Non-linear model for the plasma column macroscopic oscillations in the tokamak ISTTOK

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Introduction

Complex non-linear behaviour is a common feature in the dynamics of tokamak fusion plasmas. The detail study of such systems provides information on the underlying physics and it is important in the development of plasma control and feedback techniques. The non-linear dynamics of the plasma column macroscopic oscillations observed in the tokamak ISTTOK is studied in this paper. The tokamak ISTTOK is a small device with large aspect ratio, a minor radius r~8.5cm and a major radius of $R_0=0.46m$ [Varandas et al 1996]. The iron core transformer produces 0.25Vs of inductive flux, which allows plasma currents of 5-10 kA and plasma discharges of up to 45ms. In the present setup with toroidal magnetic field of 0.5 T, electron temperatures of 100 eV have been measured using Thompson scattering diagnostic and the energy confinement time is estimated to be around 0.5 ms.



Figures (1,2) Plasma current and line integrated density as a function of time, showing the typical plasma oscillations common in some discharges.

Magnetic diagnostics and plasma column macroscopic oscillations

Recently, a new poloidal array of 12 magnetic probes were installed inside the inconnel vacuum vessel with the objective of measuring and controlling the plasma column position in real time, replacing the original arrays off magnetic sensors [Belo, 1997], installed outside the vessel. In plasma discharges, where the current in the poloidal magnetic field coils that produce the equilibrium vertical magnetic field is low ($I_{vert} \sim 120A$) and insufficient

to balance the plasma column, typical oscillations in the plasma current are observed. In discharge #10774, analysed in detail in this paper, the plasma current varies from the higher value of 6 kA to the lower value of 1 kA as shown in figure 1. The period of the oscillations is around 10ms, which allows 3-4 oscillations during a typical plasma ISTTOK discharge of 30-40ms. During the initial phase of the discharge as the current increases the plasma moves outwards due to insufficient inward force provided by the vertical magnetic field. Limited by the outside limiter the plasma current starts to decrease and an inward movement is then observed until the plasma is again limited by the inside part of the vessel.



Figure 3 Plasma position, reconstructed using the poloidal array of magnetic sensors and the current filament numerical approximation during the various stages of the cycle (t=6.0 ms, t=11.0 ms, t=12.0 ms, t=15.0 ms, t=20.0 ms, t=22.2 ms, t=28 ms) for the ISTTOK discharge #10774.

The signal from the microwave interferometer [Vergamota, 1993], shows the density varying from a higher value of $6 \times 10^{18} \text{ m}^{-3}$ to less than $1 \times 10^{17} \text{ m}^{-3}$ as seen in figure 2. This strong variation in the measured density can be explained by the combined effect of the overall change in the plasma density and the change in the radial position of the plasma column during the current oscillations, moving the plasma away from the line of sight of the diagnostic.

Non-linear model

Small perturbations around the equilibrium can be described by the linearised expression, $I_p \approx \alpha_1 + \alpha_0 r$, where the linear coefficient is given by, $\alpha_0 = \frac{4\pi R_0 B_V}{a\mu_0} \left(\ln \frac{8R_0}{a} + \beta_p + \frac{l_i}{2} - \frac{3}{2} \right)^{-2}$ [Wesson, "Tokamaks"]. The rate of rise of the

particle content is approximated by $\frac{dn_e}{dt} = S + R \frac{n_e}{\tau_p} - \frac{n_e}{\tau_p}$, where S is the source of particles

evolving from the wall, τ_p is the particle confinement time and R is the total recycling coefficient for particles leaving the plasma and then returning to the plasma [D. Baker et al]. The energy equation is given by the balance between the ohmic heating (P_{Ohm}) and the losses caused by radiation (P_{rad}) and convection, the latter being proportional to the inverse

of the energy confinement time (τ_E) [V.V. Plyusnin et al], $n_e \frac{dT_e}{dt} = P_{Ohm} - P_{Rad} - \frac{n_e T_e}{\tau_E}$.

Using the relations described above a dynamical model for the evolutions of the plasma density (n_e) and plasma temperature (T_e) can be written as $dT_e = \lambda_s V_{loop}^2 T_e^{\frac{3}{2}}$

follows
$$n_e \frac{dI_e}{dt} = \frac{\lambda_s r_{loop} I_e^2}{2\pi^2 R_0 r^2} - \lambda_{Rad} n_e^s - \lambda_\tau \frac{n_e I_e}{\lambda_s V_{loop} T_e^{\frac{3}{2}} \cdot r^2}, \quad \frac{dI_e}{dt} = S - \lambda_p \frac{n_e}{\lambda_s V_{loop} T_e^{\frac{3}{2}} \cdot r^2}.$$

Analytical Solution

z, h and g are introduced as the energy confinement time (z), effective particle confinement time (h) and the fraction convection energy losses (g). The fraction of radiated energy is given by (1-g). These definitions together with a transformation of variables generates the dynamical system equations in terms of normalised density and temperature $n_e = \overline{n_e} \ \widetilde{n_e}$,

$$T_e = \overline{T}_e \ \widetilde{T}_e, \quad \frac{d\widetilde{T}_e}{d\varsigma} = z \left((1-g)\widetilde{n}_e^s - g\widetilde{T}_e^{-\frac{7}{2}} + \widetilde{n}_e^{-1}\widetilde{T}_e^{\frac{3}{2}} \right), \quad \frac{d\widetilde{n}_e}{d\varsigma} = h \left(1 - \widetilde{n}_e \widetilde{T}_e^{-\frac{9}{2}} \right), \text{ with the fixed}$$

point of the dynamical system given simply by $\tilde{T}_e = 1$, $\tilde{n}_e = 1$. The linearised equations

around the fixed point
$$(\widetilde{T}_e = 1, \widetilde{n}_e = 1)$$
 are given by $\begin{pmatrix} \frac{dT_e}{d\zeta} \\ \frac{d\widetilde{n}_e}{d\zeta} \end{pmatrix} = \begin{bmatrix} \frac{7g}{2} - \frac{3}{2} & gs - s - 1 \\ \frac{9}{2} & -1 \end{bmatrix} \cdot \begin{pmatrix} z \ \widetilde{T}_e \\ h \ \widetilde{n}_e \end{pmatrix}$.

The stability of the system can be analysed using the eigenvalues of the linearised equation: $\omega = \frac{1}{4} \left(-2h - 3z + 7g z - \sqrt{(2h + 3z - 7g z)^2 - 8(12h z - 7g h z + 9h s z - 9g h s z)} \right).$

Numerical Solution

A more detailed study of the evolution of the dynamical system in the non-linear phase requires a numerical solution of the model equations. A straightforward first order algorithm was used, without compromising the overall result. A comparison between the non-linear evolution of the plasma density calculated numerically using the model equations and the H-alpha light, approximately proportional to the density, is shown in figures 4 and 5 as a function of the plasma current.



Figure 4,5 The evolution of the normalised plasma density as a function of the normalised plasma current calculated using a numerical solution of the model equations and the H-alpha light intensity as a function of the plasma current are shown for comparison.

Conclusions

The model is able to reproduce the main qualitative features of the oscillations seen in the experimental data. In particular, the clear phase differences observed between the current and density oscillations. The evolution of the main plasma parameters is in good qualitative agreement with the experimental data. The stability analysis of the non-linear cycle shows that stable oscillations require a large fraction of radiation losses.

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References

- D. Baker et al GENERAL ATOMICS REPORT GA-A22345.
- P. Belo. Tese de Mestrado, Instituto Superior Técnico, Lisboa, Portugal
- V.V. Plyusin et al, 28th EPS Conference ECA vol. 25A, 2001 p601
- C. A. F. Varandas, et al.. "Engineering, Fusion Technology, 29(1):105–115, 1996.
- S. Vergamota. Tese de Mestrado, Instituto Superior Técnico, Lisboa, Lisboa, Portugal
- J. Wesson. Tokamaks. Oxford University Press, Oxford New York, 2nd edição, 1997.