Monte Carlo δf simulations of the bootstrap current in the presence of an island

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Neoclassical tearing modes (NTMs) have been found to often determine the achievable β in long-pulse discharges in tokamak devices, and are predicted to be the most significant β-limiting phenomenon for ITER. The NTM occurs when a sufficiently large resonant magnetic perturbation, the so-called "seed" island, is produced by the background MHD activity. The perturbed magnetic configuration leads to a flattening of the pressure profile inside the island and, consequently, to a loss of the bootstrap current, which in turn reinforces the perturbation and drives the instability.

The theoretical description of NTMs is based on the generalized Rutherford equation [1], in which the various mechanisms that can stabilize or destabilize the mode are taken into account. It is obtained by integrating Ampère's law across the island region, using Ohm's law to express the inductive contribution to the current density. The result is an evolution equation for the island half-width W

$$\frac{4\pi}{1.22\eta c^2}\frac{dW}{dt} = \frac{\Delta'}{2} + \frac{4\sqrt{2}}{c}\frac{qR}{sBW}\int_{-1}^{\infty} d\Omega \oint \frac{d\xi \cos\xi}{\sqrt{\cos\xi + \Omega}}\, j_{\|}^{n.i.}. \tag{1}$$

In the previous equation, η is the neoclassical resistivity, Δ' is the stability index of the equilibrium current profile [2], q is the safety factor, R is the major radius, s is the magnetic shear and B the magnetic field strength. A helical angle $\xi \equiv m\theta - n\varphi$, where m and n are the poloidal and toroidal number of the resonant rational surface and θ and φ are the poloidal and toroidal angles, respectively, has been introduced along with a normalized helical flux $\Omega \equiv (q_s'/2q_s)(\psi-\psi_s)^2/\tilde{\psi} - \cos\xi$, where ψ is the unperturbed poloidal flux, the prime denotes the derivative with respect to ψ, $\tilde{\psi}$ is the strength of the flux perturbation and the subscript s means that a quantity is evaluted at the resonant surface. Ω is defined in such a way that $\mathbf{B}\cdot\nabla\Omega = 0$ and $\Omega = -1$ at the O-point of the island and $\Omega = 1$ at the separatrix. Here, the only contribution to the non-inductive part of the current $j_{\|}^{n.i.}$ is supposed to be given by the bootstrap current j_{bs}, which must be calculated in the perturbed magnetic configuration and substituted in Eq. (1). This yields

$$\frac{4\pi}{1.22\eta c^2}\frac{dW}{dt} = \frac{\Delta'}{2} + a_2\sqrt{\varepsilon}\frac{L_q}{L_p}\frac{\beta_\theta}{W}\frac{1}{1+(W_0/W)^2}, \tag{2}$$

where a_2 is a numerical coefficient of order one, $\beta_\theta = 8\pi p/B_\theta^2$ (p is the pressure and B_θ the poloidal field), $1/L_q = d\ln q/dr$, $1/L_p = -d\ln p/dr$ and $\varepsilon = r_s/R$ is the inverse aspect ratio of the resonant surface. In the previous equation, the role of finite perpendicular transport in preventing a complete flattening of the pressure profile inside the island [3] has been taken into account and the corresponding reduction of the neoclassical drive is expressed by the term containing $W_0 = 2.55 r_s (\chi_\perp/\chi_\parallel)^{1/4} (q/ms\varepsilon)^{1/2}$. An important remark to Eq. (2) is to be made. In order to obtain an analytic expression for j_{bs} to be substituted in Eq. (1), it is supposed that the island width W is much larger than the ion banana width $w_b = \sqrt{\varepsilon}\rho_\theta$, where $\rho_\theta = v_T/\omega_{c\theta}$ is the ion poloidal gyroradius (v_T is the thermal velocity of the ions and $\omega_{c\theta}$ is ion cyclotron frequency calculated using the poloidal magnetic field). In this limit, the bootstrap current completely vanishes inside the magnetic island (if the aforementioned finite- $\chi_\perp$ effect is neglected). However, the opposite limit also deserves careful investigation, since at least in the early phase of a NTM it is often $w_b \approx W$. In this case, it can be thought that the particles trapped in the region around the island (where the pressure gradient is not flat) significantly overlap the island and might then provide the source for the bootstrap current also inside it. This effect is of course supposed to be larger for the ions than for the electrons, which have a much smaller banana width.

The role of the finite banana width of the ions is studied by solving the drift-kinetic equation for the ion distribution function

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_d\right) \cdot \nabla f = C(f) \tag{3}$$

in the presence of an island (the width of the island is kept constant and its position in the plasma is held fixed). The δf method is employed. The solution f of Eq. (3) can be written as the sum of f_0 which is analytically known and a second term δf which expresses the temporal evolution of the distribution function and is to be determined numerically. In our approach, δf is represented by the distribution in the phase space of an ensemble of markers ('particles') which evolve according to a Hamiltonian set of equations of motion. Here f_0 is assumed to be a Maxwellian f_M ; Eq. (3) then gives

$$\frac{\partial \delta f}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_d\right) \cdot \nabla \delta f = C(\delta f) - \mathbf{v}_d \cdot \nabla f_M. \tag{4}$$

Eq. (4) is integrated over a collisional time interval Δt_c in two steps. First, the markers evolve collisionfree according to the equations of motion. They are integrated using the guiding centre code HAGIS [4], which solves the equations of motion in toroidal geometry in the presence of a perturbation of the magnetic equilibrium employing Boozer coordinates. In a second step, collisions are modelled by a Monte Carlo procedure [5]. The parallel velocity of the particles obtained from the first step is modified according to the pitch-angle part of the collision operator. The change in the parallel velocity of the particles can be written $\delta v_\parallel = -v_\parallel \nu \Delta t + \gamma v_\perp \sqrt{\nu \Delta t_c}$ (so that $\delta v_\perp^2 = -(2v_\parallel + \delta v_\parallel)\delta v_\parallel$), where $\nu(v) = (3\sqrt{2\pi}/4\tau_i)(v_T/v)^3\, G(v/v_T)$, τ_i is the ion-ion collision time, $G(x) = [(x^2 - 1/2)\,\mathrm{erf}(x) + x\exp(-x^2)/\sqrt{\pi}]/x^5$ and γ are random numbers such that $\langle\gamma\rangle = 0$ and $\langle\gamma^2\rangle = 1$. The scheme is implemented in such a way that momentum is conserved.

Quantities of interest are obtained by flux surface average according to the definition

$$\langle A \rangle = \lim_{\delta\Omega \to 0} \frac{\int A d^3\mathbf{r}}{\int d^3\mathbf{r}} \Rightarrow \frac{1}{n} \langle \int A\delta f \, d^3\mathbf{v} \rangle \simeq \frac{\int_{\Omega-\delta\Omega}^{\Omega+\delta\Omega} A\delta f \, d\Gamma}{\int_{\Omega-\delta\Omega}^{\Omega+\delta\Omega} f_0 \, d\Gamma}, \tag{5}$$

where $d\Gamma$ is the phase-space volume element. Therefore, the plasma column is divided into cells, bounded between two neighbouring flux surfaces (labeled by the helical flux Ω).

The approach described previously is applied to the study of the bootstrap current in the island region for the case of a (3,2) mode in a tokamak with ITER-like and ASDEX Upgrade (AUG)-like parameters. The magnetic equilibrium is specified analytically and the unperturbed flux surfaces are circular and concentric. The parameters are chosen such that the bounce time $\tau_B = qR/v_T\sqrt{\varepsilon}$ is much shorter than the trapped-to-passing scattering time $\tau_S = \varepsilon/\nu_i$, and the plasma is hence in the banana collisionality regime $\nu_* \equiv \tau_B/\tau_S \ll 1$. In order to investigate finite-banana-width effects on the neoclassical drive of the tearing mode, the bootstrap current in the island region has been studied varying the ratio w_b/W .

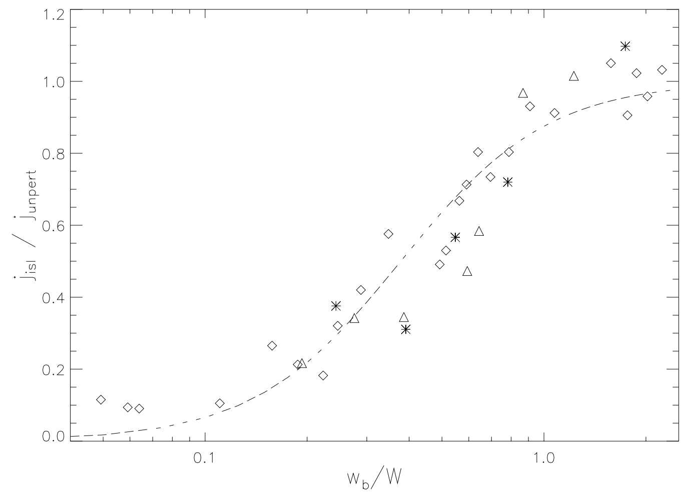

Fig. 3. Averaged current density inside the island versus the ratio w_b/W . Diamonds refer to simulations performed using ITER parameters, triangles to AUG parameters with hydrogen plasma, stars to AUG parameters with deuterium plasma.

The results are summarized in Fig. 3. The averaged current density inside the island j_{isl} (normalized to the unperturbed current density at the resonant surface) is plotted as a function of the ratio w_b/W for ITER and ASDEX Upgrade parameters. For large values of the island width, $j_{isl} \to 0$ according to the standard picture of the NTM. When the island width is reduced, j_{isl} increases until it reaches the unpertubed value for $W \approx w_b$. In this case, no perturbation of the ion bootstrap current is present in the plasma and there is no ion contribution to drive of the mode. This has significant consequences for instance for AUG, since the typical width of the seed island which triggers the mode is between 1 and 5 cm, and the width of a banana orbit is between 7 mm and 3 cm, depending on both the plasma composition and discharge parameters.

Hence, at least in its early phase the NTM is more stable than usually assumed. The data can be fitted by the curve $j_{isl}/j_{unpert} \approx 7x^2/(1+7x^2)$ showing a quadratic dependence on $x \equiv w_b/W$. This can be connected with the rough estimate that the strength of finite-banana-width effects is proportional to the area of the island overlapped by the trapped particles.

It is interesting to finally discuss the scaling of β_θ at the onset of the NTM as a function of the normalized ion poloidal gyroradius $\rho_\theta^* \equiv \rho_\theta/a$ (where a is the minor radius of the tokamak). At AUG, a linear scaling law $\beta_\theta^{crit} \propto \rho_\theta^{*1.02}$ has been observed [6]. Supposing $W_0 < w_b$, it can be assumed that the most important stabilizing effect for the ions at small island widths is that presented in this paper (the role of the polarization current is neglected). For the electrons, the finite perpendicular transport can be taken as the main stabilizing effect at small island widths. The Rutherford Eq. (2) might therefore be modified to

$$\frac{4\pi}{1.22\eta c^2}\frac{dW}{dt} = \frac{\Delta'}{2} + \frac{a_2}{2}\sqrt{\varepsilon}\frac{L_q}{L_p}\frac{\beta_\theta}{W}\left(\frac{1}{1+(W_0/W)^2} + \frac{1}{1+7(w_b/W)^2}\right), \qquad (6)$$

where the second term between parentheses has been taken according to the fit of Fig. 3. The value of β_θ^{crit} corresponding to marginal stability ($dW/dt = 0$ for some given seed $W = W_{seed}$) can be calculated directly from Eq. (6). In the limit $W_0 < w_b$ this yields

$$\beta_\theta^{crit} \propto \frac{y^3 + 7y}{2y^2 + 7}\frac{w_b}{r_s}, \qquad (7)$$

where $y \equiv W_{seed}/w_b$. The scaling $W_{seed}/r \propto \rho_\theta^{*3\alpha}$ has been predicted theoretically [7], where the exponent α depends on the details of the physical model. A fit to AUG data gives [8] $\alpha \approx 0.38$. Since it is clearly $w_b/r \propto \rho_\theta^*$, it results that y depends very weakly on ρ_θ^*. Eq. (7) hence gives $\beta_\theta^{crit} \propto w_b/r \propto \rho_\theta^*$ which is in agreement with the experimental observations of AUG. This model provides then an alternative to the polarization current model for the explanation of the observed scaling.

All previous studies have neglected the overlap of the seed island by the trapped particles. This effect is expected to be significant for both the interpretation of the experimental results as well as the extrapolations to future machines.

References

1) O. Sauter et al., Phys. Plasmas **4**, 1654 (1997).
2) P. H. Rutherford, Phys. Fluids **16**, 1903 (1973).
3) R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).
4) S. D. Pinches et al., Comp. Phys. Comm. **111**, 133 (1998).
5) A. Bergmann, A. G. Peeters, S. D. Pinches, submitted to Phys. Plasmas (2001).
6) S. Günter, A. Gude, M. Maraschek, S. Sesnic, H. Zohm and the ASDEX Upgrade Team, Nucl. Fusion **38**, 1431 (1998).
7) C. C. Hegna, J. D. Callen, J. R. La Haye, Phys. Plasmas **6**, 130 (1999).
8) S. Günter et al., Plasma Phys. Control. Fusion **41**, B231 (1999).