30th EPS Conference on Contr. Fusion and Plasma Phys., St. Petersburg, 7-11 July 2003 ECA Vol. **27A**, P-2.152

# Flux surface geometry for turbulence computation on open field lines

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**Introduction** Flux tube turbulence codes, such as DALF [1], are applicable to the region inside the magnetic separatrix, where the field lines are closed. In the scrape-off layer (SOL) of the plasma, the magnetic field lines are open and strongly deformed due to proximity of the separatrix. Using similar methods to those behind the construction of the globally consistent flux tube formulation in DALF [2,3], we construct a flux tube Hamada coordinate system which allows computations in the SOL region. The new geometry is then calculated for a real ASDEX Upgrade magnetic field configuration. From this calculation we obtain the geometrical information needed for the DALF codes. The influence of the geometry on the turbulence dynamics is briefly addressed.

**The geometry** The equations of the DALF flux tube turbulence models have several operators which are geometry dependent, namely, the $E \times B$ advection $\partial/\partial t + \mathbf{v}_E \cdot \nabla$, the perpendicular Laplacian $\nabla_\perp^2$, the parallel gradients $\nabla_\parallel$ and divergences $B\nabla_\parallel (1/B)$ and the magnetic divergence terms $\mathcal{K} = -\nabla \cdot [(c/B^2)\,\mathbf{B} \times \nabla]$. Also noteworthy here is the assumption that the plasma is magnetised, from which the perpendicular and parallel dynamics largely separate, as the first favours large wavelengths whereas the other favours smaller ones, that is $k_\perp \gg k_\parallel$. This is usually referred to as the flute mode character, and has a strong impact on the appropriate choice of the geometry model to use.

The geometry itself is established by the equilibrium magnetic field present in the plasma, which is obtained by solving the Grad-Shafranov equation, a consequence of scalar pressure equilibrium $\mathbf{J} \times \mathbf{B} = c\nabla p$. A nested set of surfaces (2-tori) is assumed, forming an axisymmetric geometrical system for which there is a natural choice of coordinates. These are a radial flux label coordinate (by definition constant within a given surface) and two angular coordinates corresponding to the poloidal and toroidal directions.

On this general coordinate system we impose a set of convenient properties that will ultimately define the coordinates themselves. In our case we impose that the flux label coordinate $\Psi$ be a monotonic function and that $\theta$ and $\zeta$ are angle like coordinates, normalised in such a way that, within a given flux surface, $\theta \to \theta + 1$ in a complete poloidal loop and $\zeta \to \zeta + 1$ in a complete toroidal loop. We also impose the contravariant components of the field to be flux functions, $\mathbf{B}\cdot\nabla\Psi = 0$, $\mathbf{B}\cdot\nabla\theta = \chi'$ and $\mathbf{B}\cdot\nabla\zeta = \psi'$, with the primes denoting the partial derivative with respect to the flux label coordinate (note that $\psi$ is different from $\Psi$ in this notation). Their amplitudes are constrained by the Jacobian of the coordinate system, which should also be constant within a given flux surface.

If we use the volume enclosed by the flux surface labeled by $V$ as the flux label coordinate, the Jacobian becomes unity and this constitutes the the standard definition of Hamada coordinates [2,4]. In the present case, since we want to describe the SOL region, where the field lines are strongly deformed and no longer closed, we can not do that. Instead we choose the magnetic poloidal flux, here represented by $\Psi$. The immediate consequence of it is the relaxation of the constraint for the Jacobian ($\mathcal{J}_H = \nabla\theta\times\nabla\Psi\cdot\nabla\zeta$) which still is a flux label, but no longer unity. Note the change in the order of the coordinates $\Psi$ and $\theta$ so that the system remains right handed, due to the assumption that the poloidal flux has its maximum value ($\Psi_0$) in the magnetic axis, and decreases towards zero, on the last closed flux surface.

The fact that the contravariant components of $\mathbf{B}$ are flux functions implies that the field lines are straight. Consequently, the field pitch is also a flux function, and in our case is given by $q(\Psi) = \psi'/\chi'$. The magnetic field can be expressed as $\mathbf{B} = \left[\nabla\chi\times\nabla\left(\zeta - q\theta\right)\right]/\mathcal{J}_H$ and this property allows us to further transform the coordinate system, aligning one of the coordinates with the field [3,5]. Namely, with the transformation $\vartheta = \theta$ and $\xi = \zeta - q\theta$ we can express the magnetic field in its Clebsch representation $\mathbf{B} = \left(\chi'/\mathcal{J}_H\right)\nabla\Psi\times\nabla\xi$ and, consequently, only one of its contravariant components, the one corresponding to the field aligned coordinate $B^{\vartheta}$, does not vanish. It follows that $\nabla_\parallel = \left(\chi'/B\right)\partial/\partial\vartheta$, which illustrates the separation between the parallel coordinate $\vartheta$ and the perpendicular ones, $\Psi$ and $\xi$. Recalling the flute mode character of the dynamics of the DALF model, the advantage becomes evident as we can afford to have a coarser grid in the $\vartheta$ direction that favours the large space scales while keeping higher resolution in perpendicular coordinates, where the spatial scales are smaller. This translates in a great improvement in computational efficiency.

Following the formalism of the flux tube approximation [2,3], we restrict the radial extent of our domain in order to neglect the $\Psi$ dependence of all physical quantities, except when their gradients enter the main non-linearities of the model, namely, the $E\times B$ advection terms and the contribution of the perturbed magnetic field to the parallel gradients. We also neglect derivatives with respect to $\vartheta$, based on flute mode character $\partial/\partial\vartheta \ll \partial/\partial\Psi \sim \partial/\partial\xi$, except in the case of parallel gradients and divergences.

An appropriate rescaling of the coordinates is performed with $x = \left(\Psi - \Psi_1\right)/\Psi'$, $y = -a\xi$ and $s = L_\parallel\vartheta$, where all the coefficients are constants. Note the minus sign in $y$

due to the change in the coordinates, in order to maintain right-handedness. This also applies for $x$, since $\Psi' < 0$. The parallel connection length is given by $L_\parallel = B_0/\chi'$ and the global shear can be defined as $\hat{s} = -\Psi' q'/B_0$. In the cylinder model these reduce to the more familiar expressions $L_\parallel = 2\pi q R$ and $\hat{s} = (r/q)\,\partial q/\partial r$, with $a$ representing the minor radius of the plasma.

Finally, the shifted metric procedure described in [2,3] is also applied. It is a local transformation which addresses the problem of the grid deformation due to the non-orthogonality of the coordinates system, specially important in the $\nabla_\perp^2$ operator. It forces the metric component $g^{xy}$ to vanish locally at a particular $s = s_k$ plane by introducing an arbitrary shift $\alpha(x)$, only dependent on $x$, to the $y$ coordinate. It is given by $\alpha_k'(x) = (g^{xy}/g^{xx})|_{s=s_k}$.

**Numerical implementation** In practice, to calculate the geometry, an intermediary coordinate system $(\eta, \Psi, \phi)$ is needed for the Hamada integrals, where $\eta$ is a parametric coordinate giving the position along the magnetic field line in the poloidal plane. The information about the magnetic field configuration for the tokamak ASDEX Upgrade is obtained from the gridding code CARRE [6]. It provides the cylindrical coordinates $R = R(\eta, \Psi)$ and $Z = Z(\eta, \Psi)$ along each poloidal flux surface, which we then interpolate using bicubic splines [7]. This also allows us to calculate $\partial R/\partial\eta$, $\partial R/\partial\Psi$, $\partial Z/\partial\eta$ and $\partial Z/\partial\Psi$ at any given spatial location. Using the transformations rules for $(R, Z, \phi) \leftrightarrow (\eta, \Psi, \phi)$ we are able to express the partial derivatives $\partial\eta/\partial R$, $\partial\eta/\partial Z$, $\partial\Psi/\partial R$ and $\partial\Psi/\partial Z$, which appear in the Hamada coordinates definitions, in terms of the previous ones. With this technique all the spatial information about the magnetic field is available and we proceed to the calculation of the integrals in $\eta$ defining $\theta$ and $\zeta$. This is done using 10-point Gauss-Legendre quadrature [7], whose requirement for high degree of smoothness explains the previous use of a third order interpolation algorithm. The Hamada contravariant metric components are then obtained from the coordinates using the definition $g^{ij} = \nabla x^i \cdot \nabla x^j$. From those, applying the coordinates transformation rules, the final flux tube metric components are eventually obtained. With these quantities calculated, all the geometrical information needed for the terms in the DALF model mentioned in the beginning is available, namely, the 7 geometrical quantities $g^{xx}$, $g^{xy}$, $g^{yy}$, $B^2$, $b^s$, $\mathcal{K}^x$ and $\mathcal{K}^y$, all dependent only on the parallel $s$ coordinate.

**Results** The results presented correspond to a given ASDEX Upgrade magnetic field equilibrium configuration with $B_0 = 1.85$T. Note that the shifted metric procedure was applied so that we show $\alpha_k'$ instead of $g^{xy}$. In terms of the influence of these quantities on the turbulence, $B^2(s)$ gives the variation of $\rho_s^2$ (drift scale) with $\theta$, important as it controls the polarization drift dynamics. The variation of $g^{xy}$ with same coordinate, gives the local shear, here calculated by $\partial\alpha_k'/\partial s_k$. The effect of the Shafranov shift is felt through $g^{xx}$, which in our case shows a clear maximum, corresponding to the rightmost side of the plasma in the figure where the flux surfaces are radially compressed. The terms $\mathcal{K}^\mu$ define the structure of the curvature effect and, finally, $b^s$ plays a role in $\nabla \cdot \mathbf{b} = \mathbf{B} \cdot \nabla (1/B)$, a flux expansion in the parallel dynamics. Besides those geometrical quantities mentioned, the Jacobian is also plotted, and we can see that it is a flux function, but not unity as in standard Hamada coordinates.

The future work involves quantifying the influence of this geometry by performing DALF turbulence simulations. One should stress here that the ratio $L_y/L_x \sim 20$ is an important result of treating properly the geometry. It plays a role when choosing the grid domain for DALF, as the truncation of the total flux tube on the $y$ coordinate requires the proper treatment [3]. The issue of the boundary conditions on the divertor plates given by Debye sheath physics is a central one.

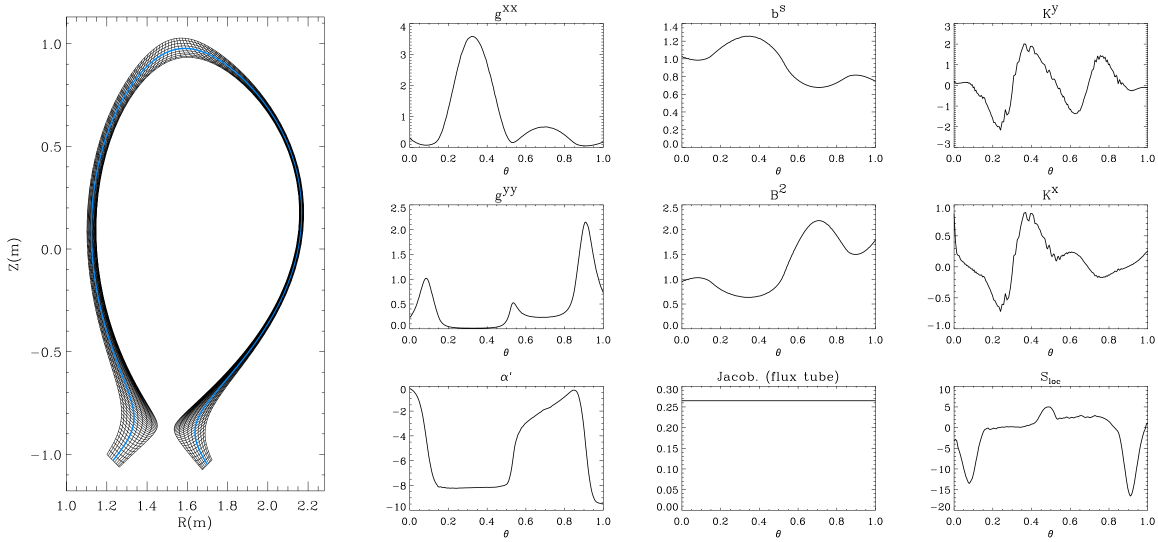


Figure 1: Flux tube shifted metric geometrical quantities (normalised [1]) necessary for DALF simulations (right), evaluated on the reference flux surface (blue) shown on the left hand side figure. There, the corresponding SOL magnetic field configuration of ASDEX Upgrade is depicted, as well as the grid calculated (equidistant in $\theta$) for the geometry construction. The origin of the $\theta$ axis corresponds to the divertor plate on the right side.

**Acknowledgments** This work, supported by the European Communities and "Instituto Superior Técnico", has been carried out within the Contract of Association between EURATOM and IST. Financial support was also received from "Fundação para a Ciência e Tecnologia" in the frame of the Contract of Associated Laboratory. T. Ribeiro also acknowledges "Fundação para a Ciência e Tecnologia" for a Ph.D grant.

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