COBRA-TERPSICHORE LOCAL STABILITY BENCHMARK FOR THE COMPACT STELLARATOR NCSX

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1. Introduction

The National Compact Stellarator Experiment (NCSX, PPPL, USA, [1]) 3-period quasi-axisymmetric configuration, li383 (Fig.1), has an aspect ratio of 4.4, the value of the rotational transform at the edge is 0.66, and the configuration has good surfaces with small internal islands. Ideal local MHD stability with high value of $\beta = 4.25\%$ was analyzed with two different 3D MHD stability codes - COBRA [2] and TERPSICHORE [3]. In this report we present some details of the Mercier (Section 2.) and ballooning (Section 3.) modes stability. The result of this comparison show that the two codes agree as on the stability boundaries.

We use a new version of 3D ideal MHD equilibrium code VMEC [4], VMEC2000, that has: an improved magnetic force balance, an accurate convergence using a new differencing scheme to compute the stream functions for high modes numbers, much finer spatial resolution and an improved assessment of the parallel current. These improvements are especially important for NCSX-li383 case because of the low aspect ratio and wide range of rotational transform profiles. VMEC2000 produces a set of nested ideal surfaces which is used as an input for many MHD ideal stability codes.

The TERPSICHORE code has been used for the numerical assessment of global and local stability properties of many stellarators [5], [6], [7]. It carries out all computations in Boozer coordinates [8]. This causes the parallel current to diverge at rational surfaces spikes to appear at rational surfaces in the Mercier criterion. To prevent the unphysical solution of the ballooning equation for the cases with poor equilibrium, the line bending term has been rewritten [9] using a representation that ensures positiveness independently of the equilibrium accuracy.

A new ideal ballooning code COBRA [2] has been developed to deal well with low aspect ratios and to avoid convergence and accuracy problems. COBRA uses the same flux coordinates as VMEC. A comparison between COBRA and TERPSICHORE results is not trivial since both codes address the high-n stability calculations in different ways, using different normalization and different underlying equilibrium solutions due to the intermediate mapping to Boozer coordinates required by TERPSICHORE. More details of this comparison can be found in the paper, reference [9], about the NCSX configuration candidate c82 and the Oak Ridge QOS stellarator concept [10].

2. Mercier stability results

A calculation of the Mercier criterion [11] as a function of flux surface for the reference value of $\beta = 4.25\%$ was made by TERPSICHORE and VMEC2000 is presented on Fig.2. We use 97 radial points, 83 toroidal and poloidal modes in VMEC2000 and 516 modes for the Boozer reconstructed equilibrium performed in TERPSICHORE. The case is Mercier stable as found by TERPSICHORE except several points around the resonant surfaces with the rational values of the rotational transform 3/6, 3/5 and 9/14 (left, middle and right spikes in the Fig. 2). These resonant spikes can be healed by using a detuning scheme [12]. The method used introduces an artificial, small positive term in denominators to avoid division by zero near the resonant surfaces.

VMEC2000 computes the Mercier criterion in VMEC magnetic coordinates without transformation to Boozer coordinates. The new method used to calculate the parallel current density in VMEC2000 produces the same first order resonant spikes at iota of 3/6 and 3/5 in the Mercier criterion as appears in the TERPSICHORE results. However, the high order resonant spike around rational value 9/14 does not appear from Mercier criterion produced by VMEC2000.

3. Ballooning mode stability

The most unstable region for ballooning modes in the NCSX-li383 case is located near the middle of the system period for the last few plasma magnetic surfaces. The ballooning eigenvalue as a function of flux surface for the most unstable forceline arclength ($\alpha = \pi/3$) made by COBRA and TERPSICHORE codes is shown on Fig.3. The TERPSICHORE code uses 715 extra-ballooning modes and 6 arclength circuits for this calculation. The COBRA code uses 10 circuits and gives the same region of ballooning instability as TERPSICHORE in spite of different normalization and form of the ballooning equation. The results of the convergence study of the TERPSICHORE ballooning code is shown in (Fig.4). The results do not depend on the number of extra-ballooning modes (we have tested also 743 and 853 modes). We have considered arclength of 6, 10 and 25 circuits and find the results to be insensitive to the arclength traversed.

4. Conclusions

The COBRA and the TERPSICHORE ideal MHD stability codes use different normalization, slightly different forms of MHD equilibria, different coordinates and compute eigenvalues from different forms of the ballooning equations. In spite of these differences, we have shown that COBRA and TERPSICHORE predict the same regions of Mercier and ballooning instability and similar critical β for the NCSX-li383 low aspect ratio stellarator. This successful benchmarking proves attests to the reliability of the ideal stability calculations.

Acknowledgments

This work was sponsored by Spanich DGES project FTN2000-0924-C03-01, by the U.S.Departement of Energy under contacts DE-AC02-76CH03073 and DE-AC05-00OR22725, by EURATOM and the Fonds National Suisse pour la Recherche Scientifique, by INTAS-99-00592 project.

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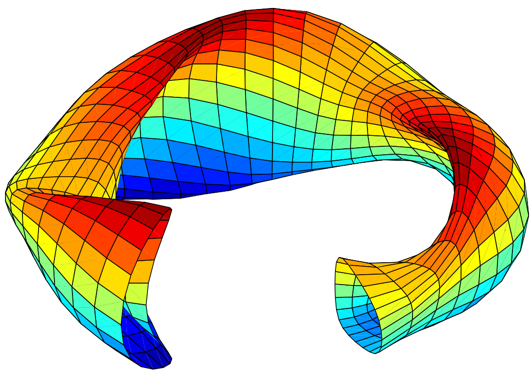

Fig. 1. NCSX-li383 plasma boundary.

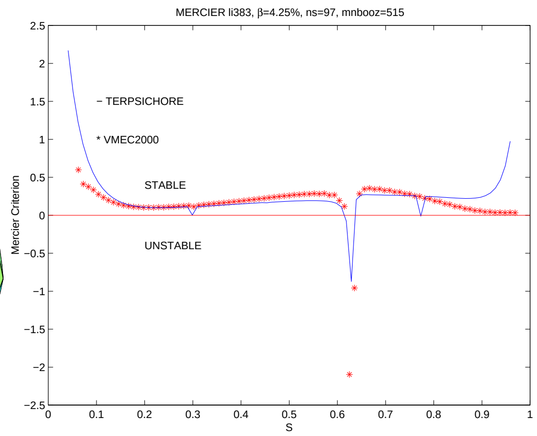

Fig. 2. TERPSICHORE and VMEC2000 Mercier criterion as a function of flux surface S for li383 case, 97 radial surfaces, 83 modes in VMEC2000, 515 Boozer reconstructed equilibirum modes in TERPSICHORE, $\beta = 4.25\%$.

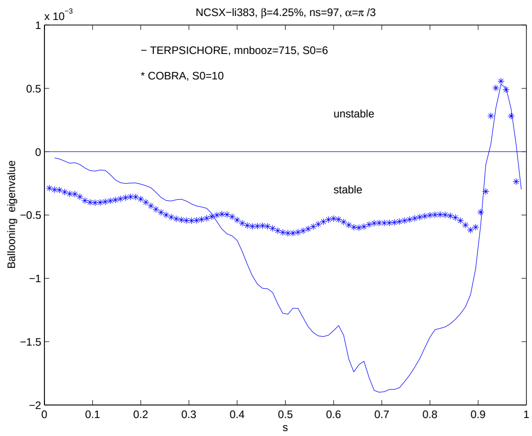

Fig. 3. Ballooning eigenvalue as a function of flux surface S for li383 case, $\beta = 4.25\%$ *by TERPSICHORE (solid line) and COBRA (stars) for the initial forcelenght angle* $\alpha = \pi/3$.

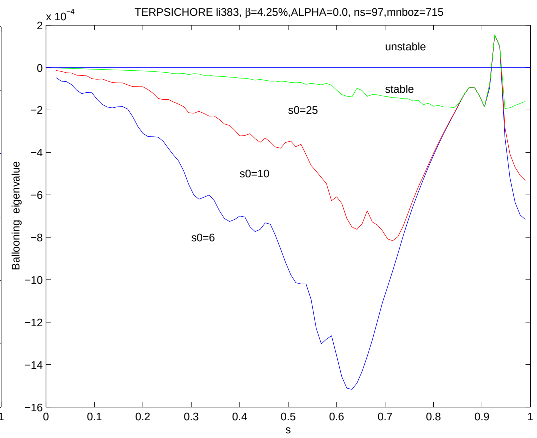

Fig. 4. Convergence study of the ballooning eigenvalues by TERPSICHORE code for different numbers of forceline circuits $(S0 = 6, 10, 25)$, *initial value of the arclengh angle is* $\alpha = 0$, *with 97 radial surfaces, 715 extra-ballooning modes and the value of* $\beta = 4.25\%$.