

Bifurcation of generic metastable tearing modes interacting with resonant magnetic fields

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1 – Introduction

The increasing efforts to improve the performance of present tokamak plasma discharges have established that neoclassical tearing modes (NTM) are a major obstacle to advanced high β_p tokamak operation, degrading plasma confinement. These modes, stable at low β_p , are destabilised by the local bootstrap current reduction over a magnetic island formed at low q rational surfaces. From the competition between the destabilising and stabilising contributions, the mode is in general metastable [1]. Non linearly, the mode can only grow provided the island width overcomes a threshold W_{thr} needing a seed island $W_{seed} > W_{thr}$ to be driven by some physical triggering mechanism. Sawtooth crashes have been suggested as a possible trigger [2], although such an hypothesis is contradicted by experimental evidence that the mode onset may appear before the crash [3].

Alternative interpretations for the triggering of some NTMs rely on the interaction with resonant magnetic fields, either arising from external magnetic fields [4] or by mode coupling [3]. The latter has recently been analysed, with reference to some JET discharges [3]. Although from numerical modelling coupling appeared to be a valid triggering mechanism, experimental evidence that the NTM is not always on a perfect frequency match with the other mode(s) when it is detected, may raise some doubts on the effectiveness of coupling. To clarify this, in this paper, rather than mode coupling and without loss of generality, we analyse in more detail the dynamic process of the bifurcation of a metastable tearing mode, driven by resonant external magnetic fields, in a scenario of differential rotation between the driving external field and the mode.

2 – Modelling the destabilisation of a metastable mode by resonant magnetic fields

In its simplest form, the stability parameter associated to NTMs can be written as

$\Delta'(W) = -|\Delta'_0| + C_b \beta_p W / (W^2 + W_d^2)$, where the first term comes from the equilibrium current density (negative), W_d is a threshold arising from finite parallel transport effects, β_p is the poloidal plasma beta and C_b is a geometrical constant. However, such an expression is just one *particular* case of a “metastable” $\Delta'(W)$. In fact, as can be seen in Fig. 1, a scenario with a tearing unstable ohmic mode, subject to a stabilising electron cyclotron current drive (ECCD) within the island (arbitrarily small wave deposition width and phased to the island O-point), is also metastable, with

$$\Delta'(W) = \Delta'_{\text{nat}}(W) - C_{\text{ECCD}} \frac{f(W)}{W^2} \tag{1}$$

where $\Delta'_{\text{nat}}(W) = \Delta'_0(1 - W/W_s)$, the second term represents the effect of local ECCD ($C_{\text{ECCD}} > 0$) and the driven current is modelled through $f(W) = \tanh(\alpha W^2)$, ensuring that for small W the driven current goes like W^2 , whereas for large values of W it saturates.

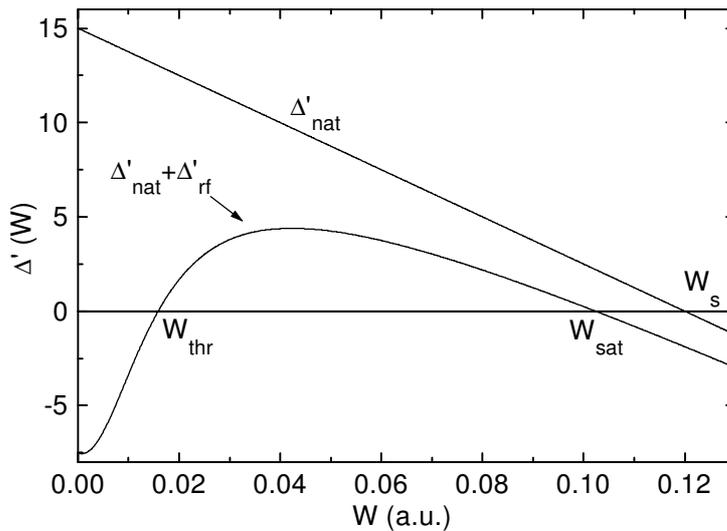


Figure 1 – Metastable mode resultant from the stabilisation of an unstable mode by local ECCD

Under the above scenario, we use a numerical reduced MHD model in cylindrical geometry to study the time evolution of the mode subject to external resonant magnetic fields rotating at a frequency different than the natural rotation frequency of the mode. In particular, we are interested in the “bifurcation” phase, where the term bifurcation designates a discontinuous variation of the island width with the external current I_E [5]. The magnetic field and plasma velocity, assuming only one harmonic (m,n) , are written as $\vec{B} = \nabla \times (\psi \vec{b}) + B_0 \hat{z}$ and $\vec{v} = \nabla \times (\tilde{u} \vec{b}) + V_z \hat{z}$, where the total flux is given by $\psi \equiv \psi_0 + \tilde{\psi}$,

(ψ_0 – equilibrium poloidal flux), only equilibrium “toroidal” velocity is assumed and $\vec{b} = nr/(mR)\hat{\theta} + \hat{z}$. The closed set of equations for the magnetic flux and potential is

$$\frac{\partial \psi}{\partial t} + \vec{B} \cdot \nabla \tilde{u} = -\eta_0(j_z + j_{\text{ECCD}}) + E_0 - v_0 \frac{\partial \psi}{\partial z} \quad (2)$$

$$\rho_0 \frac{\partial \nabla_{\perp}^2 \tilde{u}}{\partial t} = \vec{B} \cdot \nabla j_z + \rho_0 v_0 \nabla_{\perp}^2 (\nabla_{\perp}^2 \tilde{u}) - \rho_0 V_z \frac{\partial \nabla_{\perp}^2 \tilde{u}}{\partial z} \quad (3)$$

where $\nabla_{\perp}^2 \equiv \nabla^2 - \partial^2/\partial z^2$ and ρ_0 , v_0 , η_0 and E_0 are respectively, the plasma density, viscosity, resistivity and the equilibrium “toroidal” electrical field. Parabolic q and V_z profiles are assumed, with $q_0=1.33$; $q_a=3.7$ and $V_0=4\text{kHz}$; $V_a=0.2\text{kHz}$ respectively. An aspect ratio $R_0/a=3.33$, mass density $\rho_0 \sim 10^{19} \text{m}^{-3}$, $\eta_0=1/j_{z0}$ and magnetic field $B_0=1\text{T}$ are used. The plasma viscosity is assumed to be small, yielding a ratio between the resistive and viscous diffusion times scales $\tau_R/\tau_V \equiv 0.08$. We apply a gaussian ECCD distribution with amplitude $j_{\text{ECCD}} \equiv 1.75 \times 10^{-3} \tanh(4 \times 10^5 \psi_s)$, localised at the rational surface $x=x_s$, resulting on a growing mode for $\psi_s > \psi_{\text{thr}} \equiv 8 \times 10^{-6}$ and saturating at $\psi_{\text{sat}} \equiv 9 \times 10^{-5}$ (for $j_{\text{ECCD}} = 0$ one has $\psi_{\text{sat}} \equiv 1.2 \times 10^{-4}$). A normalisation for flux, radius and time, respectively, given by $\psi' = \psi \cdot (B_0 a)$, $r = x \cdot a$ and $t' = t \cdot \tau_A$ is assumed.

3 – Numerical results

An external current is ramped (starting at $t \approx 1.5\text{ms}$) to a steady value on a 10ms time scale, oscillating with a frequency of 2.5kHz (the natural rotation frequency of the mode is 3.4kHz), to destabilise the mode. When the current is below a threshold (here, 50A), the perturbed magnetic field reconnects, co-rotates with the external fields and saturates at a given amplitude. The threshold current case corresponds to a situation where the two small roots of $\Delta'(W)=0$ (with external current effects) coalesce to ψ_{bif} and is represented by the dashed curve in Fig.2a (see $\psi_{\text{bif_thr}}$). For a larger current ($I_E=57\text{A}$), the reconnected flux eventually overcomes ψ_{bif} , the mode becomes unstable and starts oscillating (see full curve in Fig.2a). These oscillations are also observed in the mode frequency (see Fig.2b) and indicate that from $\psi=\psi_{\text{bif}}$ up to $\psi=\psi_{\text{thr}}$ (where the unlocking of the mode from the forcing external fields takes place), the mode frequency is not a well defined quantity.

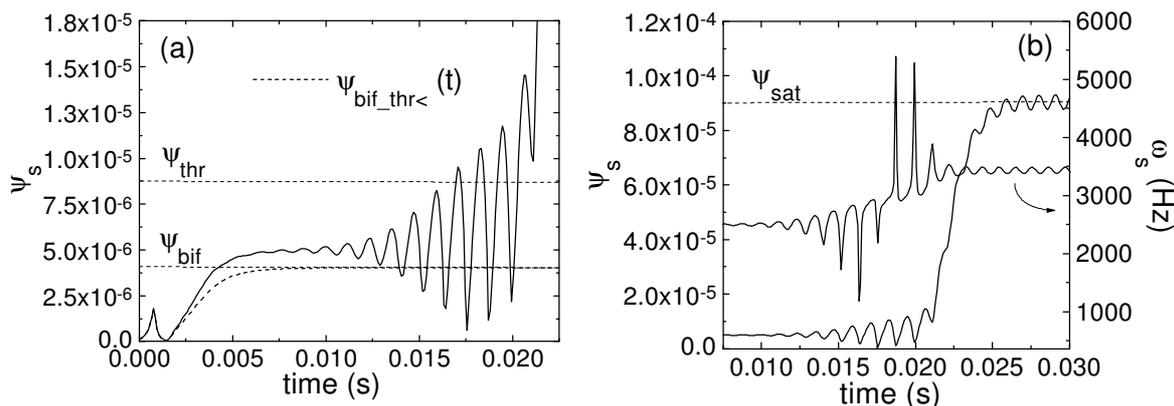


Figure 2 – (a) Early stages of the forced reconnection of a metastable mode with differential rotation. (b) Complete time evolution of mode amplitude and frequency characterising the bifurcation process

4 – Conclusions

We have shown that metastable modes, created by rf-stabilisation of unstable modes, bifurcate above a given external current threshold and that, in the presence of differential rotation between the mode and the external fields, the time evolution of the perturbed flux and mode frequency presents increasing oscillations prior to the irreversible bifurcation, rendering the mode frequency ill defined in the vicinity of the bifurcation. Therefore, if resonant external fields or mode coupling are to play a role on the destabilisation of some NTM, it is clear that the common believed perfect frequency match can only be observed before the mode bifurcates. It may be plausible that, although the mode has been driven for instance by coupling, when it is observed on the diagnostics it is no longer in frequency matching with the coupled modes. This is expected to hold specially in a scenario where the equivalent width W_{bif} is well below the diagnostic resolution. Only when both W_{bif} and W_{thr} are larger than the diagnostic resolution, the driving frequency should be clearly visible.

Acknowledgements: This work, supported by the European Communities and “Instituto Superior Técnico”, has been carried out within the Contract of Association between EURATOM and IST. Financial support was also received from “Fundação para a Ciência e Tecnologia” in the frame of the Contract of Associated Laboratory. The views and opinions expressed herein do not necessarily reflect those of the European Commission, IST and FCT. The author R.Coelho is also supported by FCT with the grant SFRH/BPD/7178/2001.

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