Optimisation of *N*=6 Helias-Type Stellarator

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Abstract. Collisionless particle confinement in stellarator configurations with helical direction of the lines *B*=const on the magnetic surfaces is investigated numerically for a six-period system. The optimisation is performed with different penalty functions that are connected with the pseudosymmetry condition and the condition that the second adiabatic invariant $J_{||}$ forms closed contours. In addition, the effect of β on the particle confinement is studied.

1. Introduction. The fulfilment of the quasisymmetry (qs) condition [1] for systems with helical direction of lines *B*=const on the magnetic surfaces leads to conservation of the invariant $\Psi + \rho_{||} F$ = const of the equations of motion in the drift approximation and, thus, to improvement of the collisionless particle confinement. It was shown [2] that qs condition cannot be fulfilled exactly in the whole plasma volume. In Ref. [3] the less restrictive condition of pseudosymmetry (ps) was suggested. The ps condition means the absence of local extrema of *B* along the magnetic field lines.

In the present paper, the possibility to fulfil the ps condition is studied numerically and the effect of its fulfilment on the collisionless particle confinement is investigated. It is shown that the fulfilment of ps condition itself is not sufficient for significant improvement of the particle confinement. Because of this, the condition of closure of the $J_{||}$ contours was added to the penalty function. The corresponding results are presented for both small and large values of β. In addition, results of neoclassical transport calculations for these optimised configurations are presented.

2. Optimisation toward pseudosymmetry. The starting point for the optimisation is an *N*=6 Helias-type configuration with a Mercier stability limit of about $\beta \approx 5\%$.

In Ref. [4] the mathematical formulation of the ps condition was presented. It requires $dB/d\theta_B \equiv 0$ along the lines $\iota\, \partial B/\partial\theta_B + \partial B/\partial\zeta_B = 0$. This condition alone was used for the construction of the penalty function. As a result, a configuration very close to ps was found. Fig. 1 shows the behaviour of the lines *B*=const on the magnetic surface *s*=0.9 near the boundary (*s* is proportional to the square of the average minor plasma radius) for the initial and the ps-optimised configurations. The confinement properties of these configurations were studied by direct calculations of the guiding centre motion of α-particles. It was shown that fulfilment of the ps condition does not lead to significant improvement of α-particle confinement. The number of the lost particles was reduced by ~10% because of elimination of locally-reflected particles.

3. Closure of the $J_{||}$ contours. The analysis of the behaviour of the $J_{||}$ contours for the ps-optimised configuration has shown that they are open, this being the reason for particle losses. Therefore, a new penalty function was used which vanishes when the $J_{||}$ contours are closed for all reflected α-particles born at 1/4 of the plasma minor radius. Depending on the geometry of the configuration and mainly on the β value, two different possibilities can be realised, with either maximum or minimum value of $J_{||}$ in the centre. In our calculations, we have considered configurations with small ($\beta \approx 0.06\%$) plasma pressure and have required a minimum for $J_{||}$ inside the plasma column. A configuration was identified in which the $J_{||}$ contours were closed for almost all trapped particles born near the magnetic axis. In Fig. 2, contours of $J_{||}$=const for the initial and optimised configurations are presented. Here the pitch angle which is worst from the viewpoint of confinement is considered.

The results of the direct calculations of α-particles lost are presented in Fig. 3. In spite of the modification of the initial geometry, the configuration found still has a magnetic well.

It was shown that for large enough β values (~ 15%), the radial dependence of $J_{||}$ is reversed. The confinement in this case is good, too. It is seen from Fig. 4 that for intermediate values of β, particle confinement is deteriorated. A similar effect occurs for configurations with poloidal direction of the lines B=const [5].

In Fig. 5, the shape of the boundary surface of the optimised configuration is displayed. The colours here are defined by the magnetic field strength which still exhibits qualitatively quasi-helical behaviour.

4. Computation of transport properties. For the $1/v$ transport, the characteristic features of the specific magnetic field geometry manifest themselves [6] through a factor ε_{eff} which is an effective ripple modulation amplitude of B and enters the expressions for transport coefficients as a factor $\varepsilon_{eff}^{3/2}$. Computational results for $\varepsilon_{eff}^{3/2}$ are presented in Fig. 6 as functions of $\sqrt{s}$. For an equivalent standard stellarator magnetic field the $\varepsilon_{eff}^{3/2}$ value turns out to be 0.01 ÷ 0.03. Therefore, the results obtained are approximately 5 times better than those for a standard stellarator although they are worse than those for the quasi-helically symmetric stellarator [1,6] by more than one order of magnitude.

The trapped particle motion can be described in terms of the second adiabatic invariant $J_{||}$. The variation of $J_{||}$ on a magnetic surface is connected with a displacement of the trapped particle orbit from the magnetic surface. This variation is characterised by a derivative $\partial J_{||}/\partial\theta_0$, where $\theta_0 = \theta_B - \iota\zeta_B$, θ_B and ζ_B being the Boozer angular coordinates. In accordance with [7], we present the results for $\partial J_{||}/\partial\theta_0$ in a normalised form as a functional dependence between dimensionless parameters η and $\gamma = \upsilon_{||i}/\upsilon_{\perp 0}$ with

$$\eta = \frac{R}{J_\perp}\frac{1}{\tau_b <|\nabla\Psi|>}\frac{\partial J_{||}}{\partial\theta_0}, \quad \frac{\partial J_{||}}{\partial\theta_0} = \frac{e}{mc}\delta\Psi, \quad \upsilon_{\perp 0} = \sqrt{J_\perp B_0} .$$

Here $J_\perp = \upsilon_\perp^2/B$, τ_b is the bounce period, $\delta\psi$ corresponds to the differential of ψ during τ_b. The normalisation of η is performed in such a way that for the standard stellarator magnetic field, the maximum η value, η_m, equals 0.5. The $\upsilon_{||i}$ quantity is $\upsilon_{||}$ at the point of a local minimum of B and the γ parameter relates to the pitch angle at this point.

Results of the η calculations are presented in Fig. 7 for an outer magnetic surface. The curves are labelled in accordance with the number of B minima along the magnetic field line. It is seen that non-zero values of η exist in the γ interval $0 \le \gamma \le \gamma_{\max}$ which corresponds to the trapped particles. In contrast to the standard stellarator, the maximum η value turns out to be smaller than 0.5 and it varies with γ. Calculations carried out for inner magnetic surfaces showed that the character of the dependencies of η on γ does not change, but that maximum values of η turn out be smaller than those for the outer magnetic surface.

Conclusions. It was shown through numerical calculations that the pseudosymmetry condition itself is not sufficient for good particle confinement. A new penalty function based on the condition of closure of the $J_{||}$ contours was introduced to the optimisation procedure. As a result of the optimisation, a configuration was found in which almost all α-particles born near the magnetic axis are confined. It was demonstrated that good collisionless particle confinement is possible in configurations far from omnigenity.

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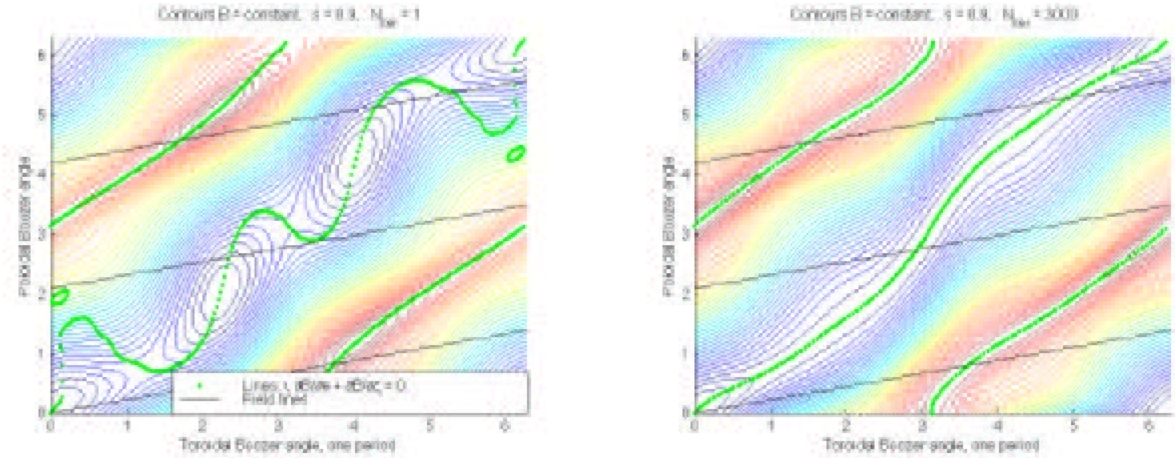

Fig.1. Optimisation of initial configuration toward pseudosymmetry. Behaviour of lines B=const on the magnetic surface s=0.9 near the boundary for initial (left) and ps-optimised (right) configurations.

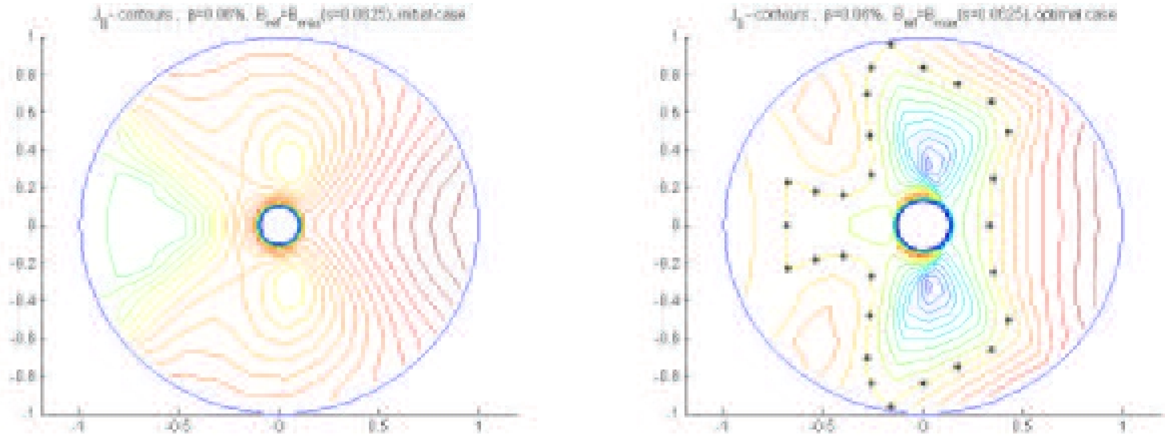

Fig. 2. Contours $J_{||} = const$ for initial and optimised configurations. The worst pitch angle from the viewpoint of confinement is considered.

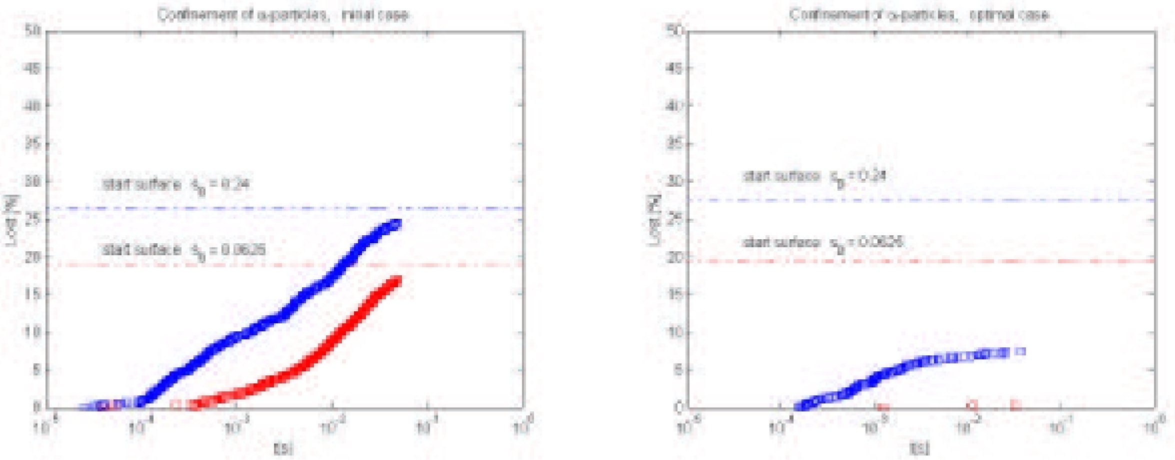

Fig.3. Confinement of α-particles for initial and optimised configurations.

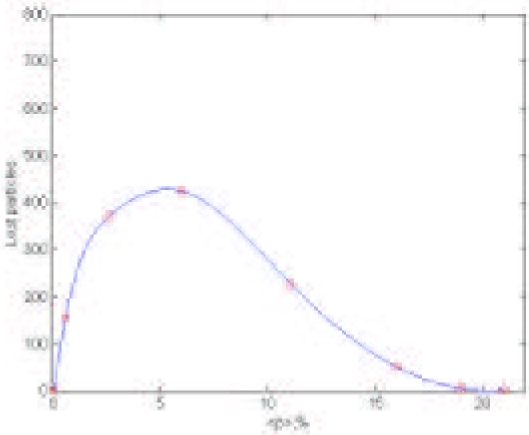

Fig. 4. Effect of <β> on the particle confinement for the system optimised with respect to $J_\|$ contours closure. The number of test α-particles is 2000.

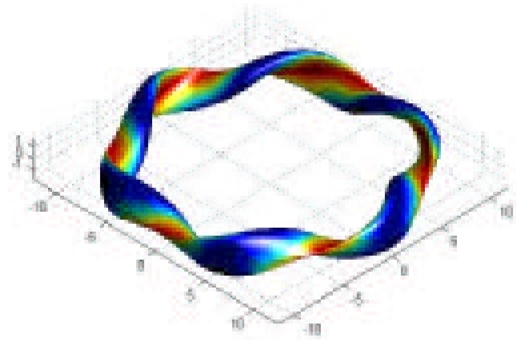

Fig. 5 Shape of the boundary surface of the configuration considered. The colour represents the magnetic field strength.

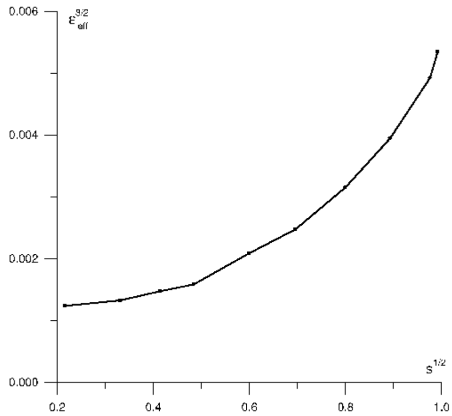

Fig. 6. Effective ripple amplitude $\varepsilon_{eff}^{3/2}$ versus averaged magnetic surface radius.

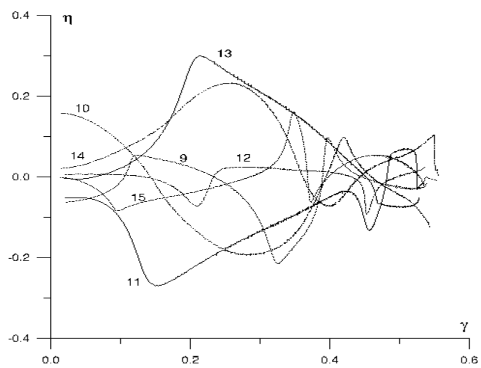

Fig. 7. Normalised bounce averaged drift velocity η for trapped particles. The curves are labelled in accordance to the number of B minima along the magnetic field line.