TEARING MODE STABILITY STUDIES ON DIII-D

D. Brennan,¹ M.S. Chu, S. Kruger,² R.J. La Haye, T.C. Luce, E.J. Strait, T.S. Taylor, and A.D. Turnbull

General Atomics, P.O. Box 85608, San Diego, California ¹ORISE, Oak Ridge, Tennessee, USA. ²SAIC, San Diego, California, USA.

I. INTRODUCTION

The prevention of tearing modes in tokamak plasmas is a major hurdle for fusion. Tearing modes are the topological rearrangement of field lines through reconnection to form magnetic islands. In tokamaks tearing modes enhance cross field transport, degrade confinement, and often lead to disruptions. Although the mechanism for the onset of tearing modes in high β plasmas is not well understood, a linear stability index for the tearing mode Δ' was described by Furth et al. [1], and developed further by others [2,3]. However, until recently [4] the tools to evaluate this parameter with realistic equilibria were not available. In general, these calculations can be extremely sensitive to the equilibrium profiles, and can even be sensitive to deviations in the profiles that are smaller than the level of detail in equilibrium reconstructions that present day diagnostics will allow [5], although experimental data is in general reproducible and robust. This paper focuses on the study of the sensitivity of Δ' to fitting parameters of high quality high-resolution kinetic equilibrium reconstructions, and presents accurate stability calculations which indicate that tearing modes previously thought to be neoclassically seeded instead may be linearly unstable at the point of onset.

II. SENSITIVITY STUDIES OF Δ' CALCULATIONS

Sensitivity studies of the stability of equilibrium reconstructions just before the onset of the modes were done with a single-parameter family of equilibria. Each family is based on fits to experimental data for a single time slice in a DIII-D discharge. Using the EFIT equilibrium code, the $FF'(\psi)$ and $P'(\psi)$ profiles are parameterized as 3-knot bicubic splines. In an attempt to retain the self-consistency of the equilib-

rium fits, the radial position of the intermediate knot is used as the parameter to be varied, keeping all other equilibrium constraints constant. The fitting procedure then minimizes the figure of merit χ^2 for each value of the knot location.

The results from calculations of Δ' of a 3/2 mode using the PEST-III code on a set of equilibria fit to a low q_{min} ELMing H–mode shot just before onset of the mode are shown in Fig. 1(b). For this family of equilibria at the 3/2 surface, there is a pole in knot position space, while the equilibrium fits are hardly changing. On the "left" of the pole all calculations are ideal unstable and Δ' is negative, while on the "right" all calculations are ideal stable and Δ' is positive. Somewhat below 0.3 in knot position the equilibrium is calculated ideal unstable such that PEST-III won't continue. Because of the assumption of an inertia free plasma, the Δ' values on the left of the pole are meaningless.



Fig. 1. The results from a survey of knot positions for equilibrium fits to a low q_{min} ELMing H-mode shot, showing (a) the χ^2 values and (b) the resulting Δ' from PEST-III.

In the simplest sense, Δ' is defined as the jump in B_r'/B_r . By Newcomb [6], if B_r changes sign anywhere the plasma is ideally unstable. If B_r changes sign at a rational surface, $\Delta' \rightarrow \infty$. For the case in Fig. 1, $q_{min} \leq 1$ near $\rho = 0$, the q=1 rational surface becomes ideally unstable with the change in knot position, and Δ' goes through a pole. One must distinguish the case where both γ' (the interchange stability index) and Δ' have a pole from the case where only one or the other has a pole. The first case represents ideal instability, while the second represents a parity selection between the interchange and tearing solutions.



Fig. 2. The calculated Δ' values using PEST-III for a family of high q_{min} AT equilibria with changing knot position, showing a robustly positive Δ' with no poles.

If q_{min} is significantly above 1 then the ideal

stability boundary described above can be avoided. In Fig. 2 are shown the results for the 2/1 mode from PEST-III in a high q_{min} (~1.6) plasma just before the onset of the mode. No poles are present in the data, and no ideal mode is calculated unstable. Furthermore, the value of Δ' is consistently positive for all knot positions. In the absence of an ideal stability boundary, Δ' is much less sensitive to the equilibrium fitting parameters than in the case of Fig. 1. Although the values are 1–2 orders of magnitude larger than expected, the sign and relative values between Δ' at various surfaces may still be meaningful, in a normalized sense.

III. COMPARISON OF Δ' CALCULATIONS TO OBSERVED MODE ONSET

The predictive capability of Δ' calculations has been tested in several cases with widely varying plasma characteristics. Here we use optimized equilibrium reconstructions from experimental data, with multiple intermediate knots placed at the locations giving a minimum in χ^2 , rather than the simplified equilibria discussed in Section II.

An experiment was conducted on DIII–D specifically to test the predictive capability of PEST-III in a low β discharge when a tearing mode is expected to be classically unstable. In Fig. 3 we see the results of this experiment and the stability calculations on a series of best equilibrium fits. The onset time agrees with the point where Δ' changes sign. This shows that good agreement can be achieved, and although sensitivity studies in this case are on going, ideal modes are not expected to play a large role in this case.

In Fig. 4 we see the time history of the n=2 and n=1 mode amplitudes for the low q_{min} shot of Fig. 1 described above, along with the calculated Δ' values for the 3/2 mode. Note that just before the n=2 mode begins growing at 3600 ms, Δ' is positive, whereas earlier in the shot it is negative. This would suggest that the mode is classically unstable at the time of onset. Although ideal modes are not unstable for these equilibria, they are near marginal stability.

In the high q_{min} shot of Fig. 2 described above, Δ' was calculated as being positive. This result is surprising because the instabilities seen in these high β plasmas are gener-



Fig. 3. Time traces of Δ' for the 2/1 mode for two conformal wall locations from PEST-III, and the measured mode amplitude of the rotating and non-rotating n=1 MHD activity.

ally thought to be neoclassical tearing modes [7,8] that are linearly stable with negative Δ' , which grow nonlinearly when seeded by forced reconnection. In fact, this discharge contains a rich assortment of MHD modes, as shown by the spectral decomposition of magnetic probe signals in Fig. 5. The poloidal mode numbers for the data in Fig. 5 are determined by comparing the mode frequency, the measured toroidal rotation profile, and the q profile.

We address only the cause of the onset of these modes, and the island will enter the neoclassical phase by the time it is saturated regardless of the mechanism for onset. Also, small islands may exist below the sensitivity threshold of the diagnostics, and by the time the island is large enough to show up on the diagnostics, it is already entering the Rutherford regime. From the Rutherford equation [2] we expect that the eventual saturated island size should be directly proportional to the Δ' at the time of onset. In Fig. 6 are shown the Δ' calculations for the best equilibrium fits to the data at various times for the shot shown in Fig. 5, also shown are the n=1 and n=2 mode amplitudes. For the 2/1mode, Δ' is the largest and peaks just before the onset of the mode at 2050 ms. Looking at Fig. 5 the 2/1 mode is the strongest mode to go unstable. The quality of the equilibrium fits was iteratively improved several times, and Δ' calculated at each stage. In fact the 2/1 results for the fit at 2050 ms deviated between $\Delta' r \sim 400$ to 2000, far more than at any other time, although the equilibrium parameters changed very slightly. This behavior is very suggestive of the proximity to a pole, and a resistive wall mode goes unstable around 2050 ms, which would cause a pole in Δ' , making Δ' large, positive and sensitive at least until the ideal boundary is crossed.

The 3/1 mode is calculated as being marginally stable and in the experiment it appears long after the 2/1 has saturated, presumably driven by coupling to the 2/1. The n=2 modes 4/2, 5/2 and 6/2 are each calculated to be more linearly stable in succession, and are observed to be each of smaller saturated size in succession, such that the 6/2 is hardly visible. The 5/2 shows



Fig. 4. The Δ' results from PEST-III on equilibrium reconstructions long before, just before, and long after a 3/2 mode onset. The Δ' value just before onset is positive, indicating a classically unstable mode.



Fig. 5. The spectral decomposition as a function of time of two toroidally separated magnetic probes in a high q_{min} DIII–D shot, showing the onset of various modes.



Fig. 6. The Δ' results from PEST-III for various modes at various times during the shot shown in Fig. 5. The Δ' values qualitatively agree in their relative values with the measured mode activity, indicating that the 2/1 mode is classically unstable at the point of onset.

up latest of all, presumably driven by n=2 coupling. The 3/2 surface is extremely small at the time of onset, and it is therefore difficult to reliably calculate the stability of the 3/2 mode due to numerical constraints.

IV. CALCULATIONS USING THE NIMROD CODE

NIMROD [9] is a three dimensional initial value code which solves an extended set of the non linear MHD equations. We intend to use NIMROD to determine Δ' and compare these results to the results from PEST-III. Calculations initially have Lundquist number $S \sim 2 \times 10^5$, and S is increased to $S \sim 2 \times 10^7$ (more typical of experiment) only after a mode begins to grow. The value of Δ' can be determined from the scaling of the growth rates with this change in S. However, NIMROD is not poloidally spectral, and the amplitudes of modes with individual m numbers must be extracted from the data by a poloidal spectral decomposition. This work is ongoing in order to determine the sign and amplitude of Δ' for the cases discussed above.

V. DISCUSSION

The proximity of the best equilibrium fit to a Δ' pole in parameter space is found to be crucial to the accuracy of the tearing mode stability calculation. Such a pole may be associated with an ideal stability boundary.

In general, with good equilibrium reconstructions, Δ' calculations seem to predict experimental observations well. For a low β case with a classically unstable tearing mode, stability calculations were very accurate in predicting the onset of the mode. A high β -low q_{min} case showed a positive Δ' for the 3/2 mode just before onset of the mode. In a high β high q_{min} case, Δ' values for all relevant modes agreed well with experimental data for the onset and relative saturated amplitudes of the modes. At later times the Δ' values approach zero, which is consistent with the consumption of free energy by the mode. This is consistent with $\Delta'(w)$ studies [10,11] where w is the island width, which show marginally negative values for the 3/2 and 2/1 long after saturation.

These results suggest that the initial island may in fact come from the positive Δ' instability, and that subsequently the growing mode transitions into the neoclassical phase with Δ' small or negative. The initial island growth occurs as the equilibrium approaches an ideal stability boundary, causing Δ' to become large as it approaches a pole. This would mean that the rate at which the ideal boundary is approached determines whether or not the island will grow large enough before the ideal mode goes unstable to enter the neoclassical phase and saturate at a large size.

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