ANALYSIS OF DETERMINATION OF REYNOLDS STRESS IN DRIFT WAVE TURBULENCE

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A major challenge in the research towards a fusion power plant is the understanding and control of the plasma turbulence leading to anomalous transport of particles and energy. It is observed experimentally and numerically that shear flows in plasmas suppresses turbulence and transport. The generation mechanism of these flows is thus of great interest. Diamond and Kim [1] presented a first theoretical approach to self-consistent flow generation in turbulent plasmas by small-scale turbulent fluctuations via the Reynolds stress, which is defined as $R_\phi = R_\phi(x) = -\langle v_x v_y \rangle_{y,z} = B_0^{-2} \left\langle \partial\phi/\partial y \;\; \partial\phi/\partial x \right\rangle_{y,z}$, where v_x and v_y are the x- and y-components (radial and poloidal directions respectively) of the $\mathbf{E} \times \mathbf{B}$–velocity and ϕ is the electrostatic potential. Measurements of the Reynolds stress can thus help to predict flows, e.g. shear flows in plasmas as demonstrated in [2]. However, the determination of the Reynolds stress requires measurements of the plasma potential, a task that is difficult in general and nearly impossible in hot plasmas in large devices.

In this work, we investigate the generation and the effect of shear flows in drift wave turbulence and the relation to the Reynolds stress. In particular, we look at an alternative way of estimating the Reynolds stress via the density fluctuations [3]. We demonstrate the validity range of this quantity, which we term the pseudo-Reynolds stress. The advantage of such a quantity is that accurate measurements of density fluctuations are much easier to obtain. We further present a numerical analysis of the importance of alignment of probes when the Reynolds stress is to be measured. To clarify the role of the self-generated shear flow on the evolution of the drift wave fluctuations, we further investigated the influence of an imposed external shear flow on the development of the drift wave fluctuations.

The model used in the numerical investigations was the 3D drift wave Hasegawa-Wakatani model [4]. One of the strengths of the model in 3D is that fluctuations and background variations are evaluated in one instance, i.e. the background and the fluctuations are separated, but the interaction between these is included. The back-reaction of the density fluctuations on the background density gradient, however, is not recoverable from Hasegawa-Wakatani simulations, which use radial periodic geometries (See e.g. [5]). Therefore, in a slab periodic in y and z (corresponding to the poloidal and toroidal directions, respectively) we use non-permeable walls in the radial direction, i.e. Dirichlet boundaries in x and fix n and ϕ to zero at $x = 0$ and $x = L_x$. Thus enabling a back-reaction of the density fluctuations on the background density gradient.

Using a background density of the form $n_0 = n_0(x) = N_0 e^{-\frac{x}{L_n}}$, the normalised, dimensionless Hasegawa-Wakatani equations are expressed as:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp\right) n + \frac{\partial \phi}{\partial y} = \mathcal{C} \frac{\partial^2}{\partial z^2}(n - \phi) + \nu \nabla_\perp^2 n \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp\right)(\nabla_\perp^2 \phi) = \mathcal{C} \frac{\partial^2}{\partial z^2}(n - \phi) + \nu \nabla_\perp^4 \phi \tag{2}$$

Note that the parallel resistivity has been included in the normalisation of the parallel length scale, while the ion gyro-radius at electron temperature, $\rho_s = \frac{\sqrt{T_e m_i}}{eB_0}$, has been used to normalise lengths perpendicular to $\mathbf{B}$ (x and y).
The simulations were performed by Fourier spectral methods using sine transforms in the x-direction, since these, intrinsically, are zero at $x = 0$ and $x = L_x$, if $k_{x,min} = 2\pi/L_x$, thus matching the radial boundary conditions. The number of modes in the simulations presented here is 256 in the perpendicular directions (x and y) and 32 in the direction parallel to the magnetic field. The domain size was $L_x = L_y = 30$ and $L_z = 20$ and the time step $dt = 3 \cdot 10^{-3}$. The viscosity parameter $\nu = 0.1$ was necessary to prevent the accumulation of energy in the short wavelengths.
The energy of the system is defined as $\mathcal{E} = \frac{1}{2}\int [(\nabla_\perp \phi)^2 + n^2]\,\mathrm{d}\mathbf{x}$. In Figure 1 the evolution of the energy of the background flow and the energy of the the drift waves is shown, and it is seen that the drift waves are suppressed as the poloidal flow builds up. One may alternatively say that the drift wave turbulence self-organises into the poloidal shear flow. To illustrate that the sheared flow also reduces the turbulent transport, the maximum shearing rate and the turbulent flux is plotted in Figure 2. The flow generation and the

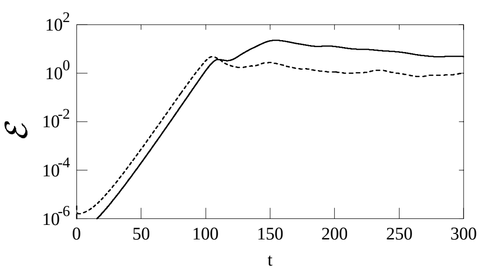

Figure 1: *The temporal evolution of the energy $\mathcal{E}(k_y = 0, k_{||} = 0)$ (full line) and the drift wave energy $\mathcal{E}(k_{||} \neq 0)$ (dashed line).*

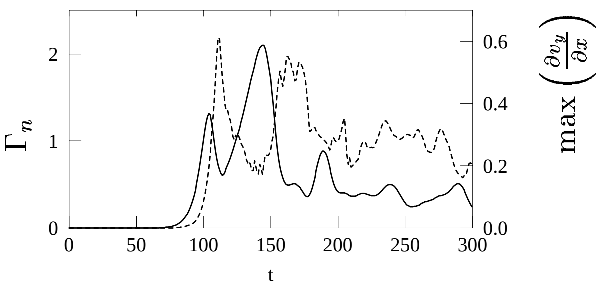

Figure 2: *The temporal evolution of the turbulent flux $\Gamma_n = -\int n \frac{\partial \phi}{\partial y} \mathrm{d}\mathbf{x}$ (full line) and the maximum shearing rate* $\max\left(\frac{\partial v_y}{\partial x}\right)$ *(dashed line).*

suppression of turbulence is caused by the back-reaction of the density fluctuations on the background density, since the fluctuations organise to flatten the background profile and the effective gradient is flattened. Consequently, the drive of the drift wave turbulence is quenched. Mathematically, the $\frac{\partial \phi}{\partial y}$-term in equation (1) originates from the convection of the background profile, and the back-reaction of the fluctuations on this is through the nonlinear convection term, where a $\frac{\partial n}{\partial x}\frac{\partial \phi}{\partial y}$-term occurs.
The property of the small-scale plasma turbulence organising into a shear flow can be explained by the theory of Reynolds stress. The Reynolds stress is a measure of the anisotropy of the turbulent velocity fluctuations, since it is generated solely from inhomogeneous correlations between v_x and v_y. These produce a stress on the mean flow and may drive a poloidal shear flow (y-direction). A self-consistent explanation of the flow generation is that for drift wave turbulence the Reynolds stress can be seen as a density flux of polarisation due to the nonlinear polarisation drift [6]. This creates a radial electric field which combined with the toroidal magnetic field generates the poloidal flow.

The Reynolds stress is interesting to measure experimentally, since large flows may be predicted [2]. This has an interest, since poloidal flows may be responsible for the L– to H–mode transition. Several effects besides self-organisation of turbulence may cause a poloidal flow such as neoclassical effects, particle trapping etc. These effects also influence the turbulence, thus the Reynolds stress may also carry a signature of those. By averaging the Hasegawa-Wakatani equation for the vorticity (2) over y and z we obtain

$$\frac{\partial \langle v_y \rangle}{\partial t} = \frac{\partial \left\langle \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial x} \right\rangle}{\partial x} + \nu \frac{\partial^2 \langle v_y \rangle}{\partial x^2} \tag{3}$$

The Reynolds stress (in normalized units) is defined as $R_\phi = -\langle v_x v_y \rangle = \langle \partial\phi/\partial y \;\; \partial\phi/\partial x \rangle$, and now the first term on the right hand side of (3) is identified as the divergence of the Reynolds stress generating the poloidal flow. Numerical simulations were performed and radial profiles of the quantities on the left and right hand side of (3) were calculated. These showed perfect correspondence as expected from (3).

Experimental determination of the Reynolds stress, however, demands an accurate measurement of the fluctuations in the electrostatic potential. This, unfortunately, is quite difficult, especially in large plasma devices [2, 7]. It has therefore been suggested that an approximate value of the Reynolds stress may be obtained from the density perturbations, since ϕ and n are strongly correlated in the drift wave limit, i.e. for $k_{||} \neq 0$. This density-based pseudo-Reynolds stress is defined as $R_n = \langle \partial n/\partial y \;\; \partial n/\partial x \rangle$. If R_n and R_ϕ are correlated it will be sufficient to measure the density fluctuations in order to get an approximate value for the shear flow generation.

We performed numerical simulations to determine whether a density-based pseudo-Reynolds stress is correlated to the real Reynolds stress. In the edge of experimental plasmas confined by a sheared magnetic field, no modes having $k_{||} = 0$ exist. Hence it is reasonable to exclude the convective cell in the Reynolds stress calculations and only use the drift wave components of n, i.e. $n(k_{||} \neq 0)$, in the calculation of R_n. From the resulting profiles (to be presented in [8]) it is seen that the pseudo-Reynolds stress gives a good qualitative hint on the flow generation - at least for times before the turbulence is fully developed. The degradation of the correspondence between the pseudo-Reynolds stress and the flow velocity can in part be explained by the fact that density fluctuations cascade to smaller scales [9], which causes problems in properly resolving the gradient of the density field numerically. This leads to the error in ∂_x growing and since the pseudo-Reynolds stress is the divergence of a product of two derivatives the error due to resolution becomes significant.

The effect of a misaligned probe array, when determining the Reynolds stress in an experiment has been studied. We imagine an array of five probes as illustrated in figure 3, with fixed distances (r) and fixed (right) angles between the probes. This is the minimal array with which to obtain the divergence of the Reynolds stress from two independent measurements of the Reynolds stress:

$$R_{\phi,1} = \frac{(\phi_2 - \phi_1)(\phi_4 - \phi_1)}{r^2} \quad \text{and} \quad R_{\phi,2} = \frac{(\phi_3 - \phi_2)(\phi_5 - \phi_2)}{r^2}$$

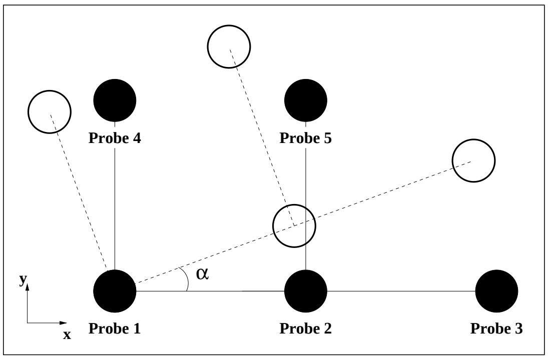

Figure 3: *The probe array with five probes. The non-filled circles are the probe array after the array is rotated by the angle* α*.*

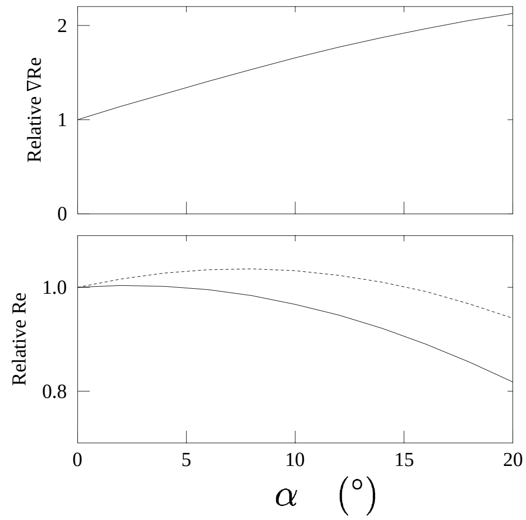

Figure 4: *Rotated probe calculations at time* $t = 176$ *with the misalignment in the plane perpendicular to the magnetic field lines. Upper graph shows the difference of the "measured" divergence of the Reynolds stress and the value for perfect alignment relative to the latter as a function of the misalignment angle* α*. The lower graph shows the change of the two Reynolds stresses.*

From Figure 4 it is seen that the calculated divergence of the Reynolds stress only changes by less than 30% if the misalignment in the plane perpendicular to the magnetic field is less than 5°. The flexibility in the alignment in the plane parallel to the magnetic field is similar. Thus alignment of the probe array is important, but it is not crucial to have a perfect alignment to obtain a reasonably good estimate of the divergence of the Reynolds stress and the acceleration of the mean shear flow.

The validity of the results presented above are naturally limited by the assumptions of the relatively simple Hasegawa-Wakatani model. Furthermore, the approximation of the pseudo-Reynolds stress is a rough method. However, these first-principles results indicate that the concept of performing accurate measurements of density fluctuations in order to predict the generation of shear flows may prove interesting.

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