EFFECTS OF NONLINEAR WAVE-PARTICLE INTERACTION ON THE ELECTRON DISTRIBUTION FUNCTION DURING ECRH *

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1. The model

The parameter validating the linear and quasilinear theory for ECRH using the 2[nd] harmonic X-mode is $\tau_f/\tau_{bE} = \delta L\,|\omega_{c0}|\,\sqrt{NE/B_0}v_\perp/(v_\parallel c)$, where τ_f and τ_{bE} are the time of transit through the ECRH beam and the bounce time of particles trapped in the wave field, respectively. δL, N, E and B_0 are the beam width, the wave refraction index, the amplitude and the main magnetic field strength, respectively. One can check that for present day experimental parameters, the condition $\tau_f/\tau_{bE} < 1$ is violated and that the problem of wave absorption has to be treated taking into account the nonlinear wave-particle interaction.

Here, a model with a uniform magnetic field directed along the z-axis and an ECRH beam propagating in x-direction is used. The system is assumed to have a period length L over z. Two cuts, A and B located on both sides of the rf interaction zone (see Fig. 1) are introduced. In this geometry, neglecting the effect of cross-field transport, the kinetic equation can be transformed to a 2-D integral equation (mapping equation) for the particle flux density through the cuts A and B extended in velocity space, $\Gamma = v_\parallel J f$. Here, J is the phase space Jacobian, and f is the particle distribution function. The mapping of the particle position in phase space between the cuts, $\Gamma = \hat{P}_c\hat{P}_{rf}\Gamma$, is incorporated with two propagators, $\hat{P}_c$ for Coulomb collisions and $\hat{P}_{rf}$ for wave-particle interaction. The integral representation of these mapping relations is given as

$$\Gamma_{\rm A}(v_\perp, v_\parallel, t) = \int_0^\infty dv_{\perp 0}\int_0^\infty dv_{\parallel 0}\int_{-\infty}^\infty dt_0\, p_{+-}(v_{\perp 0}, v_{\parallel 0}; v_\perp, v_\parallel, t-t_0)\Gamma_{\rm A}(v_{\perp 0}, v_{\parallel 0}, t_0) + \int_0^\infty dv_{\perp 0}\int_{-\infty}^0 dv_{\parallel 0}\int_{-\infty}^\infty dt_0\, p_{--}(v_{\perp 0}, v_{\parallel 0}; v_\perp, v_\parallel, t-t_0)\Gamma_{\rm B}(v_{\perp 0}, v_{\parallel 0}, t_0) \quad (1)$$

where p_{+-} is a transition probability density (TPD) from $v_{\perp 0}$, $v_{\parallel 0} > 0$ on cut A to $v_\perp$, $v_\parallel < 0$ on cut A and p_{--} is a TPD from $v_{\perp 0}$, $v_{\parallel 0} < 0$ on cut B to $v_\perp$, $v_\parallel < 0$ on cut A. The expression for $\Gamma_{\rm B}$ has a symmetric form. The operator $\hat{P}_{rf}$ is discretized,

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$$\Gamma_{\rm B}\left(I_\perp, v_\parallel\right) = \int_0^\infty {\rm d}I'_\perp P_{\rm AB}\left(I_\perp, I'_\perp; v_\parallel\right)\Gamma_{\rm A}\left(I'_\perp, v_\parallel\right), \qquad v_\parallel > 0, \tag{2}$$

$$\Gamma_{\rm A}\left(I_\perp, v_\parallel\right) = \int_0^\infty {\rm d}I'_\perp P_{\rm BA}\left(I_\perp, I'_\perp; v_\parallel\right)\Gamma_{\rm B}\left(I'_\perp, v_\parallel\right), \qquad v_\parallel < 0, \tag{3}$$

$$P_{\rm AB}\left(I_\perp, I'_\perp; v_\parallel\right) = \sum_{i,j=1}^{\infty} \delta\left(I_\perp - I'_\perp + I'^{(j)}_\perp - I^{(i)}_\perp\right) P^{ij}_{\rm AB}\left(v_\parallel\right) \times \Theta\left(I'^{(j+1)}_\perp - I'_\perp\right)\Theta\left(I'_\perp - I'^{(j)}_\perp\right), \tag{4}$$

where the transition probabilities $P^{ij}_{\rm AB}$ are numerically obtained by following the particle orbit in the wave electric field $\mathbf{E} = E_0{\rm Re}\left[\mathbf{f}F(\mathbf{r})e^{i(\mathbf{kr}-\omega t)}\right]$. Here, E_0, $\mathbf{f} \equiv \mathbf{E}/|\mathbf{E}|$ and $F(\mathbf{r}) = \exp\left(-\alpha(z^2+y^2)/2\right)$ are the wave amplitude, the polarization vector and the form factor for the Gaussian beam shape, respectively. The Hamiltonian is of the form

$$H = m_0c^2\gamma - \frac{v_E k_\perp^{n_0-1}\left|2m_0\omega_{c0}n_0 I_\perp\right|^{\frac{n_0}{2}}}{2(n_0-1)!\left(2m_0\omega_{c0}\right)^{n_0-1}}|f^-|F(\mathbf{R})\sin\left(k_\parallel z + \psi\right) - \omega I_\perp. \tag{5}$$

Here, $n_0 = 2$ and $k_\parallel = 0$ (perpendicular injection). $\psi = n_0\phi - \omega t + \psi_0$ is the wave-particle phase, ϕ the particle gyrophase, $v_E \equiv eE/(m_0\omega)$, m_0 is the particle mass, and ω_{c0} is the gyrofrequency at rest. Eq. (5) is the conventional form of the Hamiltonian where the expansion over a small electron Larmor radius has been used where only the resonant term is retained [2].

For computing the transition probabilities, the phase space is discretized with respect to the canonical action $I_\perp$ by introducing levels $I^i_\perp$. The TPD $P^{ij}_{\rm AB}$ from the band between levels $I^j_\perp$ and $I^{j+1}_\perp$ on cut A to the band between the levels $I^i_\perp$ and $I^{i+1}_\perp$ on cut B is defined as the overlapping area of the image of the band from cut B mapped to cut A along the orbits with the band on cut A normalized with the total band area (Fig. 1). This information is obtained numerically and stored on a grid of $v_\parallel$ values. In the considered case, the change of $v_\parallel$ is negligible and the Hamiltonian system can be transformed, by neglecting small parameters, to the form where $v_\parallel$ is an invariant of motion. Of course, in general the Hamiltonian itself is an invariant of motion. In the case of the adiabatic model, the $P^{ij}_{\rm AB}$ have a simple form, $P^{ij}_{\rm AB} = \frac{1}{2}(\delta_{i,j} + \delta_{i,k_0-j})$ for $k_{min} < i,j < k_{max}$ and $P^{ij}_{\rm AB} = 1$ otherwise. Here, $I^{k_0}_\perp = I_\perp(v^{res}_\perp)$ with $v^{res}_\perp$ being the position of the resonance zone in the linear case and $I^{k_{min},k_{max}}_\perp = I_\perp(v^{min,max}_\perp)$ defines the boundaries of the nonlinearly broadened resonance zone.

2. Computation results

The integral equation is solved using the Monte Carlo (MC) algorithm. The propagator $\hat{P}_c$ is sampled with a conventional MC method (see e.g [1]). Examples of transition probabilities, $P_{\rm AB}^{ij}$, used for sampling the rf propagator are shown in Fig. 1 for particles with a high $v_\parallel$ when the quasilinear theory is valid, with a medium $v_\parallel$ and for particles with a low $v_\parallel$ when the adiabatic theory is formally valid. The structure on the right side is due to the particle phase memory in the beam - an effect discarded by adiabatic theory. In Fig. 3 a typical particle distribution function in the nonlinear case is shown. This distribution function is asymmetric and a plateau-like structure is formed around the resonance zone. Here, unlike in results of the quasilinear theory, regions with positive derivative with respect to the perpendicular velocity appear. With such a distribution function, the absorption coefficient (Fig. 4) is computed. It is given as $\alpha_{NL} = \left(\int {\rm d}z P_{\rm abs}\right)\left(\int {\rm d}z S_x\right)^{-1}$, where S_x denotes the Poynting flux and $P_{\rm abs}$ the absorbed power density. The absorption coefficient is strongly reduced as compared to the linear theory due to both the distortion of the particle distribution function and the nonlinear nature of the wave-particle interaction. As a result, the absorbed power profile, for a typical set of parameters (see [3]), appears to be essentially broader than predicted by linear theory with an optical depth reduced by a factor of 10.

3. Conclusion

A numerical model for ECRH which consistently takes into account nonlinear wave-particle interaction has been developed. The results of computations show that the distortion of the particle distribution function from Maxwellian is strong for parameters typical for present day ECRH experiments. This leads to a reduction of the absorption, consequent broadening of the absorption profile and incomplete absorption. The assumptions made within this computation are not essential and the consideration of a realistic situation is straightforward. Especially, for realistic magnetic field configurations, the propagator $\hat{P}_c$ can be sampled using the mapping technique [4]. It should be mentioned that the distortion of the particle distribution function is essentially different from what is expected from the quasilinear theory where a Fokker-Planck equation is assumed to be valid. The effect of power redistribution in velocity space can especially be important for current drive. Therefore, the nonlinear rf interaction has to be properly described.

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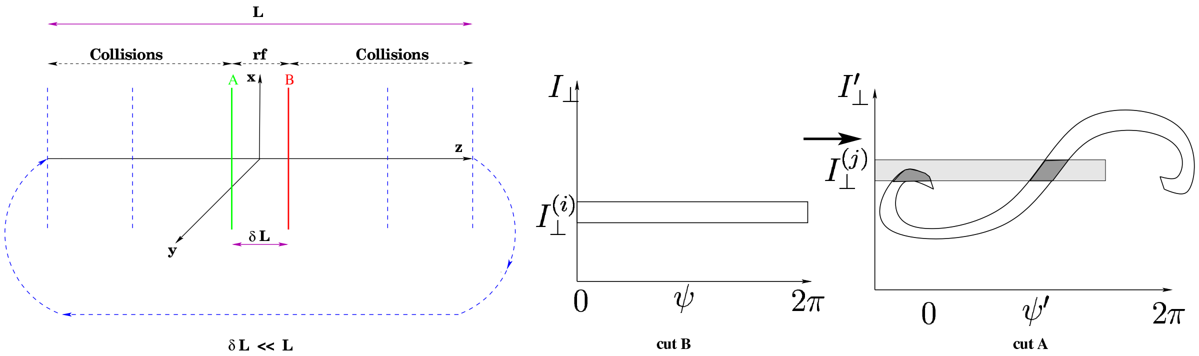

Figure 1: Geometry (left) and scheme for discretized transition probability densities.

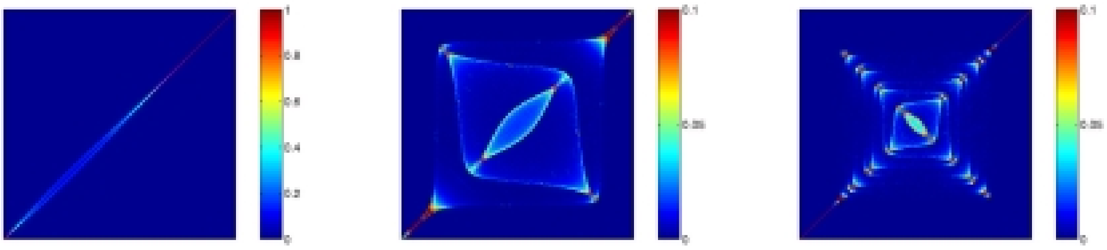

Figure 2: Color density plot of transition probability densities for canonical perpendicular action when traveling through the wave beam. Increased nonlinearity from left to right.

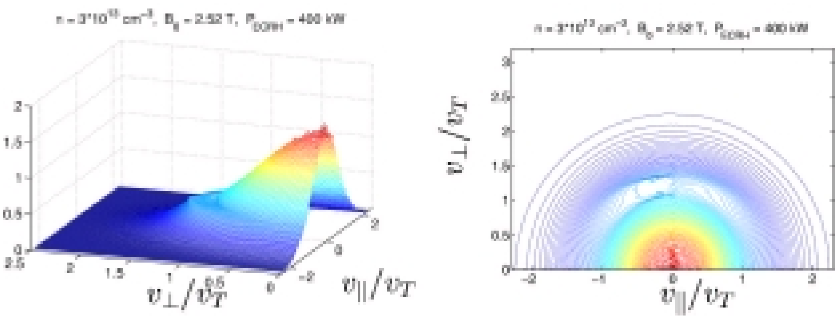

Figure 3: Distribution function and its contour.

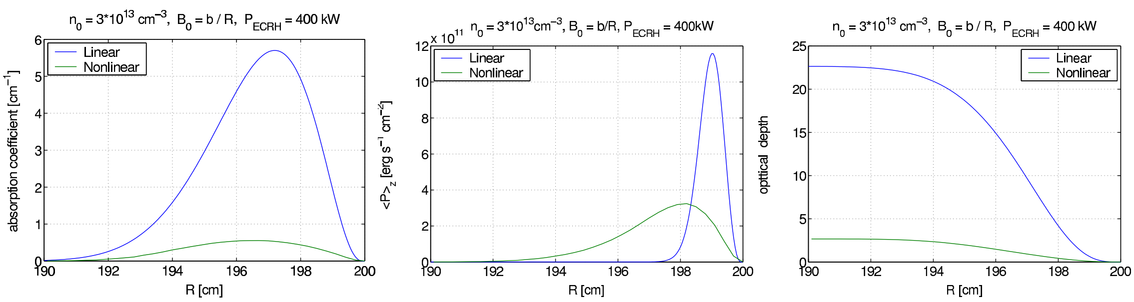

Figure 4: Absorption coefficient, field line integrated absorbed power density profile and optical depth. $B_s = 2.5$ T, $R_s = 200$ cm, $b = B_s R_s$. Density profile is parabolic with maximum n_0 at plasma major radius R_s.