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Lower-Hybrid Ray-Tracing Calculations in Toroidal Plasmas with Magnetic Ripple: Nonlinear Oscillations and Spectral Gap

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Introduction

Lower-Hybrid (LH) current drive (CD) is essential for the next advanced tokamaks as a method to control the current-density-profile, to ramp-up the plasma current, and to sustain steady state-plasmas [1, 2]. Even though extensive studies have been done to model LHCD in tokamaks [3], there are still unsolved issues to be addressed before propagation and damping of LH waves in toroidal fusion plasmas is fully understood. One of those issues is the so-called spectral-gap problem [4]. In fact, a large difference between the parallel refractive index ($n_{\|}$) of the launched LH wave and the much-higher value for the damped wave must exist to explain effective LHCD via Landau damping on many electrons. Several mechanisms have been proposed to explain this issue but none has become a complete solution to the problem. The effects of the toroidal inhomogeneity of the magnetic field (magnetic ripple) on LH ray propagation have been proposed as one of the possible mechanisms [5, 6], but analytical results have shown that ray tracing, including both magnetic ripple and toroidal effects, fails to provide a closure to the spectral-gap issue [7]. However, a numerical confirmation has not yet been proposed and such is the aim of this paper. To validate this hypothesis, an analysis of the LH ray dynamics for a consistent large-aspect-ratio toroidal equilibrium with magnetic ripple is presented within the Hamiltonian formalism for the cold-plasma dispersion relation.

Model

To study LH wave propagation in fusion plasmas a detailed description of the plasma equilibrium is necessary. When toroidal inhomogeneities (magnetic ripple) caused by the discrete set of the magnetic coils that produce the main toroidal magnetic field in tokamaks are neglected, the plasma equilibrium is usually given by the Grad-Shafranov (GS) equation [8]. This equation is a reduction of the Ideal Magnetohydrodinamical (MHD) equations for the case of toroidal axisymmetry. Once the magnetic ripple is considered, the solution of the Ideal MHD equations for the plasma equilibrium increase in complexity and leads to a unsolved 3D equilibrium problem. In the present study, we will make some simplifications to achieve a simple but consistent equilibrium for large aspect-ratio plasmas that includes magnetic ripple. First we will use a simple solution for the GS equation within this approximation in which to the lowest order in the inverse aspect ratio, a/R_0, (with a and R_0 the minor and major radius,respectively) of the toroidal plasma, the magnetic field can be written as: $\mathbf{B} = B_0 R_0/R\ \mathbf{e}_\phi + r\, B_0/(R\, q)\, \mathbf{e}_\theta$ in simple toroidal coordinates $\mathbf{x} = (r, \theta, \phi)$, where B_0 is the magnetic field on axis, R the distance to the vertical axis, and $q\,(r)$ the safety factor. It is assumed within this study that $q\,(r) = q_0 + (q_a - q_0)\,(r/a)^2$, where q_0 and q_a are values on the plasma axis and at the edge, respectively. This equilibrium is characterized by nested concentric magnetic surfaces with a toroidal geometry.

The magnetic-ripple effects, which are typically small, are introduced as a perturbation to the toroidal axisymmetric equilibrium and the resulting magnetic field, $\mathbf{B} = \bar{\mathbf{B}} + \tilde{\mathbf{B}}$, changes from regular nested toroidal field lines to weak periodical modulated quasi-toroidal field lines. In the present study we will neglect the finite β effects and the plasma current corrections due to magnetic ripple. This corrections will be discussed and published [9]. From the MHD equations and within the assumptions made the magnetic-field perturbation has the following form:

$\tilde{B}_r = -\delta_a \bar{B}_\phi\,(a) \big/ 2\ R_0/R\ I_1\,(r\,N/R_0)/I_0\,(a\,N/R_0)\sin\,(N\,\phi)$,

$\tilde{B}_\theta = \delta_a \bar{B}_\phi\,(a) \big/ 2\ \sin\,(\theta)/N\ I_0\,(r\,N/R_0)/I_0\,(a\,N/R_0)\sin\,(N\,\phi)$, and

$\tilde{B}_\phi = \delta_a \bar{B}_\phi\,(a) \big/ 2\ \ I_0\,(r\,N/R_0)/I_0\,(a\,N/R_0)\cos\,(N\,\phi)$, where δ_a, and $\bar{B}_\phi\,(a)$ are the ripple amplitude and unperturbed magnetic field at the low-field-side plasma edge, and N the number of magnetic-field coils. The magnetic surfaces can be parameterized by a radial flux surface coordinate ρ, obtained by integrating $\tilde{B}_r \big/ \left(\bar{B}_\phi + \tilde{B}_\phi\right)$ along ϕ. The electron density will be constant on a flux surface and, for the present work, we will assume it has the form $n_e\,(\rho) = n_{ea} + (n_{e0} - n_{ea})\left[1 - (\rho/a)^2\right]^\alpha$, with n_{e0} and n_{ea} the values taken at the plasma axis and edge, respectively, and α an arbitrary value that will depend of the experimental profile.

The LH rays are traced using the usual Hamiltonian ray equations obtained from the WKB theory, $d\mathbf{x}/dt = -(\partial D/\partial \mathbf{k})/(\partial D/\partial \omega)$, $d\mathbf{k}/dt = (\partial D/\partial \mathbf{x})/(\partial D/\partial \omega)$ where $\mathbf{x}$ is the coordinate vector, $\mathbf{k}$ the canonical conjugate wave momentum, ω the wave frequency, and $D\,(\omega, \mathbf{r}, \kappa) = 0$ the local electromagnetic dispersion relation for the LH waves. The integration of these equations is done in cylindrical coordinates (R, Z, ϕ) to avoid the usual numerical problems associated with the radial-type coordinate systems.

Results and Discussion

To illustrate the effects of magnetic ripple on the propagation of the LH wave for large aspect-ratio tokamaks we choose a set of parameters that represent typical LHCD experiments performed in TRIAM-1M [4, 7]. This is a tokamak with a major radius $R_0 = 0.84$ m, minor radius $a = 0.12$ m, and $N = 16$ toroidal-field coils. The relevant experimental parameters are: $B_0 = 7.0$ T, $q_0 = 1.0$, $q_a = 15.0$, $I_p = 0.04$ MA, $T_{e0} = 0.6$ KeV, $\omega/2\pi = 8.2$ GHz, $n_{e0} = 1.5 - 4.0 \times 10^{19} m^{-3}$, $n_{ea} = 0.15 - 0.40 \times 10^{19} m^{-3}$ for magnetic field on axis, safety factor on axis and plasma edge, plasma current, central electron temperature, and electron density range on axis and plasma edge, respectively. The magnetic ripple amplitude at the edge for TRIAM-1M is not known but is resonable to admit that $\delta_a = 10\%$ is sufficient high to maximize the ripple effects and, therefore, to evaluate safely the effects of this inhomogeneity if they are small. The simulations are done for LH rays launched between two discrete toroidal-field coils, at the plasma edge, with poloidal mode number $m = 0$, and values of $n_{\parallel 0} = 2.0, 3.0, 4.0$, which represent typical LH power-spectrum values. The rays are numerically integrated along 100 toroidal turns for the above specified range for the electron density. In Fig.1 we plot the maximum and minimum excursion of the normalized wave index $n_\parallel/n_{\parallel 0}$ along the ray trajectories as a function of n_{e0} for two values of δ_a, 5% and 10%. It is clear from the figure that the maximum $n_\parallel/n_{\parallel 0}$ increases near the ressonances as expected [7], however, even for the strongest ressonance and for the worst case ($\delta_a = 10\%$), the excursion is, at most, 100% of the launched $n_{\parallel 0}$. This upshifing of the $n_\parallel$ is insufficient to bridge the spectral gap.

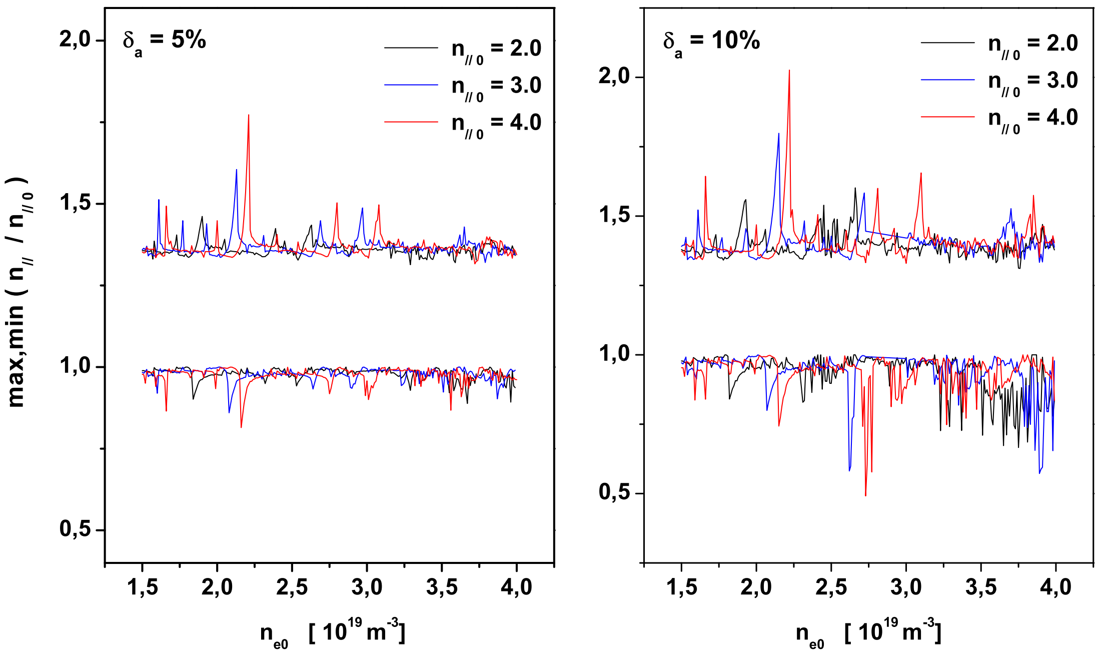

Figure 1: Maximum downshift and upshift of the normalized LH wave parallel refractive index ($n_\parallel/n_{\parallel 0}$) for representative values of $n_{\parallel 0}$. Propagation occurs for 100 toroidal turns for TRIAM-1M LHCD plasmas with magnetic ripple of 5% and 10%.

Conclusions

Progress on the effects of magnetic ripple on LH wave propagation has been made with the formulation of a model for the propagation of these waves in large-aspect-ratio toroidal plasmas in the presence of magnetic ripple. A simple three-dimensional equilibrium model was formulated that includes, consistently to the first order in the inverse-aspect-ratio, the poloidal and toroidal inhomogeneities of the magnetic field. Numerical results for TRIAM-1M tokamak, a typical medium-to-high-aspect-ratio tokamak, show that the joint effects of toroidicity and magnetic ripple can change the $n_\|$ of the launched LH wave yet, even at resonances, the upshift is bounded and insufficient to fully explain the spectral-gap issue. This results confirm previous analytical numerical studies [7].

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