A tool for the parallel calculation and ready visualization of the Choi–Williams distribution: application to fusion plasma signals

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Introduction

The Choi-Williams distribution has been successfully used to analyse some nonstationary fusion plasma signals, for which the short-time Fourier transform spectrogram, wavelets, and the Wigner distribution do not constitute the best approach [1–3]. Nevertheless, from the practical point of view, a possible drawback of using the Choi-Williams distribution, particularly when compared with the spectrogram, is its larger computation time. Here, a tool is presented that addresses this problem by performing independent parts of the calculation of the Choi-Williams distribution simultaneously, using several processors. For convenience, the spectrogram can also be calculated and displayed.

Fast Calculation of the Choi-Williams distribution

The Choi-Williams distribution [1, 4] of a signal $s(n)$ sampled at frequency f_S, calculated at instant $t_n = n/f_S$ and frequency $f_m = mf_S/(2l_\tau)$ ($m = 0,\ldots,l_\tau - 1$), can be written in as

$$\mathrm{CW}(t_n, f_m;\sigma) = 2\sum_{\mu=-(l_\mu-1)/2}^{+(l_\mu-1)/2}\sum_{\tau=0}^{l_\tau-1}\alpha^{n,m,\sigma,\mu}(\tau)e^{-\frac{i2\pi\tau m}{l_\tau}},$$

where σ controls the level of artifacts and resolution of $\mathrm{CW}(t_n, f_m;\sigma)$, and $\alpha^{n,m,\sigma,\mu}(\tau)$ is given by

$$\alpha^{n,m,\sigma,\mu}(\tau) = s\left(n+\mu+\tau-\frac{l_\tau-1}{2}\right)s^*\left(n+\mu-\tau+\frac{l_\tau-1}{2}\right)h_\mu(\mu)h_\tau\left(\tau-\frac{l_\tau-1}{2}\right)I\left(\mu,\tau-\frac{l_\tau-1}{2},\sigma\right)e^{i\pi m(l_\tau-1)/l_\tau}.$$

The windows $h_\mu(n)$ and $h_\tau(n)$ are zero except for $|n| \le (l_\mu - 1)/2$ and $|n| \le (l_\tau - 1)/2$, respectively, and the function $I(\mu,\tau,\sigma)$ can be written in exact or approximate forms,

$$I(\mu,\tau,\sigma) = \int_{-\pi}^{+\pi} e^{i\xi\mu-\frac{\xi^2\tau^2}{\sigma}}d\xi = \sqrt{\frac{\sigma}{4\pi\tau^2}}e^{i\xi\mu-\frac{\mu^2\sigma}{4\tau^2}}\,\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi\tau}{\sqrt{\sigma}}+i\frac{\mu}{2\tau}\sqrt{\sigma}\right)\right] \approx \sqrt{\frac{\sigma}{4\pi\tau^2}}e^{i\xi\mu-\frac{\mu^2\sigma}{4\tau^2}}.$$

Using this approximation, valid for low σ, is useful for faster computations, but then $\mathrm{CW}(t_n, f_m;\sigma)$ no longer becomes the Wigner distribution in the $\sigma \to \infty$ limit [1]. The above expression for $\mathrm{CW}(t_n, f_m;\sigma)$ shows another way to speed up the calculations: the summation in τ can be computed as the fast Fourier transform (FFT) of $\alpha^{n,m,\sigma,\mu}(\tau)$. Nevertheless, the extra summation in μ still makes $\mathrm{CW}(t_n, f_m;\sigma)$ much slower to calculate than the spectrogram. Since $\mathrm{CW}(t_n, f_m;\sigma)$ can be calculated independently at any desired instant t_n, further optimization can be achieved by parallel computation.

Tool description

The two distributions (Choi-Williams distribution and spectrogram) are calculated by a parallel Fortran 90 code using MPICH2 [5–6], an implementation of the Message Passing Interface (MPI) library standard. An interactive IDL [7] graphical user interface (GUI) allows the user to load signals, choose the calculation parameters, launch and interrupt the code, and plot the distributions. Figure 1 shows the GUI displaying the analysis of a JET magnetic signal [3]. The displayed plots can subsequently be changed by zooming in and out or restricting the range of plotted values. Signals can be loaded from a file on disk or from a MDSplus server [8]. Files can have two comma-separated columns, with the time base in the first column and the signal samples in the second, or just one column with the samples, in which case the user must introduce the sampling frequency and starting instant of the signal. Currently, two MDSplus servers are configured: ISTTOK and JET. To load a MDSplus signal, the user first selects the server from the server droplist, and then either chooses a

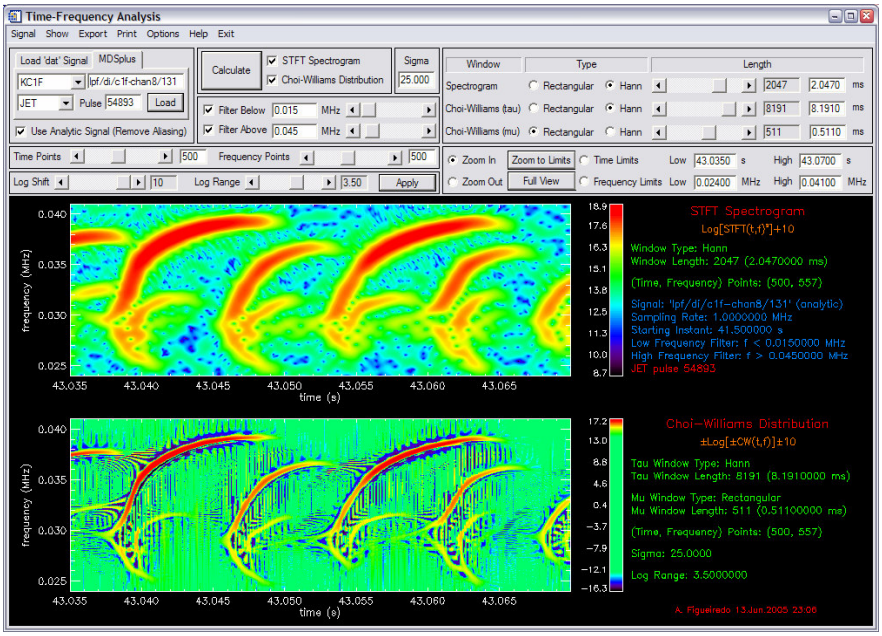

Figure 1. The MS Windows version of the IDL graphical user interface, displaying the Choi-Williams distribution and the spectrogram of a previously studied magnetic signal from the JET tokamak [3].

predefined signal from the signal droplist, or types any valid MDSplus signal name. After introducing the pulse number the signal can be loaded by pressing a button. After loading, a MDSplus signal can be given a short name and added to the signal droplist of the chosen server. Before pressing the calculate button, the user can choose which distributions to calculate, the value of σ, to filter the signal, to use the analytic signal for aliasing removal [1], the window types and lengths, the time and frequency intervals, and the number of points inside those intervals where the distributions will be calculated. Notice that l_τ does not have to be a power of 2 for optimal FFT performance. The number of FFT bins, and therefore the number of frequency points where $\mathrm{CW}(t_n, f_m; \sigma)$ can be calculated, are not restricted by l_τ since the summation in τ can be extended by zero padding the windowed signal. The GUI has menu commands to visualize the signal and its time base on screen, as well as the window functions used in the calculations. Commands also exist to print all the plots produced by the GUI and export them to JPEG or PostScript files. MS Windows and Linux versions of the GUI are available. There are also nonparallel versions of the Fortran 90 code for MS Windows and Linux, so the tool can be used in single-processor computers. In this work, all parallel calculations have been done in Orionte, a cluster with 16 nodes (Intel P4 3 GHz CPU, 1 GB RAM) running Linux (Gentoo, 2.4.28 kernel [9]) and communicating through a 1 Gbps network [10] — see figure 2. Currently, only the Linux version of the GUI running in Orionte can control the parallel code, the GUI being remotely accessed using the X protocol. Menu commands allow the user to choose a parallel calculation and the number of processors to be used.

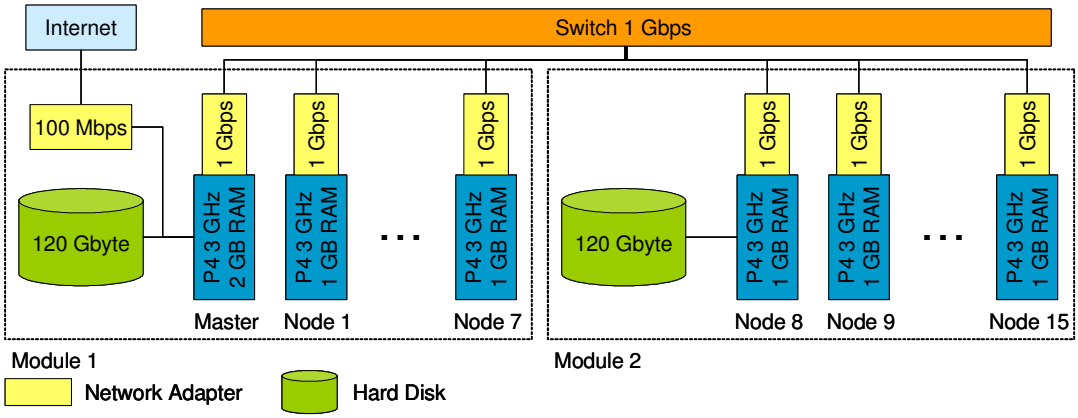

Figure 2. Basic structure of the cluster Orionte: each of the 2 currently installed modules supports 8 nodes.

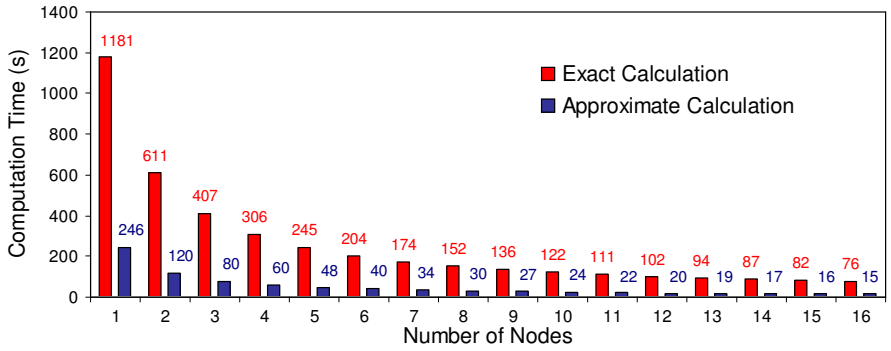

Figure 3. Computation times for the exact and approximate forms of the Choi-Williams distribution. The computation time is inversely proportional to the number of used processors, the computation with 16 nodes being approximately 16 times faster than with a single one.

Results

The calculation of $\mathrm{CW}(t_n, f_m; \sigma)$ can be carried out independently at any instant t_n. Therefore, the fortran 90 code can be parallelized very efficiently by distributing the calculations of $\mathrm{CW}(t_n, f_m; \sigma)$ at the selected instants by some or all of the available cluster nodes. To assess the performance of the parallel code and show the advantages of the parallel implementation of the Choi-Williams distribution, calculations have been made using 1 to 16 processors. The same signal and parameters have been used in all runs: a magnetic signal from a set of pick-up coils with a 1 MHz bandwidth installed in a poloidal cross section of the ISTTOK tokamak [11], recorded with a 2 MHz sampling rate. The computation times for 1 node have been obtained with the nonparallel code. Results are shown in figure 3 for the exact and approximate forms of $\mathrm{CW}(t_n, f_m; \sigma)$, where it can be seen that, as expected, the computation time is inversely proportional to the number of nodes. The tool has also been applied to signals from Langmuir probes during polarization experiments on the ISTTOK tokamak [12–14]. Figure 4 shows the Choi-Williams distribution and the spectrogram of the edge density during one such electrode biasing experiment (ISTTOK pulse 11483).

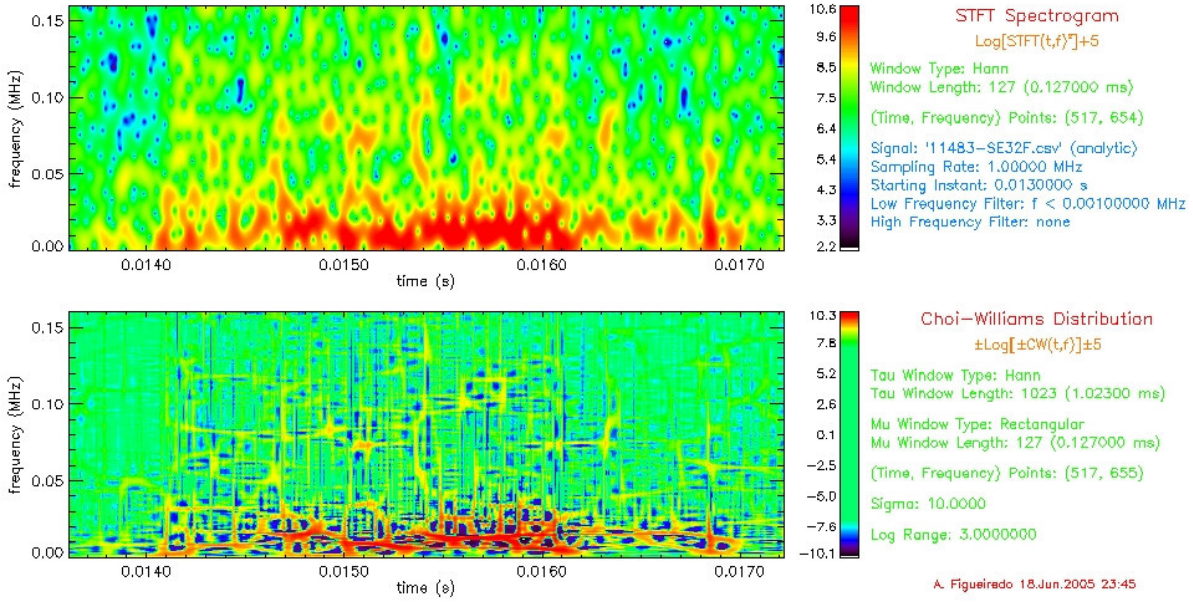

Figure 4. Time-frequency analysis of the edge density during an electrode polarization experiment on ISTTOK [12–13]. Low frequency oscillations around 15 kHz are observed with a positive bias voltage.

Conclusions

A parallel fortran 90 code has been built that computes the Choi-Williams distribution and the spectrogram. The code has been run in a 16-node cluster, whereby it has been demonstrated that it is a powerful instrument to calculate the Choi-Williams distribution when a large computational effort

is required. In addition, a specially designed graphical user interface has been created that greatly simplifies the use of the fortran 90 code. Together, the code and the graphical interface constitute a powerful, user-friendly tool, which can be effectively used in the time–frequency analysis of fusion plasma signals. With the attained parallel computing power, the calculation of the exact form of the Choi-Williams distribution, instead of the approximate one, has become possible within a reasonable time. The new tool is at present being used to analyse ISTTOK data, namely to characterize fluctuations observed in the signals of magnetic pick-up coils and Langmuir probes from limiter and electrode polarization experiments.

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