Δ'_B can be evaluated by solving the Ideal $s - \alpha$ equation. It is positive throughout the first stability region and $\to \infty$ on the Ideal MHD stability boundary. The resulting dispersion relation takes the form:

$$\Delta'_B\epsilon = -\frac{\hat{\gamma}^{1/2}Q^{1/4}}{8}\frac{\Gamma\left[\frac{\sqrt{Q}-1}{4}\right]}{\Gamma\left[\frac{\sqrt{Q}+5}{4}\right]} \tag{2}$$

Equation (2) determines the stability of resistive modes close to an Ideal MHD stability boundary. For smaller values of Δ'_B, in the range $\epsilon^{1/3} < \epsilon\Delta'_B < 1$, the simpler dispersion relation, $\Delta'_B\epsilon = 0.68Q^{\frac{1}{4}}\hat{\gamma}^{\frac{1}{2}}$, follows from the small Q limit of Eq.(2). However, since the mode width (in ballooning space) is determind by $Q \propto (\epsilon\Delta'_B)^4$, Δ'_B controls this width. As $\epsilon\Delta'_B \to 0$ the eigenmode becomes increasingly broad (increasingly localized in real space), and appears to become singular in the limit $\epsilon\Delta'_B = 0$. In fact, these dispersion relations break down when $\epsilon\Delta'_B < \epsilon^{1/3}$, and a more sophisticated analysis is required. To resolve this limit, we have developed a new ordering for the two length scale averaging procedure, which extends to $Z \sim \epsilon^{-4/3}$, beyond the usual resistive region which has $Z \sim \epsilon^{-1}$. In this region the eigenmode equation takes the form:

$$\frac{d^2F}{dY^2} - FY^2\left[\frac{Q}{\delta^2} + \alpha^4Y^2G(q,\gamma/\omega_s)\right] = 0 \tag{3}$$

where $\delta = (\epsilon^2/\hat{\gamma})^{1/3}$, $Y = Z\delta^2$. The function $G(q,\gamma/\omega_s)$, which determines the mode width in the limit $Q \to 0$, is positive for real values of γ. Eigenvalues for Eq.(3) can be obtained variationally by employing a trial function (with variational parameter σ) of the form $F = Y^{1/2}K_{1/4}(\sigma Y^2)\left[1 - Y[\delta\Delta'_B - \sqrt{(\sigma/2)}\Gamma(3/4)/\Gamma(5/4)]\right]$, which correctly matches to the ideal region. In the $\delta\Delta'_B \to 0$ limit the dispersion relation is

$$Q + 2.46\delta^2(\alpha^4G)^{2/3} = 0. \tag{4}$$

This resolves the mode structure for $\Delta'_B \sim O(1)$, but the growth rate is still approximately determined from $Q \simeq 0$, since $\delta \ll 1$.

The growth rates of resistive modes are typically small and fall within the drift ordering, $\gamma \leq \omega_{*j}$ [7]. A dispersion relation containing diamagnetic effects can be obtained by the same averaging method, applied to the linearised two-fluid equations [8], to generate new expressions for the functions Q and G in Eqs.(1) and (3). Neglecting temperature gradients and temperature perturbations, the resulting expression for Q is:

$$Q = \frac{\hat{\gamma}(\hat{\gamma}+i\hat{\omega}_{*i})(\hat{\gamma}+i\hat{\omega}_{*e})}{s^2}\left[1+\frac{2q^2\hat{\omega}_s^2}{\hat{\gamma}(\hat{\gamma}+i\hat{\omega}_{*e})+\hat{\omega}_s^2}\right] - \frac{\alpha^2}{2s^2}\left[\frac{\hat{\gamma}(\hat{\gamma}+i\hat{\omega}_{*e})}{\hat{\gamma}(\hat{\gamma}+i\hat{\omega}_{*e})+\hat{\omega}_s^2}\right] \quad (5)$$

where $\hat{\omega}_{*j} \equiv \omega_{*j}\tau_A/\epsilon$ and $\hat{\omega}_s \equiv \omega_s\tau_A/\epsilon$. The coefficient G and $\delta = (\epsilon^2/(\hat{\gamma}+i\hat{\omega}_{*e})]^{1/3}$ also become complex. For $\Delta'_B \sim 1$, Eq.(3), with the 2-fluid expressions for Q and G, must be solved. In leading order the dispersion relation, $Q \simeq 0$ determines lowest order complex eigenvalues, $\gamma = \gamma_0$. However these only correspond to acceptable localized eigenmodes if the condition $Re[\delta^{12}G(\gamma_0)] > 0$ is satisfied. The lowest order dispersion relation takes the form:

$$(\hat{\gamma}+i\hat{\omega}_{*i})\left[(1+2q^2)\hat{\omega}_s^2+\hat{\gamma}(\hat{\gamma}+i\hat{\omega}_{*e})\right] - \alpha^2/2 \simeq 0. \quad (6)$$

In the absence of α this predicts three waves; an ion drift wave and a pair of toroidally modified electron drift acoustic waves. As found in [7] in the $\omega_s = 0$ limit, the pressure gradient drives instability of the low frequency branch of the drift acoustic mode, with $\hat{\gamma} \simeq \alpha^2/(2\hat{\omega}_*^2)$ when $\omega_{*j} \gg \gamma$. This is typically a much weaker growth rate than that predicted in the absence of diamagnetic effects, $\hat{\gamma}^3 = \alpha^2/2$. However, solution of Eq.(5) shows that the growth rate of the drift RBM is actually enhanced by sound wave propagation when the low frequency electron drift acoustic mode is degenerate with the ion drift wave. This occurs when $\omega_{*i}(\omega_{*i}-\omega_{*e}) = (1+2q^2)\omega_s^2$ and therefore at a particular value of the toroidal mode number, n. For this mode number the growth is $\hat{\gamma} \simeq 0.3\alpha/\hat{\omega}_*^{1/2}$.

The effect of sound wave propagation on the RBM [2] and drift-RBM [7] is shown in Fig.1. Case (a) shows the reduction in the RBM growth, $\hat{\gamma}$, at fixed α, as $\hat{\omega}_s \simeq \sqrt{\beta}(S/n^2q^2)^{1/3}$ is increased. The parameters used are $\alpha = 1$, $q = 4$ and $\omega_{*j} = 0$. Curve (b) shows the growth rate for the drift-RBM, when $\hat{\omega}_{*i} = -\hat{\omega}_{*e} = 4$. Figure 2 shows the dependence of the growth rates, $\tilde{\gamma} = \gamma\tau_A(S/q^2)^{1/3}$, as a function of toroidal mode number, n, for fixed plasma parameters: $\alpha = 1$, $q = 4$, $S = 10^6$, $\beta = 0.0006$, with $\omega_{*j} = 0$ for case (a) and $\hat{\omega}_{*i} = -\hat{\omega}_{*e} = 2$ at $n = 1$ for case (b). A peak in $\tilde{\gamma}$ for the drift RBM is seen at low n. Evaluating the sign of $Re[\delta^{12}G]$ establishes that the drift resistive eigenfunction is localized in Y for $n > 1$.

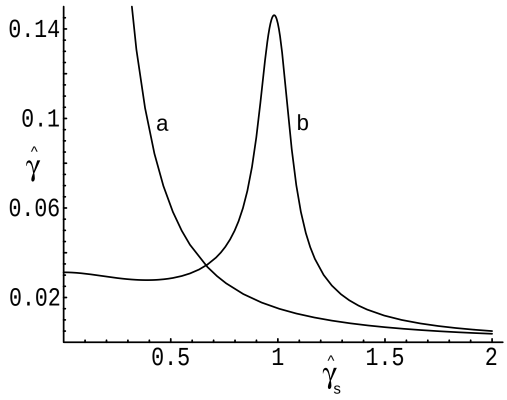

Figure 1:

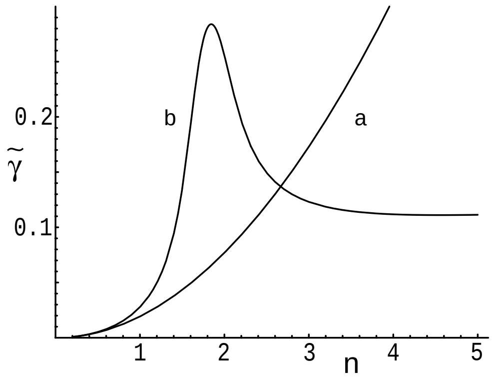

Figure 2:

3. **Conclusions**

Analytic resistive ballooning dispersion relations, appropriate for collisional plasma conditions in the edge pedestal of a tokamak have been derived. These extend the original analyses [2,7] by describing the effects of matching to the ideal region and coupling to drift-acoustic modes. Diamagnetic and sound effects interact to produce a peak in γ at intermediate values of the toroidal mode number n. For parameters typical of the edge pedestal in Alcator C-Mod this peak occurs at $n = 2$. An eigenmode equation for drift-visco-resistive ballooning modes has also been derived and preliminary results show that perpendicular ion viscosity strongly modifies the radial localization of the drift RBM.

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