Path –Sum Calculations for rf Current Drive

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Abstract. Path sums and Gaussian short-time propagators are used to solve two-dimensional Fokker-Planck models of lower-hybrid (LH) and electron-cyclotron (EC) current drive (CD), and are shown to be well suited to the two limiting situations where the rf quasilinear diffusion coefficient is either relatively small, $D_{rf} \approx 0.1$, or very large, $D_{rf} \to \infty$, the latter case enabling a special treatment. Results are given for both LHCD and ECCD in the small D_{rf} case, whereas the limiting situation is illustrated only for ECCD. To check the accuracy of path-sum calculations, comparisons with finite difference solutions are provided.

INTRODUCTION

As is well known, electron kinetics plays a fundamental role in studies of current drive (CD) in plasmas using rf power, such as with electron-cyclotron (EC) and lower-hybrid (LH) waves, being usually modeled by some appropriate form of the Fokker-Planck (FP) equation [1]. For such complex models there is no recourse other than to resort to some kind of numerical calculation, the most widespread approach consisting in directly solving the FP equation by means of finite differences [2]. Still, alternative approaches such as the use of Monte Carlo and propagators have found increased acceptance, as they are known to offer a very simple and clear picture of the kinetics involved and lead, moreover, to a straightforward numerical implementation. Of these, the use of Gaussian short-time propagators to numerically evaluate solutions to FP equations as path sums has been gaining some interest as a valuable alternative to Monte Carlo, over which they have advantages both in terms of accuracy and computational efficiency [3].

The fundamental building block of the path-sum approach is the short-time propagator solution to the FP equation satisfying a Dirac δ-like initial condition which, for appropriately short times τ, is known to be a Gaussian distribution function [3]. With this method, any initial distribution function $f(\vec{v}, t_0)$ (in velocity space $\vec{v}$) is propagated during the time interval τ by multiplying the distribution vector $f(\vec{v}_l, t_0)$, which is a discrete representation in an appropriate grid at time t_0, by the propagator matrix $T(\vec{v}_l, \vec{v}_m, \boldsymbol{t})$. The latter gives the probability for the transition from the point $\vec{v}_l$ in cell l to the entire cell m, so that for any finite time the process reduces to the simple multiplication of matrices by vectors.

PATH-SUM SOLUTIONS FOR RF CURRENT DRIVE

The path-sum approach was employed to solve a few well-known two-dimensional (2-D) models of LHCD and ECCD. For particle-particle collisions, the usual assumptions were used [1,2]: a homogeneous and azimuthally symmetric plasma (about the apllied magnetic field, B_0); and collisions restricted to the non-relativistic limit, and such that the test electrons interact with a nonevolving Maxwellian distribution of bulk electrons at a given temperature T to which they tend to equilibrate. In the case of the wave-particle interaction, the common quasilinear diffusion limit was considered for both LH and EC waves [2].

A frequently used approach in ECCD and LHCD calculations is to utilize a model form for the quasilinear diffusion coefficient $D_{rf}(w)$ in which $D_{rf}(w) = D$ inside the resonant region (delimited by w_1 and w_2) and $D_{rf}(w) = 0$ otherwise, where w is the velocity component parallel to B_0 and D is a constant [1,2]. However, to avoid the limitations imposed on the path-sum approach by sharp discontinuities in the quasilinear diffusion coefficient (which are felt in the case of LHCD alone), $D_{LH}(w)$ is considered to be divided in three parts: the normal region where $D_{LH}(w) = D$, and two symmetric (sine-type) ramps of width dw, such that the resulting $D_{LH}(w)$ forms an overall smooth function [3].

The path sum method is illustrated here with the following cases: a) LHCD with $D = 0.1$, $w_1 = 4$, $w_2 = 5$, and $dw = 1$; b) ECCD with $D = 0.1$, $w_1 = 4$, and $w_2 = 5$; c) ECCD with $D = 0.25$, $w_1 = 4$, and $w_2 = 5$; and d) ECCD with $D \to \infty$, $w_1 = 4.15$, and $w_2 = 5$. All these cases were solved in the spherical coordinates v and $\boldsymbol{q}$, on a grid with $v \times \boldsymbol{q} = 300 \times 192$ cells in the region delimited by $v_{\max} = 10.0$, and using a time step $\boldsymbol{t} = 0.1$.

The ECCD example with $D \to \infty$ enables a special treatment built on a method already proposed for finite-difference calculations that is based on the observation that, in such a case, the waves induce a flattening of the distribution in the direction in which they accelerate the particles, i.e. $\vec{S} \cdot \partial f / \partial \vec{v} = 0$, where $\vec{S}$ is a unitary vector in that direction [4]. Accordingly, in the path-sum approach, the resonance is divided into an appropriate number of strips aligned along $\vec{S}$ (the direction perpendicular to B_0 in the ECCD case) and, either the propagator (accounting for collisions and waves) is computed allowing for this leveling effect of the waves, or else the flattening is introduced directly in the distribution function that results from the evolution produced by the propagator (which, now, considers the effect of collisions alone). Note that the latter scheme is less demanding on computer resources (both in terms of time and memory), since it leads to much smaller propagator matrices, even if the flattening operation, which must be performed each time step, takes some time.

To check the accuracy of propagator calculations, whose outcome appears in Figs. 1, 2, and 3, tests are provided against the corresponding finite-difference solutions, for all but the ECCD case with $D \to \infty$, which is to be compared to results appearing in the literature [4]. In this case, the lower limit of the resonant region was chosen to be

$w_1 = 4.15$, in order to match the results given in Fig.3 of Ref. [4], where the flattening of the distribution does not start exactly at $w_1 = 4.0$ (but somewhat higher), and the reported steady-state current is 3×10^{-3}. With $w_1 = 4.15$, path sums lead to a steady-state current of 2.91×10^{-3}, whereas for $w_1 = 4.0$ they lead to an increased value of 5.59×10^{-3}.

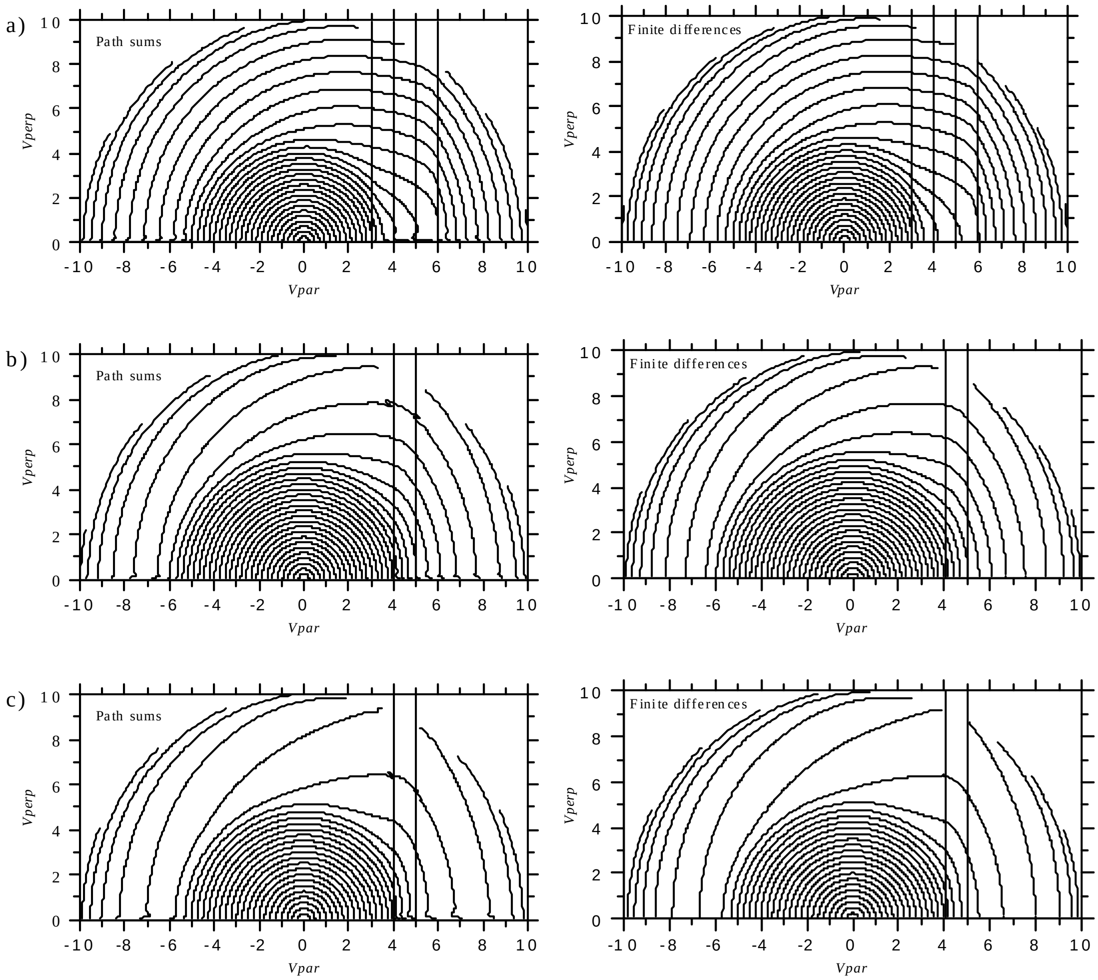

FIGURE 1. Steady-state distributions for cases a), b), and c).

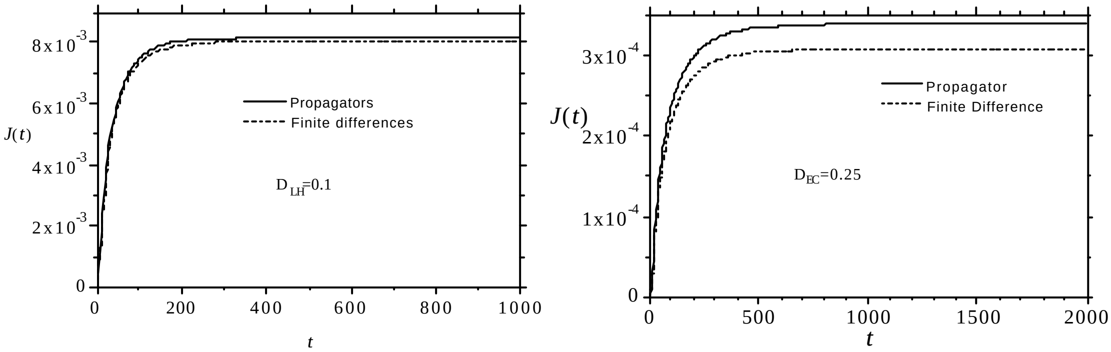

FIGURE 2. Evolution of the rf current for cases a) and c).

For the cases in Fig. 1, the steady-state currents obtained from path sums and finite differences were, respectively: a) 8.10×10^{-3} and 8.03×10^{-3}; b) 9.38×10^{-5} and 8.67×10^{-5}; c) 3.38×10^{-4} and 3.07×10^{-4}.

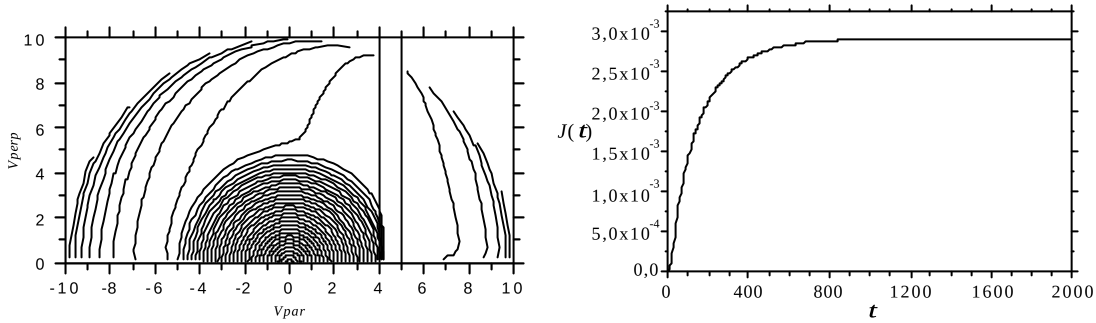

FIGURE 3. Steady-state distribution and evolution of the rf current for case d).

CONCLUSIONS

The comparison between path-sum and finite-difference calculations shows that Gaussian short-time propagators are well suited in the two limiting situations where the rf quasilinear diffusion coefficient is either relatively small or very large. Unfortunately, for intermediate values, $D_{rf} \approx 1$, and owing to time-step restrictions and concomitant constraints on grid spacing, the handling of very large propagator matrices is required when solving 2-D FP equations, which may all add up to extremely demanding computational costs.

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