Anomalous ExB Ion Diffusion and Radial Electric Field Generation in a Fluctuating Electrostatic Potential

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Recently [1,2,3], we simulated potential structures, appearing due to low-frequency tokamak edge plasma turbulence, by means of a spatially periodical stationary potential. We studied its effect on the ion dynamics and found strong diffusion of plasma impurities. This diffusion generates an electrostatic potential and, consequently, a radial electric field and a plasma rotation. Moreover, we found there both diffusion of typical random-walk character and dynamics close to the Lévy walk [4]. The latter is intrinsically connected with superdiffusion [5,6]. The diffusion appearing in our model is basically caused by finite Larmor radius effects, whereas models, discussed in the literature, are mainly based on the drift approximation. All of this motivates us to study the phenomenon in question under more general conditions. Accordingly, the aim of this contribution is to discuss the differences in the chaotic dynamics following from the full and the drift solutions for the models considered, and further to estimate to which extent Lévy walk dynamics we found in the stationary case will survive in the time-dependent potentials. Of course, it will be possible to use the potential given, e.g., by the Hasegawa-Mima model, as has been done by Manfredi and Dendy [7], but nevertheless we would like to take advantage of our simple model [1]. Therefore, we present a modification thereof which is close to the model of Misguich and Nakach [8].

The model previously mentioned describes the spatially periodical potential in cartesian coordinate x, y in the form $U = U_0[2 + \cos kx + \cos ky]$, and the magnetic field as $\mathbf{B} = (0, 0, B_z)$.

Using further the dimensionless variables $\xi = kx, \quad \eta = ky, and \quad \theta = \frac{Zq_pB}{Am_p}\, t = \omega_c\, t$, with ($\omega_c$ is the proton cyclotron frequency), the equations of motion take the form

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}\theta^2} = R\sin\xi + \frac{\mathrm{d}\eta}{\mathrm{d}\theta}; \quad \frac{\mathrm{d}^2\eta}{\mathrm{d}\theta^2} = R\sin\eta - \frac{\mathrm{d}\xi}{\mathrm{d}\theta}; \quad R = \frac{Am_pU_0k^2}{Zq_pB_z^2}, \tag{1}$$

where R is the dimensionless parameter governing the ion motion.

Together with this full system, we shall also discuss the drift modification for the same dynamics, namely, the equations

$$\frac{\mathrm{d}\xi}{\mathrm{d}\theta} = R\sin\eta; \qquad \frac{\mathrm{d}\eta}{\mathrm{d}\theta} = -R\sin\xi \tag{2}$$

in the same dimensionless system of coordinates.

In spite of their simplicity, the motion given by the equations (1) can be very different for different values of R. For small values of R, particles drift along equipotential curves [1,2]. For larger R, the motion starts to be chaotic and can then be described as a random walk between neighbouring hills and valleys (Fig. 1a). For even larger R, the motion become yet complicated, as presented in Fig. 1b. Here the particles move intermittently between ballistic and quasitrapped motion. The latter one allways appears in the valley regions. Such motion can be described as Lévy walks [1,2].

In [3], this phenomenon was proposed for the generation of radial electric fields in the tokamak edge plasma region. In [9], we presented the first results of the self-consistent solution of this effect, based on particle-in-cell simulation. We found that for typical plasma and turbulence parameters, the radial electric field appears with amplitudes of the order 10^3 V/m and poloidal velocities of the order 10^4 m/s.

Obviously, our model suffers mainly from its stationary form. To be closer to reality, and using the advantages of our simple model, we have modified our potential as follows.

We assume the potential of the form

$$U = U_0[2 + \cos(\Omega t - kx) + \cos(\Omega t - ky)] \tag{3}$$

and again use dimesionless variables ξ, η and θ defined above. The corresponding Hamiltonian reads

$$\tilde{H}(\xi, \eta, p_\xi, p_\eta) = \frac{1}{2}((p_\xi + \frac{1}{2}\eta)^2 + (p_\eta - \frac{1}{2}\xi)^2) + R[2 + \cos(\Omega\theta - \xi) + \cos(\Omega\theta - \eta)] \tag{4}$$

This model is very close to the model of Misguich and Nakach [8], who simulated the turbulent potential with three electrostatic waves and used the drift approximation. For the case of only two waves, they found the motion to be integrable; only for the interaction of three waves, the motion starts to be chaotic. We shall see that, describing the dynamics by the full system of equations, chaotic motion can also by generated by just two waves.

In our model we shall study whether the phenomena we found previosly will also persist in the time-dependent potential (3), i.e., in the potential which is closer to reality.

The following figures present the first results of our test-particle simulation. The first of these, Fig. 2a, describes the dependence of the diffusion coefficient on the parameter R. Fig. 2b presents the dependence of the diffusion coefficient on the parameter Ω. Both dependencies are calculated from the full solution of the dynamical equations. Figure 3 presents the initiall position of a sample set of particles and their final position after 150 cyclotron periods.

We see a remarkable difference between the full and the drift versions. For the time-independent case ($\Omega = 0$), the drift approximation yields no chaos at all [8], whereas the full simulation brings chaos, e.g, for $R = 0.6$. Therefore, the system exhibits chaotic behaviour in the full version. An example of Lévy walks presents Fig. 4.

Moreover, the time-dependent case yields stochasticity for lower values of R than the stationary case, where according to [1] the stochastic regime requires $R > 1$. It is therefore possible that in the time-dependent case the stochastic regime will be attainable not only for impurities, but also for the main plasma constituents, i.e., for protons. In this case, the generation of the radial electric field will be probably influenced as well.

Fig. 5 brings diffusion coefficients for three waves system $U(\xi, \eta, \theta) = [2 + \cos(\Omega\theta - \xi) + \cos(\Omega\theta - \eta) + \cos(\pi\Omega\theta - \xi)$ for full and drift solution. The differences are rather small.

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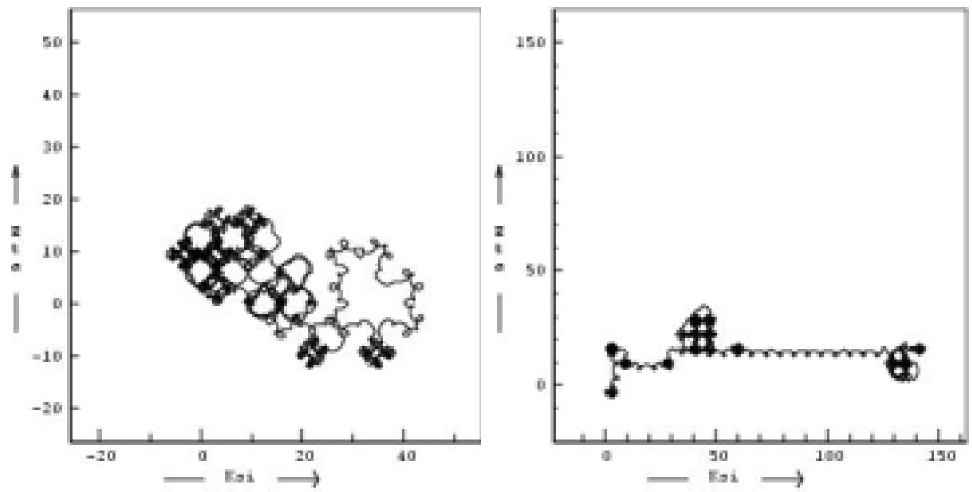


Figure 1: Random walk and Lévy walk diffusion

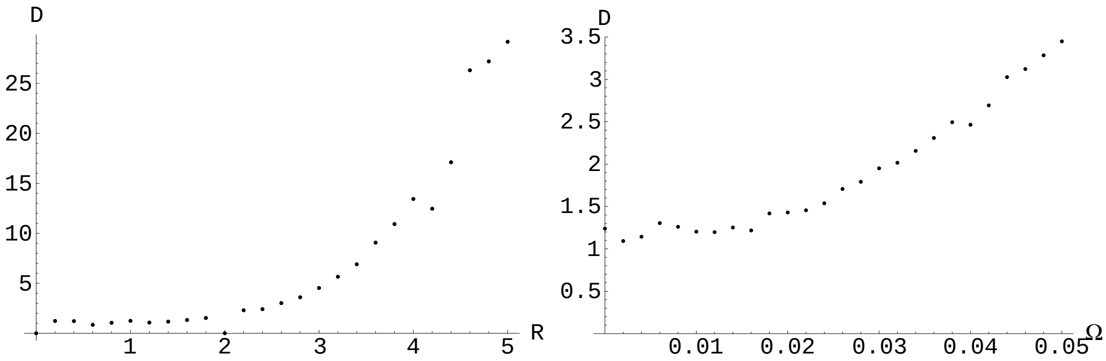


Figure 2: Dependence of the dimensionless diffusion coefficient on a) $R, (\Omega = 0.01)$ and b) $\Omega, (R = 1)$

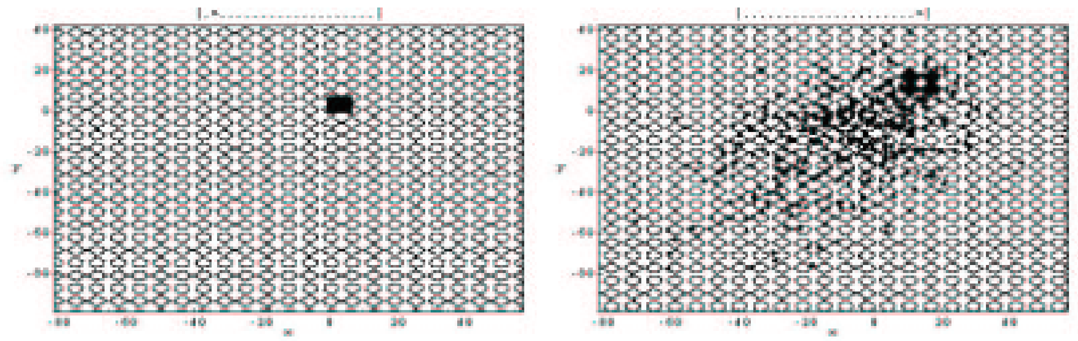
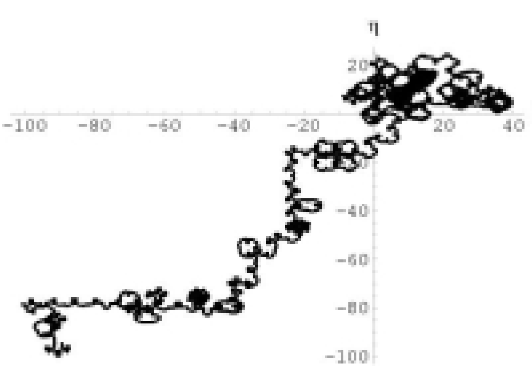


Figure 3: Diffusion of particles in the ξ, η coordinate system for $R = 0.6, \Omega = 0.01$ computed for 10^3 particles



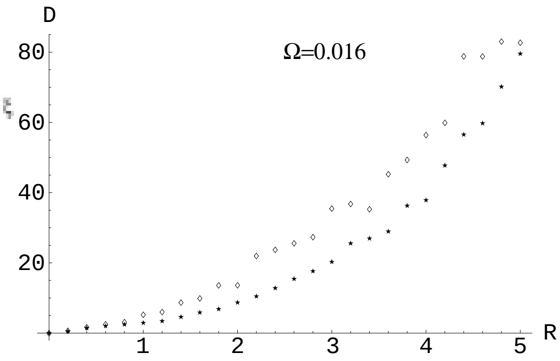


Figure 4: Example of the Lévy walks for time-dependent potential for $\Omega = 0.01$ and $R = 0.8$. Figure 5: Diffusion coefficient for drift and full solution ($\diamond$=drift, $\star$=full)