FAST ION TRANSPORT PROCESSES IN SPHERICAL TOKAMAKS INDUCED BY NON-CONSERVATION OF THE MAGNETIC MOMENT

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INTRODUCTION

The magnetic moment, $\mu^0 = V_\perp^2/2B$, of fast ions confined in conventional axisymmetric tokamaks is known to be well conserved due to a relatively small adiabaticity parameter, ε (ratio of ion gyro-radius, ρ_L, to the gradient scale length of the magnetic field, R). For energetic ions confined in standard tokamaks we have $\varepsilon < A^{-3/2}q^{-1} << 0.1$ [1] with A denoting the plasma aspect ratio and q the safety factor. However, for NBI ions in spherical torus (ST), where typically is A < 1.5 [2] at the plasma periphery, the adiabaticity parameter is greater than 0.1 and hence the magnetic moment of these ions is not conserved.

NON-ADIABATIC VARIATIONS OF MAGNETIC MOMENT

Following [3] we distinguish between adiabatic and non-adiabatic variations of the magnetic moment, which, in the case of arbitrary ε, are given by

$$\dot{\mu}^0 = \sum_{-2}^{2} M_k\left(V_\parallel, V_\perp, \mathbf{r}\right)\exp\left(ik\vartheta\right), \quad \dot{\vartheta} = -\omega_B\left(\mathbf{r}\right) + \sum_{-2}^{2} N_k\left(V_\parallel, V_\perp, \mathbf{r}\right)\exp\left(ik\vartheta\right), \quad M_0 = N_0 = 0 \quad (1)$$

where ϑ is the gyro phase, $V_\parallel$ and $V_\perp$ are the particle velocity components parallel and perpendicular to **B**, and **r** defines the position of particle. Taking into account the smallness of M_k and N_k , $N_k/\omega_B \sim M_k/(\mu^0\omega_B) \sim \varepsilon$, and neglecting the interaction of gyro oscillations given by $\exp(ik\vartheta)$ and fast oscillations of M_k, N_k one obtains the corrected expression for the magnetic moment in the adiabatic approximation

$$\mu^a = \mu^0 + \delta\mu^a, \quad \delta\mu^a = \mu^1 + \mu^2 + \ldots, \quad \mu^k = \mu^0 O\left(\varepsilon^\kappa\right). \quad (2)$$

The term $\delta\mu^a$ compensates gyro-oscillations of μ^0 and is purely oscillative with the gyro-averaged component being zero. Thus the nonadiabatic variation, $\delta\mu^n$, per time interval $\tau_b > \Delta t = 2\pi n/\omega_B$ (n - integer) is given by $\delta\mu^n = \int_{\Delta t} dt\dot{\mu}^0$. Typical time variations of μ^0 for fast ions are illustrated in Figs. 1a, 1b with also the complete orbit of a 40 *keV* circulating deuteron in NSTX (β = 40%, I/B=1MA/0.3T) shown (Fig. 1c). Starting point is at R = 0.55m, Z = 0 with $\dot{R} = 0, \dot{Z} < 0, V_\varphi/V = 1$. The magnitude of adiabatic gyro oscillations of μ^0 is of order 100%; rather strong nonadiabatic variations $\delta\mu^n/\mu_0^0 \simeq 10\%$ are observed when the particle passes the minimum B along the orbit (at the outboard part of the midplane). These variations occur during a short time period $\Delta t \sim \tau_B \ll \tau_b$ as demonstrated by the time behavior of the corrected magnetic moment $\mu = \mu^0 + \mu^1$. Non-regularity of the $\delta\mu^n$-variations over several bounce times indicates stochastic behavior. Note that the particle energy E and the toroidal canonical momentum P_φ are conserved with high accuracy ($\Delta E/E \sim \Delta P_\varphi/P_\varphi \le 10^{-10}$).

On account of the purely oscillative nature of $\delta\mu^a$ the nonadiabatic variation of μ on time scales $\Delta t \geq \tau_b$ corresponds to $\left\langle \mu^0 \right\rangle = \tau_b^{-1} \int_{\Delta t = \tau_b} dt \mu^0 \cong \mu^a$.

Role of resonances

Because of the local nature of $\delta\mu^n$ the nonadiabatic variations of μ^0 during $\Delta t \gg \tau_b$ are very sensitive to the resonances between the gyro motion and bounce oscillations,

$$\omega_B - l\omega_b = 0, \quad l = \text{integer} . \tag{3}$$

Ions with $\mu^a \approx \mu_l\left(E, P_\varphi\right)$, where μ_l is the value of magnetic moment corresponding to the resonant condition, Eq. (3), can in $\Delta t \gg \tau_b$ accumulate the $\delta\mu_b^n$ ‘s experienced per each bounce resulting in superbanana oscillations with amplitudes $\delta\mu_{sb}^n \gg \delta\mu_b^n$ and characteristic superbanana periods $\tau_{sb} \gg \tau_b$. This is seen in Fig. 2a where the superbanana oscillation of $\left\langle \mu^0 \right\rangle \cong \mu^a$ is displayed for an 80 *keV* deuteron starting at R = 1 *m*, Z = 0 in NSTX (β = 40%, I/B=1MA/0.3T) with $\dot{R} = 0, \dot{Z} < 0, V_\varphi / V = 0.975$. The value $V_\varphi / V = 0.975$ results in the superbanana oscillation of maximum amplitude (in the vicinity $\mu \leq \mu_{10}$). As evident from Fig. 2b, the orbit starting with $V_\varphi / V = 0.96$ corresponds to a near resonant one ($\mu \cong \mu_{10}$). Decreasing V_φ / V to 0.93 yields the superbanana orbit of maximum magnitude in the vicinity $\mu \geq \mu_{10}$. Finally, further decrease of V_φ / V results in leaving the resonant range. At $V_\varphi / V = 0.925$ we see a typical non-resonant orbit with relatively small variations $\delta\mu^n$, Fig.2c.

Superbanana (super-adiabatic) variations ($\varepsilon < \varepsilon_{cr}$)

The diversity of bounce averaged superbanana oscillations for $0.925 \leq V_\varphi / V \leq 0.975$ for 80 *keV* deuterons in NSTX (β = 40%) is shown in Fig. 2d. The range $0.93 \leq V_\varphi / V \leq 0.975$ corresponds to a class of superbananas with their maximum amplitude in the vicinity of l=10 and indicates that the fraction of resonant particles is about 4%. Fig. 3 displays the superbanana orbits corresponding to two neighbouring resonance levels (l=11 and l=12) for 80 *keV* deuterons in NSTX (β = 23%). Accounting for the difference between values V_φ / V associated with neighboring resonances (about 0.1) and considering the fractions of resonant particles, f_r, which are ~ 0.03 and ~0.02 for l=11 and l=12, we estimate the fraction of all resonant particles to be ~(20-30)%. Regularity of the superbanana oscillations indicates the existence of a new adiabatic invariant (superadiabatic behaviour)

$$\mu = \mu_a + \delta\mu^n , \tag{4}$$

which, however, only occurs if $\varepsilon < \varepsilon_{cr}$, i.e. when the magnitudes of superbanana oscillations $\delta\mu_{sb}^n$ are less then the distance between the neighboring resonant levels, $\Delta\mu\left(= \mu_{l+1} - \mu_l\right)$.

Transition to stochasticity ($\varepsilon > \varepsilon_{cr}$)

If the condition for overlapping of resonances [4],

$$\varepsilon > \varepsilon_{cr}, \quad \delta\mu_{sb}^n\left(\varepsilon_{cr}\right) = \Delta\mu \equiv \mu_{l+1} - \mu_l , \tag{5}$$

is satisfied, μ is no longer an invariant and transition to stochasticity will take place. This is possible for co-going NBI ions passing the outer part (R~1.5-1.6m) of NSTX. A typical stochastic orbit of an 80 *keV* deuteron in NSTX is given in Fig. 4 with the non-adiabaticity induced radial drifts of the guiding centre of the bounce orbit being of the order of the ρ_L.

NON-ADIABATICITY INDUCED TRANSPORT

In the phase space domain corresponding to the stochastic regime ($\varepsilon > \varepsilon_{cr}$) non-adiabaticity will result in strong pitch angle diffusion at a rate $D_{st}^n / \mu^2 = \left(\Delta\mu_b^n/\mu\right)^2 \omega_b$ that, for NSTX-like parameters ($\omega_b \cong 10^6 s^{-1}$, $\Delta\mu_b^n/\mu \sim 10^{-2} - 10^{-1}$), gives $D_{st}^n/\mu^2 \cong \left(10^2 - 10^4\right) s^{-1}$ $\gg \nu_\perp$ where $\nu_\perp \sim (1-3)\, s^{-1}$ is the rate of Coulomb pitch angle scattering. This scattering will result in enhanced radial diffusion. Further, Fig. 5 demonstrates the collisionless transformation of a barely trapped 3.5 *MeV* alpha in DTST (I/*B*=10*MA/*2*T*) into a marginally circulating one, again associated with a radial shift of ~ Δr_b.

In the super-adiabatic regime ($\varepsilon < \varepsilon_{cr}$) non-adiabaticity will give rise to additional pitch angle diffusion with the rate $D_{coll}^n/\mu^2 = \nu_\perp \left(\Delta\mu_{sb}^n/\mu\right)^2 f_r/(\Delta\xi_r)^2$, where $\Delta\xi_r$ is the width of the resonance region in $V_\parallel/V$. For NBI ions in NSTX ($\Delta\mu_{sb}^n/\mu \sim 0.05 - 0.1$, f_r ~ 0.2-0.3, $\Delta\xi_r$ ~ $\Delta(V_\varphi/V)$ ~ 0.02-0.03) this rate exceeds $\nu_\perp$. Due to the dependence of ω_b on E the slowing down can result in radial convection of resonant orbits in the super-adiabatic regime, $\varepsilon < \varepsilon_{cr}$. Fig. 6 indicates a strong outward shift ($\Delta R \cong 10$ *cm*) of the resonance orbit of a 3.5 *MeV* alpha upon the energy decrease ΔE = 50 *keV* in DTST (I/*B*=10*MA/*2*T*). Hence non-adiabaticity may result as well in rather strong radial convection due to slowing down (at the rate $\Delta R \nu_s E/(a\Delta E) \geq \nu_s$).

CONCLUSIONS

Non-conservation of the magnetic moment is expected to significantly affect the confinement of fast ions in spherical tori. Pronounced non-adiabaticity, $\varepsilon > \varepsilon_{cr}$, will result in strong collisionless pitch angle scattering and also in the collisionless transformation of trapped orbits into circulating ones as well as vice versa. In the case of weak non-adiabaticity, $\varepsilon < \varepsilon_{cr}$, non-conservation of μ may result in enhanced collisional radial diffusion and convection of fast ions gyrating resonantly with bounce oscillations. We note that the non-adiabaticity effects described may be modified by magnetic field ripples [5, 6] not accounted here.

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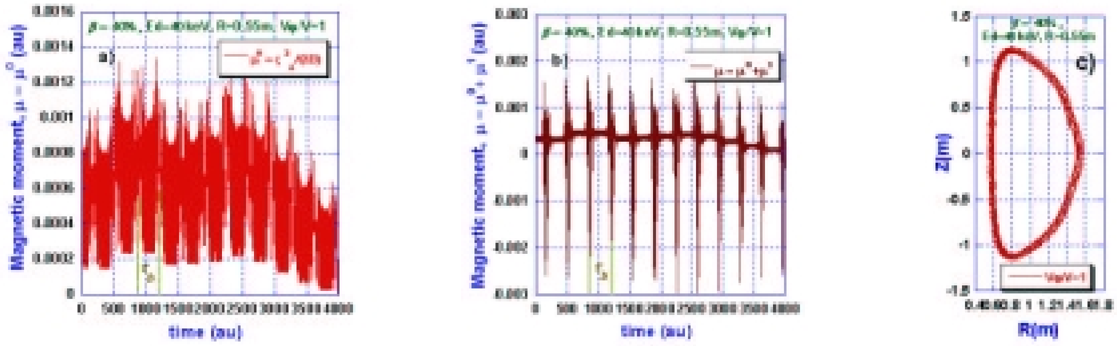

Fig.1 μ variations and gyro orbit of 40 *keV* deuteron in NSTX (β=40%, I/B=1MA/0.3T)

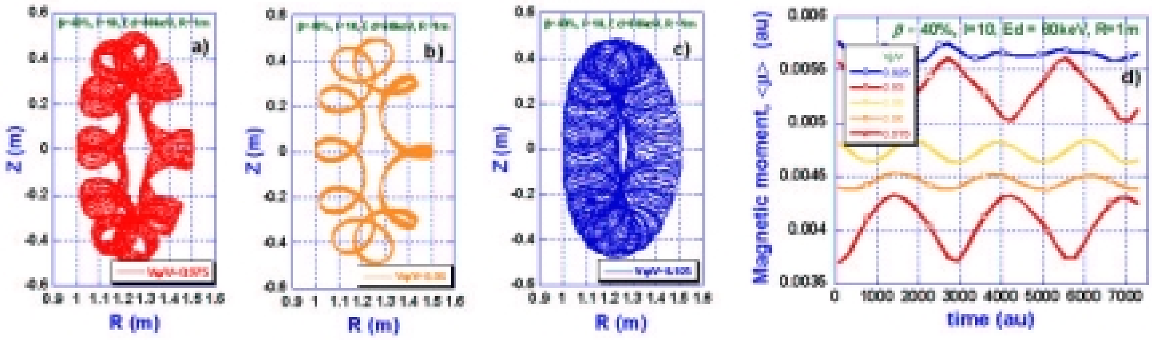

Fig.2 Oscillations of $<\mu>$ of 80 *keV* deuterons in the vicinity of l=10 resonance level *vs* $V\varphi/V$

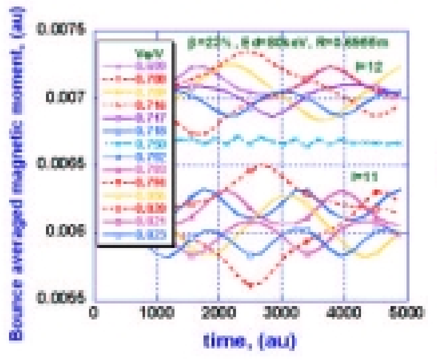

Fig.3 Variations of $<\mu>$ for **l**=11, 12 in NSTX (β=23%, I/B=1MA/0.3T)

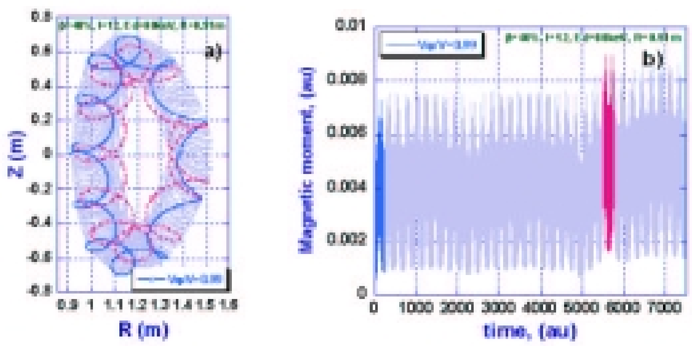

Fig.4 Nonadiabaticity induced radial drift of deuteron in NSTX (β=40%, I/B=1MA/0.3T)

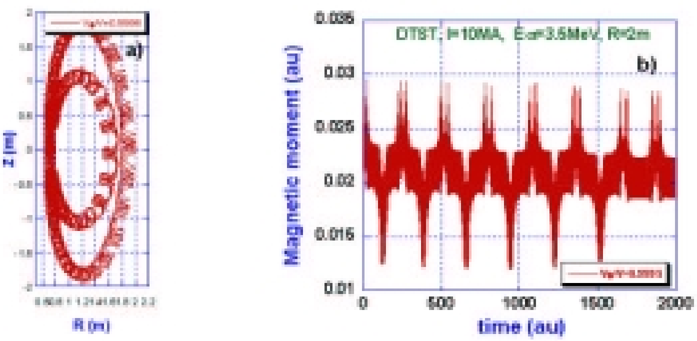

Fig.5 Collisionless orbit transformation of 3.5 alphas in DTST (I/B=10MA/2T)

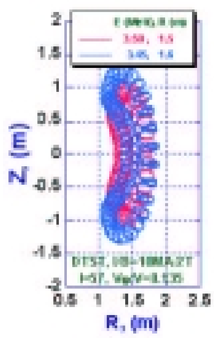

Fig. 6 Outward shift of resonance level of 3.5 alphas in DTST