LARMOR GYRATION CONTRIBUTION TO THE TRANSPORT PHENOMENA IN WEAKLY COLLISIONAL TOROIDAL PLASMAS

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INTRODUCTION

Consistent gyrophase averaging of the Fokker-Planck equation for magnetically confined, weakly collisional, axisymmetric toroidal plasma is performed using Lagrangian coordinates [1] transformed from Eulerian space and employing a multiple timescale approach [2]. The drift kinetic equation applicable for the case of ions having poloidal gyroradii in the order of the plasma inhomogeneity scale length is derived. The collision term may be represented as a sum of the conventional collision operator and the additional contributions arising from the incorporation of finite Larmor radius effects and leading to diffusion in both velocity and real space.

INITIAL EQUATIONS

The kinetic equation for the distribution function of energetic particles is written as

$$\frac{\partial f}{\partial t} + \dot{x}^i \frac{\partial f}{\partial x^i} = C_{\mathbf{x}}(f) + S \tag{1}$$

where x^i are arbitrary phase-space coordinates, $C_{\mathbf{x}}(f)$ is the collision operator and S a fast ion source. The collision term in Eq. (1) is a differential operator of the form

$$C_{\mathbf{x}}(f) = \nabla_{\mathbf{p}}\left(\mathbf{d} + \overleftrightarrow{\mathbf{D}}\nabla_{\mathbf{p}}\right) f, \quad \mathbf{d} = \nu_S \mathbf{V},\ \overleftrightarrow{\mathbf{D}} = \nu_\perp \mathbf{V}^2 \overleftrightarrow{\mathbf{I}} + \left(\nu_{\parallel} - \nu_\perp\right)\mathbf{V}\otimes\mathbf{V} \tag{2}$$

with $\mathbf{d}$ being the vector of the "dynamic friction force", $\overleftrightarrow{\mathbf{D}}$ the diffusion tensor, both defined in the velocity space, and $\overleftrightarrow{\mathbf{I}}$ is the unit dyad, $\mathbf{V}$ the particle velocity; ν_S, $\nu_\perp$ and $\nu_{\parallel}$ are the characteristic collision frequencies of slowing down, transverse and parallel diffusion.

The conventional theory is essentially Eulerian in nature, working with the independent velocity and spatial variables that are not constants of the orbital motion. However, many of the concepts in neoclassical theory involve orbital properties and are Lagrangian [1] in nature. Here we present a transport theory for energetic particles using what amounts to the Lagrangian picture.

TRANSFORMATION OF COORDINATES AND AVERAGING PROCEDURE

Considering the motion in a strong, but slowly varying magnetic field **B**, it is convenient to carry out gyrophase averaging and to focus attention on the motion of the guiding centre. Following conventional drift theory, one can define a set of three orthogonal unit vectors $(\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2)$ such that $\mathbf{e}_0 = \mathbf{B}/B, [\mathbf{e}_i \mathbf{e}_j] = \mathbf{e}_k$. If we express the particle position by the guiding centre position (**R**) and the Larmor rotation ($\boldsymbol{\rho}$) [3], $\mathbf{r} = \mathbf{R} + \boldsymbol{\rho}$, then the Larmor rotation can be represented by

$$\boldsymbol{\rho} = \rho\left(\mathbf{e}_1 \sin\alpha - \mathbf{e}_2 \cos\alpha\right), \quad \rho = V_\perp / \Omega, \qquad \mathbf{V} = V_\parallel \mathbf{e}_0 + V_\perp\left(\mathbf{e}_1 \cos\alpha + \mathbf{e}_2 \sin\alpha\right), \tag{3}$$

where Ω is the particle gyrofrequency and α the gyro phase; $V_\parallel$ and $V_\perp$ are the longitudinal and transverse components of $\mathbf{V}$.

As velocity variables one can choose $\{V,\xi,\alpha\}$ with $\xi = V_\parallel/V$, and as spatial Eulerian variables one may introduce the flux coordinates $\{\Phi,\theta,\varphi\}$, where Φ is the toroidal flux, θ and φ the poloidal and toroidal angles, so that the magnetic field can be represented in the form

$$\mathbf{B} = \nabla\Phi \times \nabla\theta - \iota\nabla\Phi \times \nabla\varphi, \tag{4}$$

with $\iota(\Phi)$ being the rotational transform.

Deriving the drift kinetic equation within conventional theory [4] one first arranges the LHS of Eq. (1) in orders of the gyro radius and then averages over the gyro phase, whereas the collision operator on the RHS of Eq. (1) is evaluated only at the guiding center position **R**. For a more consistent way of evaluation we transform from Eulerian (**x**) to Lagrangian variables (**z**) taking into account the corrections arising from the Larmor gyration, and then perform the gyrophase averaging of both the LHS of the Fokker-Planck equation and of the transformed collision operator.

The most practical way for a correct averaging is the use of constants-of-motion variables: the energy ε, the magnetic moment μ and, in the case of axisymmetric configurations ($\partial/\partial\varphi = 0$), the longitudinal adiabatic invariant P_φ. The Fokker-Planck equation can be rewritten in Lagrangian variables $\mathbf{z}=(\varepsilon,\mu,\alpha;P_\varphi,\theta,\varphi)$ as

$$\frac{\partial f}{\partial t} + \dot{\varepsilon}\frac{\partial f}{\partial \varepsilon} + \dot{\mu}\frac{\partial f}{\partial \mu} + \dot{\alpha}\frac{\partial f}{\partial \alpha} + \dot{P}_\varphi \frac{\partial f}{\partial P_\varphi} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\varphi}\frac{\partial f}{\partial \varphi} = C_z(f) + S \tag{5}$$

with $\varepsilon = \dfrac{V^2}{2} + \dfrac{e}{m}\Phi,\ \mu = \dfrac{V_\perp^2}{2B},\ P_\varphi = \Psi - \dfrac{R_0^2 B}{\Omega}\mathbf{V}\cdot\nabla\varphi,\ \dot{\alpha} = \Omega + A(\alpha),\ A(\alpha) \ll \Omega,$ and Ψ being the poloidal flux and R_0 the major radius. For the averaging procedure, we should distinguish between oscillating and non-oscillating terms in Eq. (5). Any function of spatial variables can be given as one at the guiding center position plus the correction arising from Larmor oscillations such as

$$F\left(t,\mathbf{q},\varepsilon,\mu,\alpha\right) = \left\langle F\left(t,\mathbf{Q},\varepsilon,\mu\right)\right\rangle + \tilde{F}\left(t,\mathbf{q},\varepsilon,\mu,\alpha\right), \quad \langle F\rangle = \frac{1}{2\pi}\oint F d\alpha. \tag{6}$$

Hence the collision term is written as a sum of the conventional collision operator and the additional contributions arising from the inclusion of finite Larmor radius effects:

$$C_{\mathbf{z}}\left(\langle f\rangle+\tilde{f}\right)=C_{\mathbf{P}}\left(\langle f\rangle+\tilde{f}\right)+C_{\mathbf{Q}}\left(\langle f\rangle+\tilde{f}\right). \tag{7}$$

The largest term (0$^{\text{th}}$ order in ρ) in Eq. (5) is

$$\Omega\frac{\partial}{\partial\alpha}\langle f\rangle=0, \tag{8}$$

which means that $\langle f\rangle$ does not depend on α. Assuming axial symmetry of the configuration and accounting for the fact that energy and the longitudinal adiabatic invariant are well conserved, the next order equation can be derived. Regular gyro-oscillations of the magnetic moment will vanish by gyro-averaging implying the conservation of $\langle\mu\rangle$. However, non-conservation of the magnetic moment may occur due to resonance between cyclotron and bounce oscillations [5].

In the case of adiabatic behavior the LHS of Eq. (5) will have only two dimensions corresponding to gyration and bounce oscillations,

$$\frac{\partial\langle f\rangle}{\partial t}+\Omega\frac{\partial\tilde{f}}{\partial\alpha}+\langle\dot{\theta}\rangle\frac{\partial\langle f\rangle}{\partial\theta}=C_{\mathbf{z}}(\langle f\rangle+\tilde{f})+S, \tag{9}$$

where on the RHS only the corresponding order contribution should be considered. With the source term being independent on Larmor gyration, averaging over α yields finally

$$\frac{\partial\langle f\rangle}{\partial t}+\langle\dot{\theta}\rangle\frac{\partial\langle f\rangle}{\partial\theta}=\left\langle C_{\mathbf{z}}(\langle f\rangle+\tilde{f})\right\rangle+S. \tag{10}$$

Evidently, the oscillating part in Eq. (9) is determined by the difference

$$\Omega\frac{\partial}{\partial\alpha}\left(\tilde{f}\right)=C_{\mathbf{z}}\left(\langle f\rangle+\tilde{f}\right)-\left\langle C_{\mathbf{z}}\left(\langle f\rangle+\tilde{f}\right)\right\rangle, \tag{11}$$

from where the part $\tilde{f}$ of the distribution function may be found, which is associated with fast Larmor gyration. Since it is small in comparison with $\langle f\rangle$, i.e. $\tilde{f}/\langle f\rangle\sim O(\nu_S/\Omega)$, we neglect $\tilde{f}$ in our further calculations.

COLLISION INDUCED DIFFUSION

The transformation of diffusion coefficients to the **z**-space is accomplished by

$$d_{\mathbf{z}}^{i}=d_{\mathbf{x}}^{k}\frac{\partial z^{i}}{\partial x^{k}},\; D_{\mathbf{z}}^{ij}=D_{\mathbf{x}}^{kl}\frac{\partial z^{i}}{\partial x^{k}}\frac{\partial z^{j}}{\partial x^{l}}\;\; (k,l=1,2,3;\; i,j=1,2,3,4,5,6) \tag{12}$$

where $\sqrt{g_{\mathbf{z}}}=\left|\frac{\partial(\varepsilon,\mu,\alpha;P_{\varphi},\theta,\varphi)}{\partial(V,\xi,\alpha;\Phi,\theta,\varphi)}\right|^{-1}\cdot\sqrt{g_{\mathbf{x}}}=\frac{BR_0^2}{\iota J\sqrt{\varepsilon-\mu B}}$ and J is the total poloidal current. Gyrophase averaging yields two expressions, one for the conventional collision operator,

$$\left\langle C_{\mathbf{P}}\left(\langle f\rangle\right)\right\rangle=\frac{1}{\sqrt{g_{\mathbf{z}}}}\frac{\partial}{\partial z^{k}}\sqrt{g_{\mathbf{z}}}\left(\left\langle d_{\mathbf{z}}^{k}\right\rangle+\left\langle D_{\mathbf{z}}^{kl}\right\rangle\frac{\partial}{\partial z^{l}}\right)\langle f\rangle,\quad k,l=1,2 \tag{13}$$

where

$$\left\langle d_{\mathbf{z}}^{1}\right\rangle = \nu_S \cdot 2\varepsilon,\quad \left\langle d_{\mathbf{z}}^{2}\right\rangle = \nu_S \cdot 2\mu,\quad \left\langle D_{\mathbf{z}}^{11}\right\rangle = \nu_{\parallel} \cdot 4\varepsilon^2,\quad \left\langle D_{\mathbf{z}}^{22}\right\rangle = \nu_{\perp}\left(\frac{\varepsilon}{\mu B}-1\right)\cdot 4\mu^2, \tag{14}$$

and one representing the additional Larmor-contribution

$$\left\langle C_{\mathbf{Q}}\left(\langle f\rangle\right)\right\rangle = \frac{1}{\sqrt{g_{\mathbf{z}}}}\frac{\partial}{\partial z^k}\sqrt{g_{\mathbf{z}}}\left(\left\langle d_{\mathbf{z}}^{k}\right\rangle + \left\langle D_{\mathbf{z}}^{kl}\right\rangle \frac{\partial}{\partial z^l}\right)\langle f\rangle \tag{15}$$

with the non-zero terms

$$\begin{gathered} D_{\mathbf{z}}^{14} = D_{\mathbf{z}}^{41} = -2\nu_{\parallel}\rho_0 \varepsilon J\sqrt{1-\frac{\mu B}{\varepsilon}},\quad D_{\mathbf{z}}^{24} = D_{\mathbf{z}}^{42} = 2\nu_{\perp}\rho_0 \mu J\sqrt{1-\frac{\mu B}{\varepsilon}}, \\ D_{\mathbf{z}}^{44} = \left(\nu_{\perp}\frac{\mu B}{\varepsilon}+\nu_{\parallel}\right)\rho_0^2 J^2,\quad d_{\mathbf{z}}^{4} = -\nu_S \rho_0 J\sqrt{1-\frac{\mu B}{\varepsilon}};\quad \rho_0 \equiv \frac{V}{\Omega}. \end{gathered} \tag{16}$$

Here $z^1 = \varepsilon,\ z^2 = \mu,\ z^4 = P_{\varphi}$. Thus, deriving the drift kinetic equation consistently throughout, i.e. considering Larmor motion on both sides of the Fokker-Planck equation, has resulted in additional contributions to the transport coefficients.

CONCLUSIONS

For a magnetically confined, weakly collisional, axisymmetric toroidal plasma the consistent gyrophase averaging of the Fokker-Planck equation, i.e. considering Larmor motion also in the collision operator, has lead to a drift kinetic equation containing additional terms indicating enhanced collision induced diffusion both in velocity and real space. This modification of diffusion and convection transport is suggested to be important for the description of NBI ion behavior in spherical tokamaks and present-day stellarators, as well as for charged fusion products in future toroidal devices, where the ions can have poloidal gyro radii in the order of the plasma inhomogeneity scale length.

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