Theory of toroidal rotation in Alcator C-Mod ohmic H-mode plasmas

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I. Introduction

Recently, bifurcation from the low (L) to the high (H) confinement mode has been obtained in ohmic Alcator C-Mod plasmas. After a delay of the order of the energy confinement time τ_E following the bifurcation, the toroidal velocity measured on the magnetic axis by means of the Doppler shift of Argon lines was observed to increase from a negligible negative to a finite positive value (of the order of 40 kms^{-1}) [1]; a velocity is positive if in the co-current direction. Under certain conditions, an internal transport barrier (ITB) also forms in the H-mode discharge; during the ITB phase, electrons and impurities peak continuously for $r/a \leq 0.5$ (a is the minor plasma radius) while the central toroidal rotation decreases significantly [2].
We show that a recent neoclassical theory of toroidal and poloidal rotation appropriate to high collisionality plasmas (as those in the reference discharges) with large gradients (as occur in H-mode pedestals) explain well the observations reported in [1]. Further, the observations in [2] are explained by a straightforward extension of the equations obtained in [3], where it had been explicitly stated that "*With the ordering adopted here, the contributions arising from the density evolution and from the radial flux of angular momentum are negligible in the framework of the neoclassical theory **of a one ion species plasma***", and ion inertia had been neglected accordingly.

II. Theoretical model

There is no accelerating force in the experiments under consideration and the role of cx neutrals will be neglected presently (discussion will be included in a forthcoming paper). Under those conditions, Eq.(1) of [3] can readily be integrated twice to yield

$$[U_{\varphi,i}]_{r_s}^{r} = 0.107q^2(B_\varphi / B_\theta)\int_{r_s}^{r}\exp[\int_{r'}^{r}(m_i N_i U_{r,i} / \eta_{2,i}]dr''](1+Q^2/S^2)^{-1}U_{\theta,i}\partial_{r'}\ln T_i dr' \quad (1)$$

$$Q/S = 0.51\Lambda[(eB_\varphi / \partial_r T_i)U_{\theta,i} - 0.625(1+2\eta_i^{-1})], \quad (2)$$

$$\Lambda = (qR\nu_i / c_i)(a_i)_p / L_{T_i}. \quad (3)$$

$(a_i)_p$, $c_i = (T_i/m_i)^{1/2}$, ν_i and $\eta_{2,i} = 1.2\ P_i\nu_i/\Omega_i^2$ are the ion Larmor radius in the poloidal magnetic field, the thermal velocity, the collision frequency and the perpendicular viscosity, respectively; we recall that $\Lambda \equiv 0$ in "conventional" neoclassical theory. The exponential factor, where $U_{r,i}$ is the ion radial velocity, arises when keeping the radial flux of angular momentum into account [see Eq.(8') of [3] and the following discussion]. Since $U_{\theta,i} \propto \partial_r T_i$, (1) shows that the toroidal velocity experiences a "kick" across the pedestal -where $\partial_r T_i$ is large- and remains relatively constant in the core -where $\partial_r T_i$ is small-. This also holds for the discharge with ITB, as the latter affects mostly the density profile. It is interesting to note that (i) the toroidal velocity is a *global* (by opposition to local) property of the discharge and (ii) the toroidal velocity in the core is barely dependent on core transport; *thus it is irrelevant if core transport is dominated by turbulence*! We shall assume that the toroidal velocity in ohmic discharges vanishes at the radius r_s of the last closed flux surface (LCFS). The poloidal velocity is provided by Eq.(4) of [3]. It can easily be shown that $U_{\theta,i}$ vanishes for a peculiar value Λ_0^2 of Λ^2. We shall further assume that peculiar value to corresponds to the LCFS parameters; thus

$$\Lambda_0^2 = \Lambda_s^2 = [0.299(1+1.6\eta_i^{-1})^2 - 0.102(1+2\eta_i^{-1})^2]^{-1} \quad (4)$$

The above hypothesis requires that the parallel flow be directed into both the right and the left divertor legs all across the scrape off layer (SOL); it is in any case consistent with the measured density, temperature and gradients at the LCFS. (We have investigated the alternative hypothesis whereby the radial electric field would vanish at the radius of the LCFS; that leads to a contradiction, namely Λ_s^2 should be negative!) Here, the solution of Eq.(4) of [3] is merely interpolated linearly between the values in the core (where $\Lambda \to 0$ and -in agreement with [4]- $U_{\theta,i} \to -1.83\ \partial_r T_i/eB_\varphi$) and at $r = r_s$:

$$U_{\theta,i} = -1.83(1-\Lambda^2/\Lambda_s^2)\partial_r T_i / eB_\varphi \quad (5)$$

Inserting (5) into (2) shows that $1+Q^2/S^2$ is a cubic function of Λ^2. To proceed further, we fit that cubic by a quadratic (we match at $\Lambda^2=\Lambda_s^2$ and $\Lambda^2 = \Lambda_M^2$ where $1+Q^2/S^2$ is maximum). We further assume the pedestal temperature, density and effective charge profiles (the latter enters the collision frequency) to be of the form

$$T_i(r) = T_i(r_{inf})[1-\tanh(r-r_{inf})/\Delta_T], \tag{6}$$

$N_i(r) \propto [T_i(r)]^{1/\eta_i}$ and $Z_{eff,i}(r) \propto [T_i(r)]^z$, with z such that

$$\Lambda = \Lambda_{inf}[2-(T_i/T_{i,inf})] \tag{7}$$

(We note that $L_T^{-1} = \Delta^{-1}[(T_i/T_{i,inf})-2] \leq 0$). *In the limit of negligible radial particle flux ($U_{r,i}\to 0$)*, the integral in Eq.(1) can then be carried out, leading to

$$U_{\varphi,i}(r) = \frac{0.098q^2}{(\alpha^2-4\beta)^{1/2}}\frac{\Lambda_s^2}{\Lambda_{inf}^2}\left[\ln[\frac{\Lambda_{inf}^2}{\Lambda_s^2}(2-\frac{T_i}{T_{i,inf}})^2-a_-]^{1-a_-}[a_+-\frac{\Lambda_{inf}^2}{\Lambda_s^2}(2-\frac{T_i}{T_{i,inf}})^2]^{a_+-1}\right]_{r_s}^{r}\frac{(\partial_r T_i)_{inf}}{eB_\theta} \tag{8}$$

where α, β and $a_\pm$ are numerical coefficients depending on η_i and Λ_s^2, itself a function of η_i [Eq.(4)]. It has been verified numerically that the toroidal velocity calculated for the core does not depend strongly on any of the simplifications introduced for analytical purpose: the temperature hyperbolic tangent model (6), the linear fit of the poloidal velocity profile (5), the quadratic fit of $1+Q^2/S^2$ and the choice (7) of the function Λ/Λ_{inf}.

III. Results and comparison with experiment

1. H-mode discharge without ITB

The characteristic parameters of the discharge analyzed in [1] are: $\eta_i \cong 1.6$, $T_{i,inf} \cong 165$eV, $N_{i,inf} \cong 1.87\text{x}10^{20}\text{m}^{-3}$, $Z_{eff} \cong 1.4$, hence $Z_{eff,i} \cong 1+0.4\sqrt{2} \cong 1.57$, $\Delta = (L_{Ti})_{inf} \cong 0.6\text{x}10^{-2}$m, B = 5.2T, B_θ =0.625T (keeping account of the vertical elongation κ=1.62), q = 3.4; thus $(qR\nu_i/c_i)_{inf} \cong 1.73$ (the pedestal plasma is in the high collisionality regime), $\Lambda_s^2 \cong 1.47$, $\Lambda_{inf}^2 \cong 0.46$, α = 3.25, β = -2.49, $a_- = -0.26$ and $a_+ = 1.56$. According to (7), the ratio of the temperatures at the LCFS and at the inflexion point is ≅ 0.2, which corresponds to experimental data. According to (8), the toroidal velocity in the core ($r - r_s << \Delta$) is ≅ 39 kms^{-1}, which also corresponds well to the experimental value of 35 kms^{-1}. The radial electric field can be calculated from the toroidal and poloidal rotation velocities and the pressure gradient according to the relation

$$E_r = B_\theta U_{\varphi,i} - B_\varphi U_{\theta,i} + (eN_i)^{-1}\partial_r P_i \tag{9}$$

Profiles are merely described here, because of lack of space: the toroidal velocity increases monotonically from the LCFS to the core, where it reaches the above asymptotic value of 39 kms^{-1}; the poloidal velocity increases from the LCFS inwards up to a maximum value of 6 kms^{-1} at the radius corresponding to $T_i/T_{i,inf}$ = 1.25 and then decreases together with the temperature gradient to a negligible value in the core; finally, the radial electric field assumes the characteristic shape observed in DIII-D H-mode discharges with NBI heating [5]: E_r = –13 kVm^{-1} is slightly negative at the LCFS, decreases to a minimum value of –53 kVm^{-1} at r_{inf} and then increases up to ≅ +24 keVm^{-1} in the core, where it stays approximately constant.

2. H-mode discharge with ITB

Since $r < r' < r_s$, the exponential factor in the integrand of Eq.(1) is smaller than unity if $U_{r,i} > 0$, i.e., if the main ion flow is outward. This is probably the case for the discharge described in Figs.(3) and (4) of [2] (the latter is ICRF heated; similar behavior is observed with ohmic heating alone): indeed, although both electrons and impurities peak continuously for r/a ≤ 0.5, one can legitimately conjecture that electron accumulation is only a side effect and that -in line with neoclassical predictions- impurity influx is primarily balanced by the main ion outflux. Reduction of the toroidal velocity will occur if

$$-\int_{r'}^{r<<r_s} (m_i N_i U_{r,i}/\eta_{2,i})dr'' \approx 2\Delta(U_{r,i}/1.2a_i^2\nu_i)_{inf} \geq 1 \tag{10}$$

where a_i is the genuine Larmor radius. Introducing $\nu_i = 0.69\text{x}10^5 s^{-1}$ and $a_i = 0.35\text{x}10^{-3}$m at the inflexion radius, this condition becomes $U_{r,i} \geq 0.84$ ms^{-1}. As a point of comparison, the experimental value of the edge electron radial velocity estimated from their accumulation rate in the core is $-U_{r,e} \cong 0.25$ ms^{-1}: thus our theoretical model can explain the peculiar behavior of H-mode discharges with ITB formation and concomitant impurity accumulation, if the latter is primarily balanced by an outward main ion flux.

[1] Rice, J.E., et al., Phys. Plasmas **7** (2000) 1825.

[2] Rice, J.E., et al., Nucl. Fusion **41** (2001) 277.

[3] Claassen, H.A., Gerhauser, H., Rogister, A.L., Yarim, C., Phys. Plasmas **7** (2000) 3699.

[4] Hazeltine, R.D., Phys. Fluids **17** (1974) 961.

[5] Burrell, k.H., et al., Plasma Phys. Control. Fusion **34** (1992) 1859.