# Formation of Plasmoid Chains due to Resonant Magnetic Perturbations


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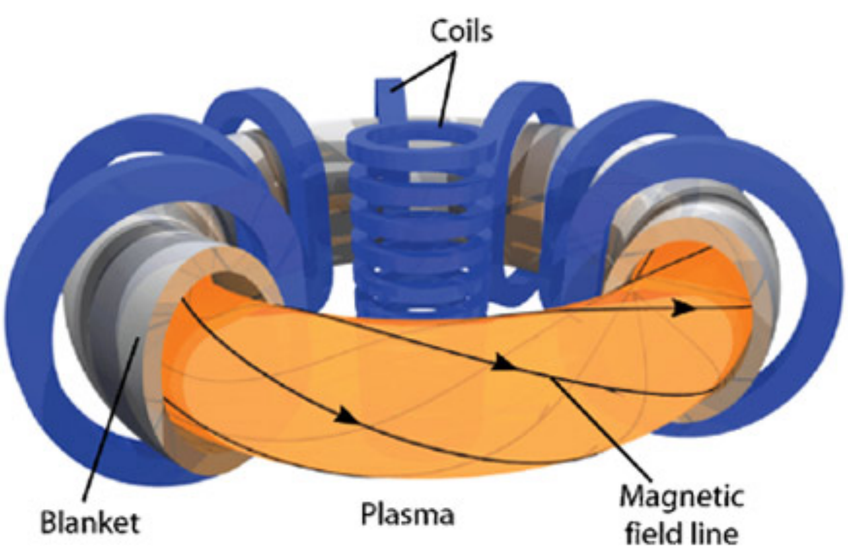
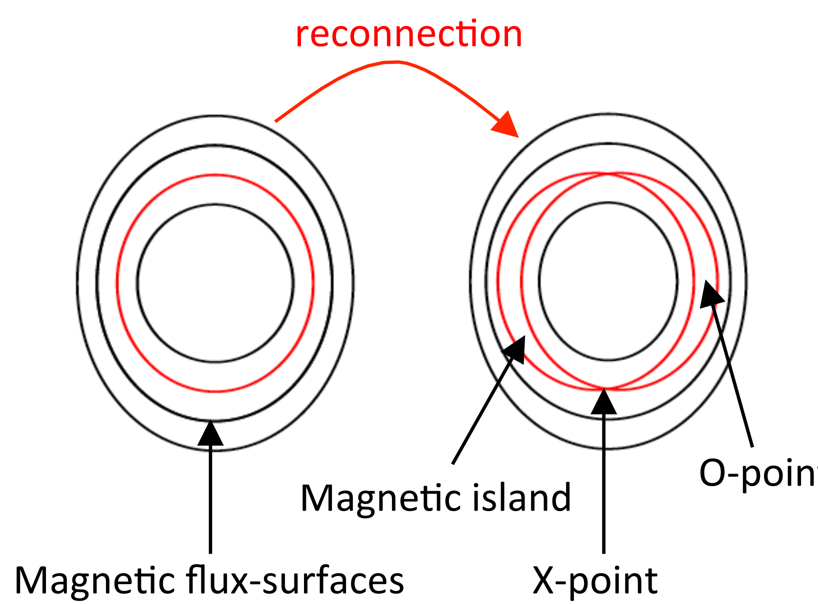
16th European Fusion Theory Conference, Lisbon, Portugal, 5-8 October 2015

# Magnetic Reconnection

- Magnetic reconnection is a process whereby the magnetic field line connectivity is modified due to the presence of a localized diffusion region.
- This gives rise to a change in magnetic field line topology and a release of magnetic energy into kinetic and thermal energy.



# Spontaneous vs Forced Reconnection

- Magnetic reconnection in a given system is conventionally categorized as Spontaneous or Forced/Driven.

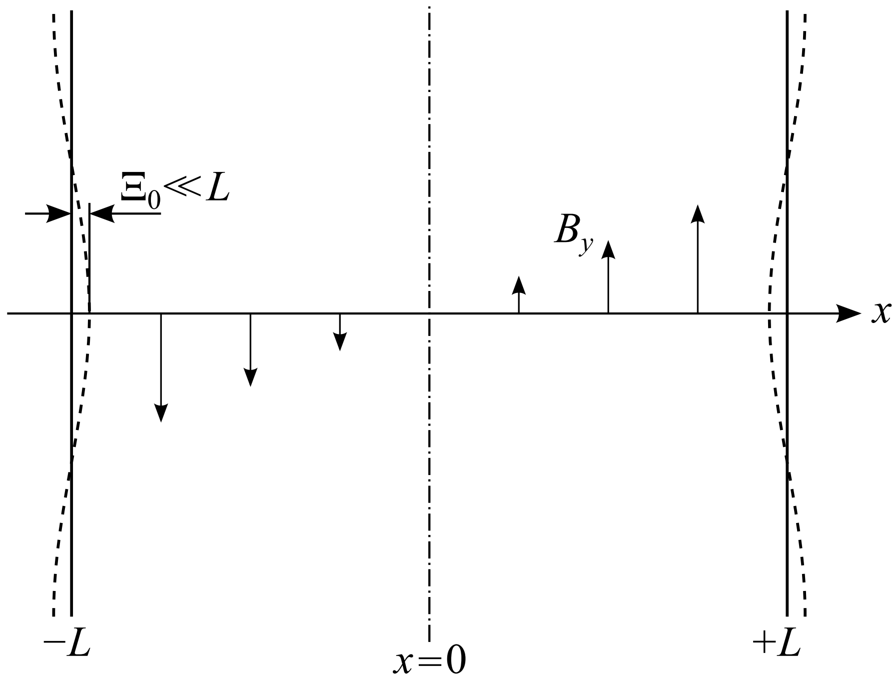
- Spontaneous magnetic reconnection refers to the cases in which the reconnection arises by some internal instability of the system or loss of equilibrium.

- Most typical paradigm ⟶ Tearing mode

- Forced/Driven magnetic reconnection refers to the cases in which the reconnection is driven by some externally imposed flow or magnetic perturbation.

- Most typical paradigm ⟶ Taylor problem

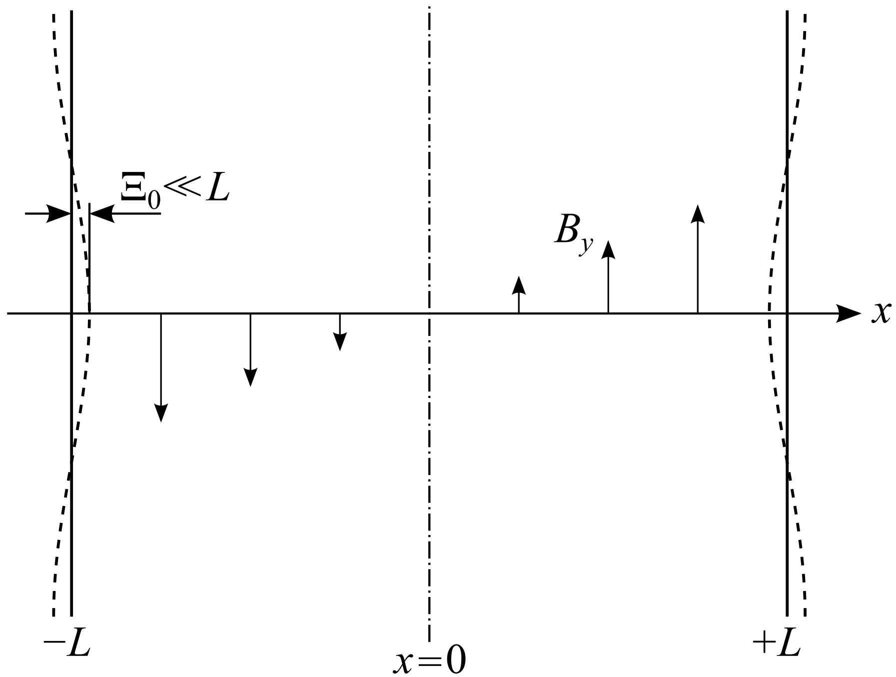
# Forced Reconnection: Taylor Problem




- Assume a tearing-stable slab plasma with an equilibrium magnetic field of the form

$$\boldsymbol{B} = B_z \boldsymbol{e}_z + B_0 \left( \frac{x}{L} \right) \boldsymbol{e}_y$$

# Forced Reconnection: Taylor Problem

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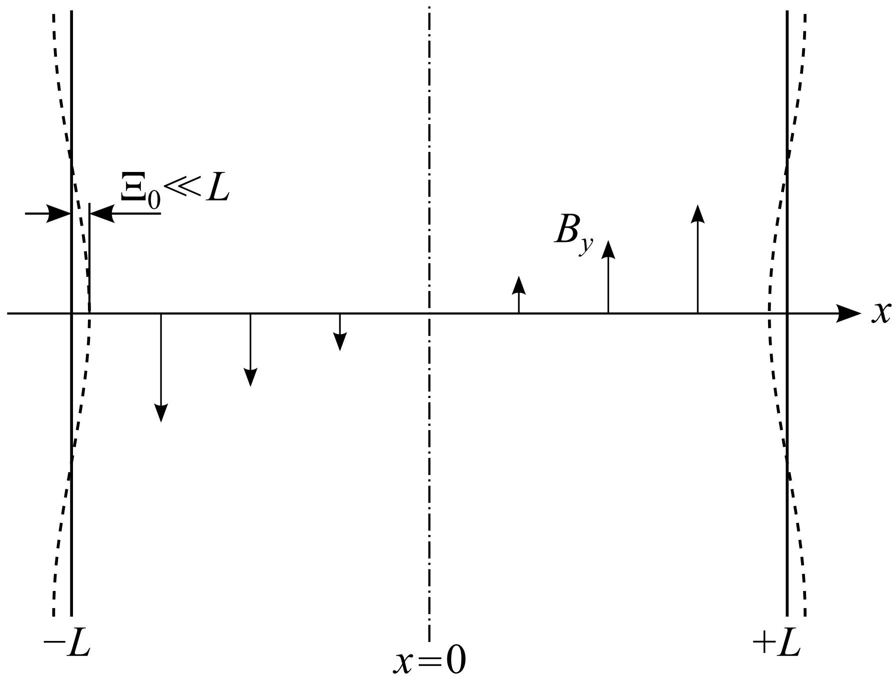
- Suppose that the conducting walls are subject to a sudden displacement

$$x_w \to \pm L \mp \Xi_0 \cos(ky)$$

# Forced Reconnection: Taylor Problem

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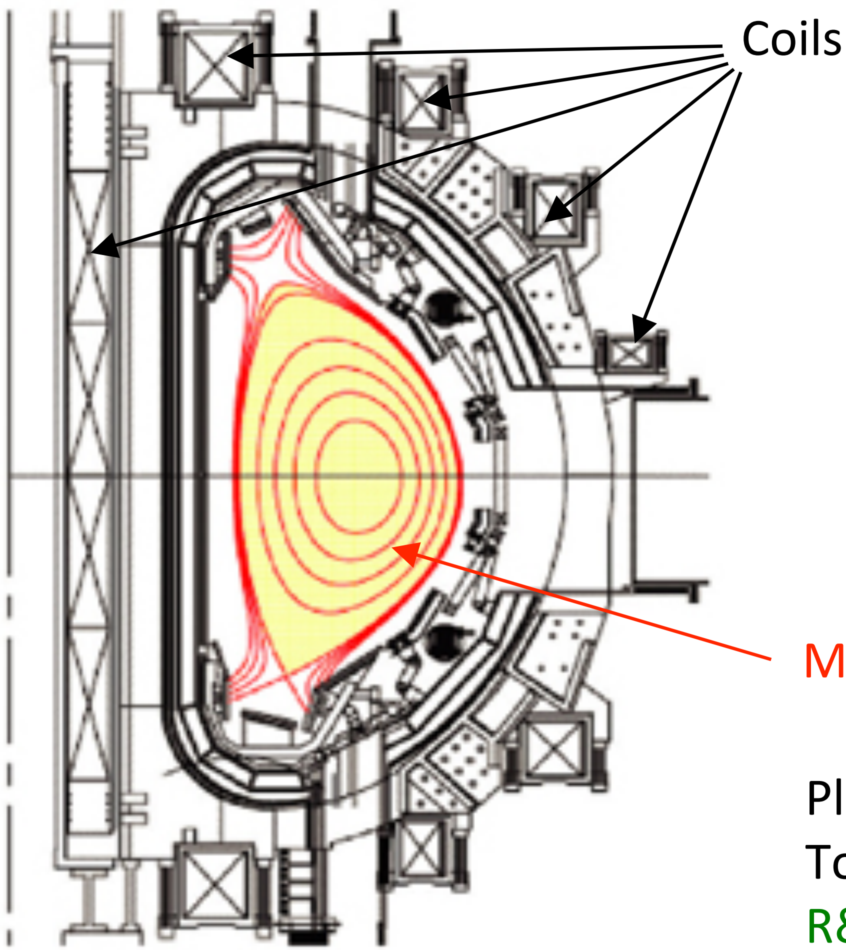
$$x_w \to \pm L \mp \Xi_0 \cos(ky)$$

- Determine the evolution of the forced reconnection process!

# Forced Reconnection: Laboratory

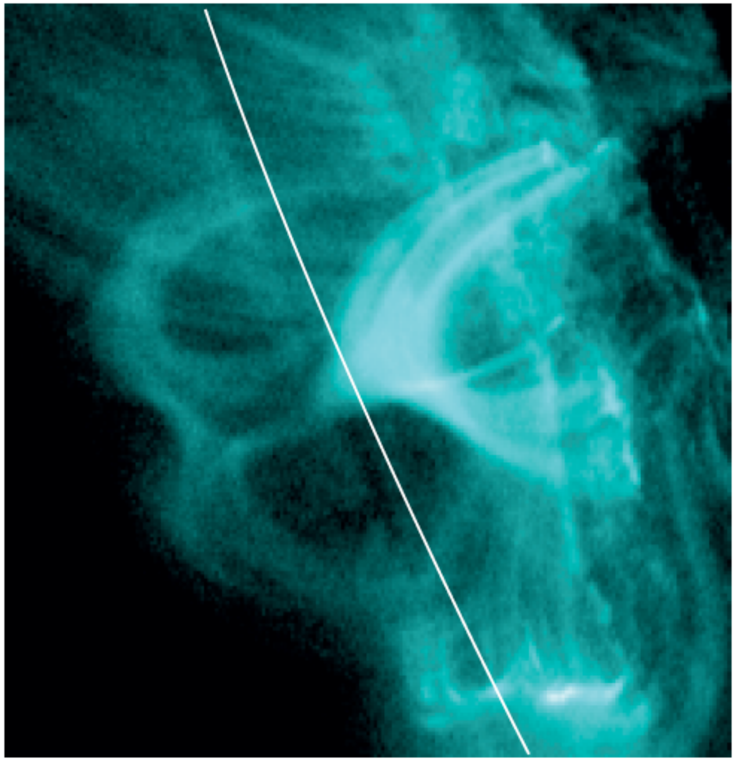



Small non-axisymmetric magnetic perturbations generated by field-coil misalignments can drive magnetic reconnection in tearing-stable plasmas

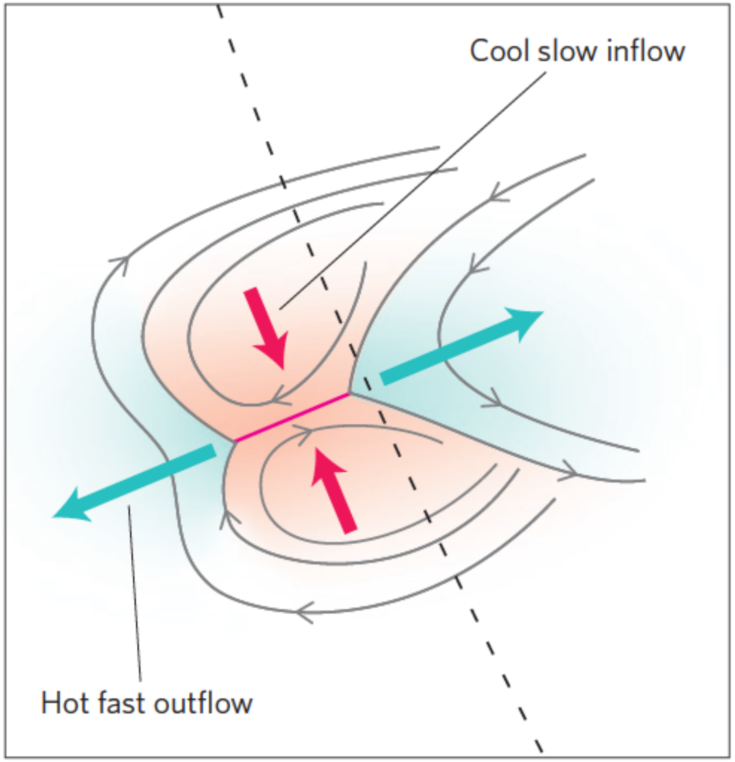
Plasma configuration in the Japanese Tokamak JT-60SA [Kikuchi *et al.*, JAEA R&D Review (2007)]

# Forced Reconnection: Astrophysics

- Forced magnetic reconnection have been investigated also in astrophysical contexts, e.g., the coronal heating problem.



Su *et al.*, Nature Phys. (2013)



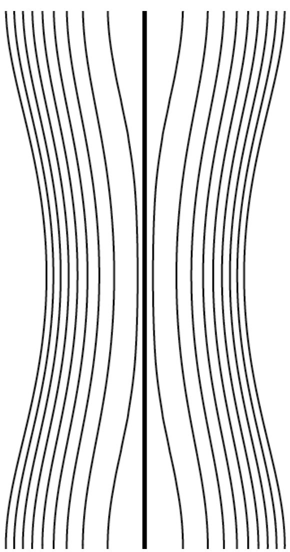

Forbes, Nature Phys. (2013)

# Hahm & Kulsrud Solution

- Hahm & Kulsrud [Hahm & Kulsrud, PoF (1985)] found two equilibria consistent with the boundary deformation.

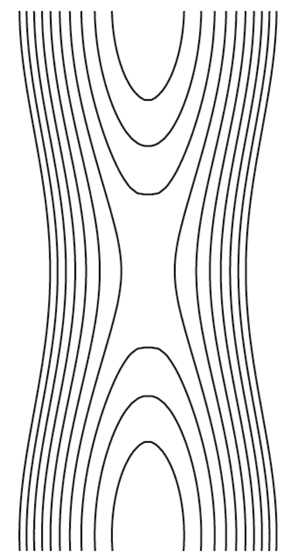
$$\mathbf{B} = B_z \mathbf{e}_z + \nabla\psi \times \mathbf{e}_z \quad \text{with} \quad \psi(x,y) = \psi_0(x) + \psi_1(x)\cos(ky)$$

(I) $\psi_1(x) = B_0 \Xi_0 \dfrac{\sinh|kx|}{\sinh(kL)}$ (II) $\psi_1(x) = B_0 \Xi_0 \dfrac{\cosh(kx)}{\cosh(kL)}$



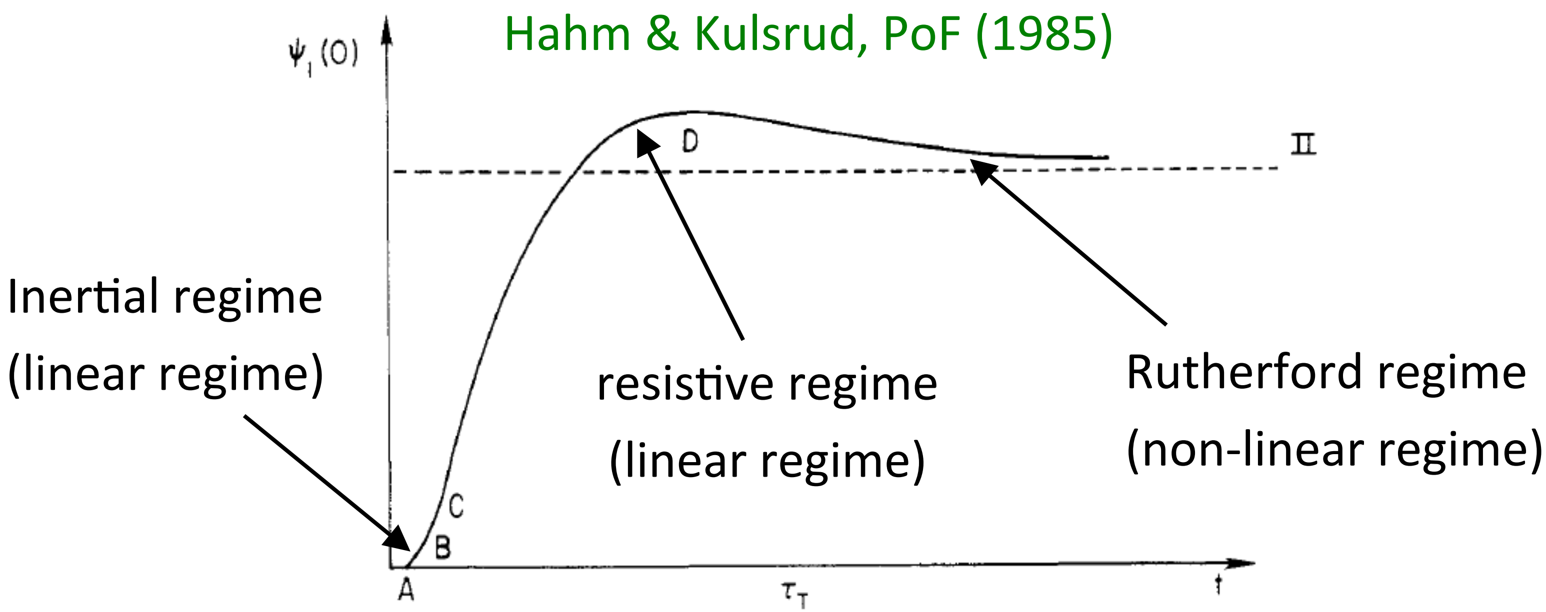
Magnetic reconnection allows the transition from equilibrium (I) to equilibrium (II)

But how the reconnection process evolves?



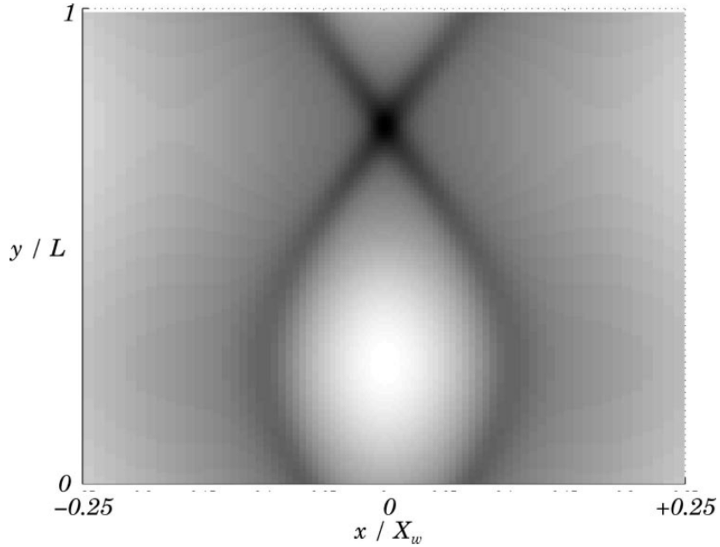
# Hahm & Kulsrud Solution

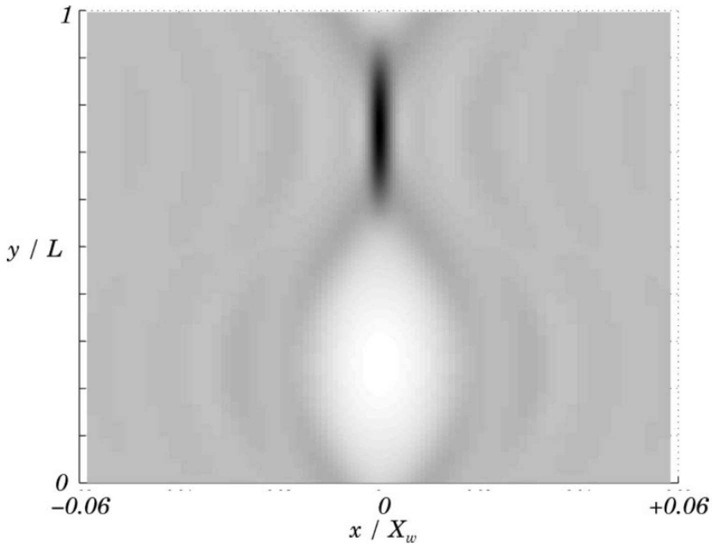
- In order to determine the explicit evolution of the forced reconnection process, Hahm & Kulsrud solved an initial value problem (within the resistive MHD framework).
- They found the following time evolution of the reconnected magnetic flux:



# Wang & Bhattacharjee Solution (+ Fitzpatrick)

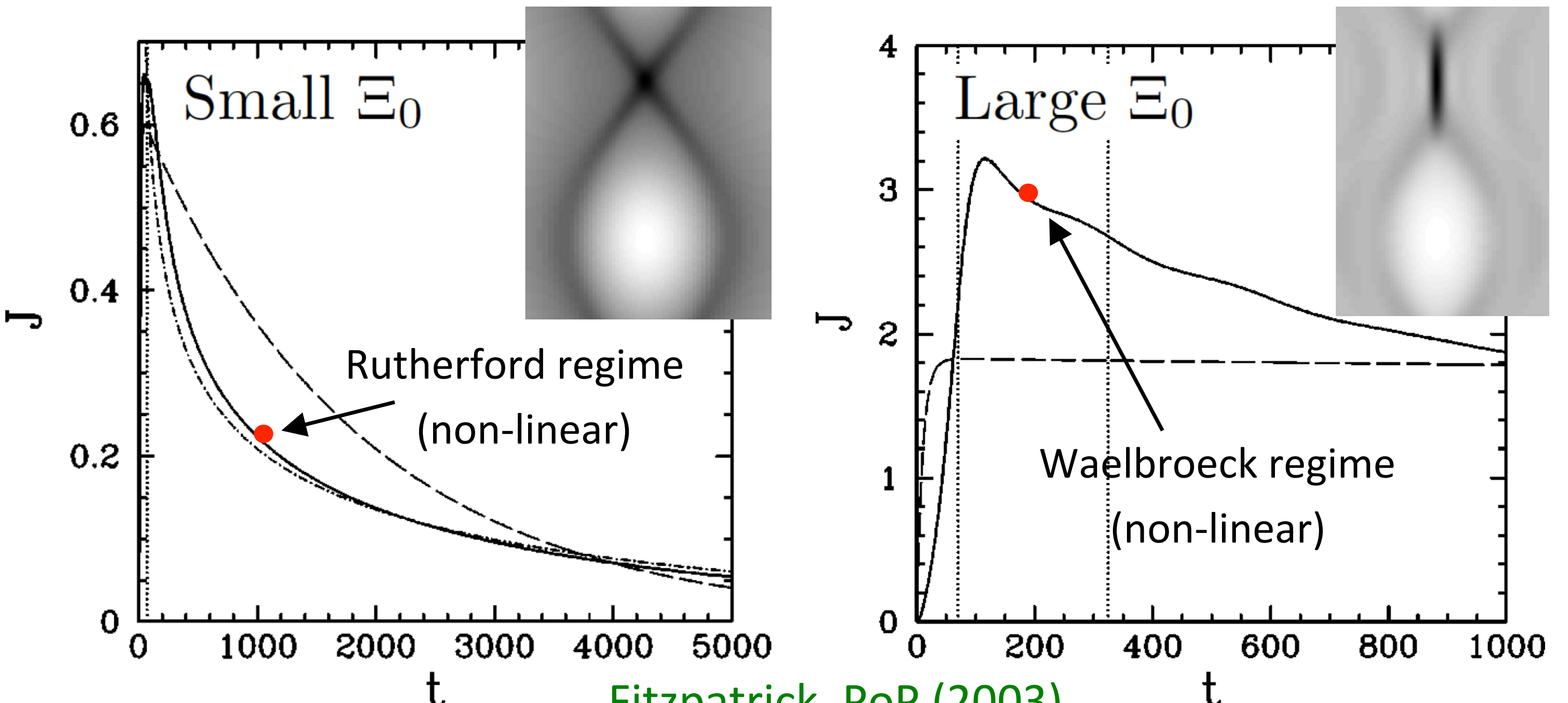
- Wang & Bhattacharjee [Wang & Bhattacharjee, PoF B (1992)] showed that the Rutherford regime may be preceded by a nonlinear phase W [Waelbroeck, PoF B (1989)] with a current sheet
- Fitzpatrick [Fitzpatrick, PoP (2003)] reconsidered the works by H&K and W&B within the visco-resistive MHD framework.



# Wang & Bhattacharjee Solution (+ Fitzpatrick)

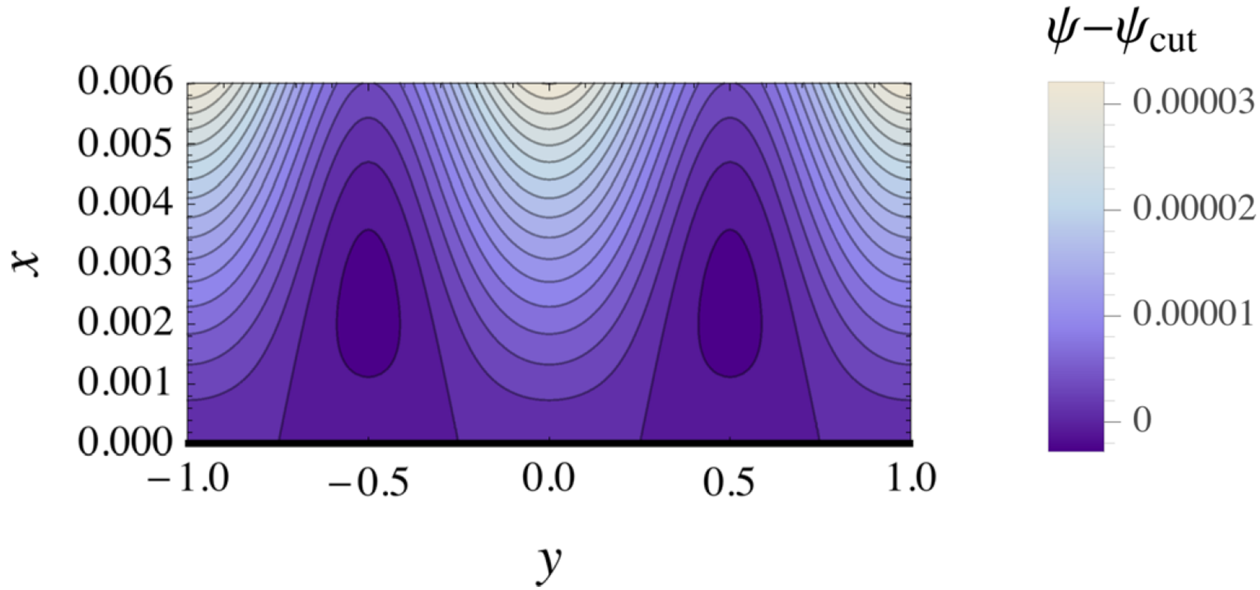
- The occurrence of the Hahm & Kulsrud evolution or the Wang & Bhattacharjee evolution depends on the perturbation amplitude.




Fitzpatrick, PoP (2003)

# More Recent Works

- Dewar and coworkers [Dewar *et al.*, PoP (2013)] found that Taylor's model admits static equilibrium solutions with plasmoids.

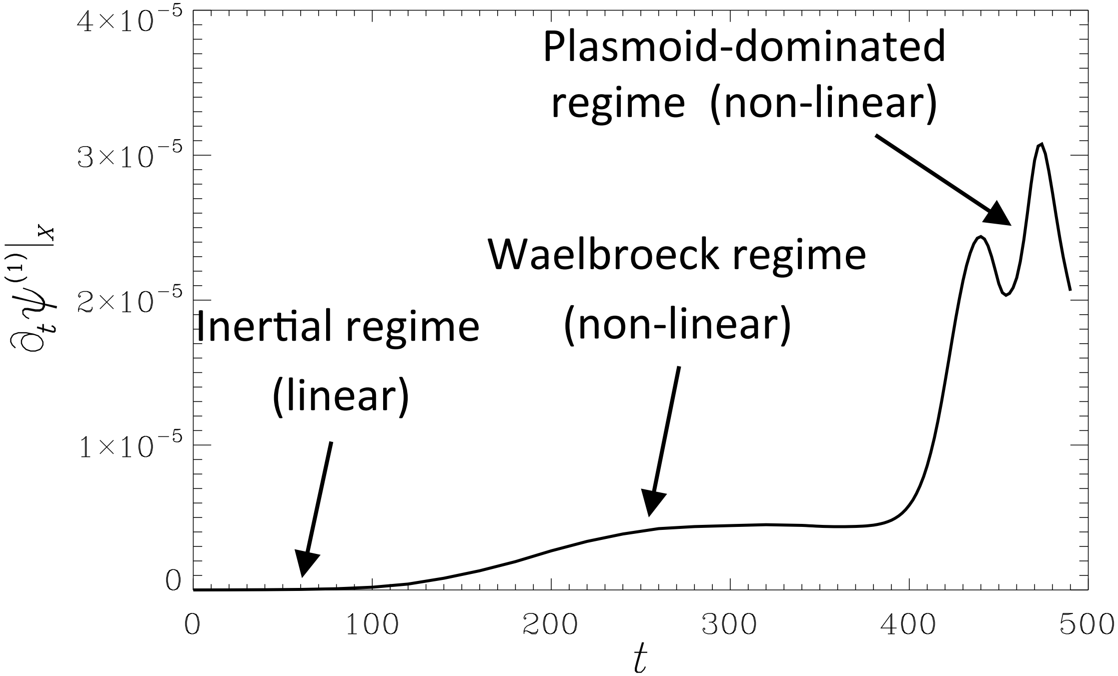



- But no attempt is made to determine a physical reconnection sequence.

⟶ It is not a study of reconnection!

# More Recent Works

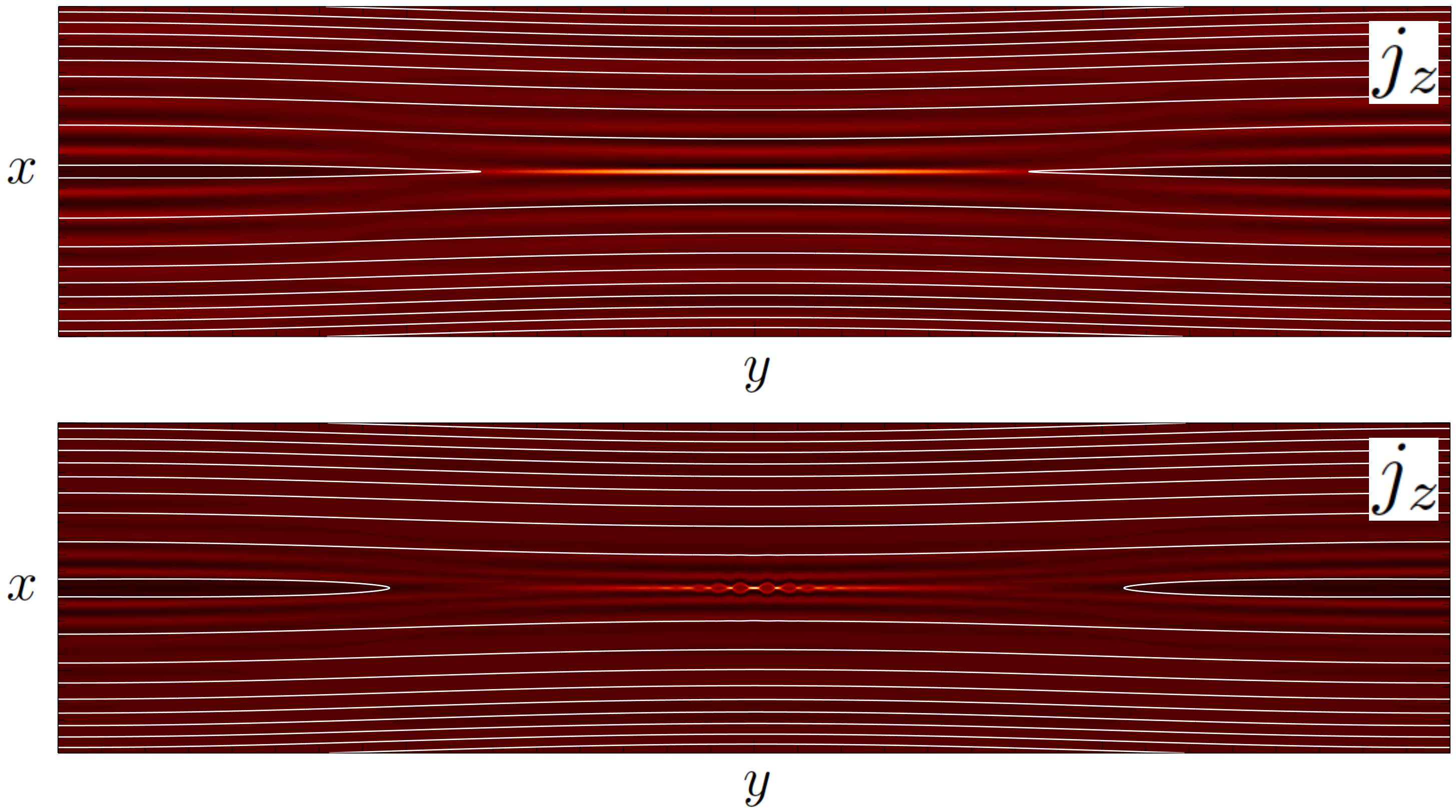
- Another evolution of the forced reconnection process has been pointed out and demonstrated in [Comisso *et al.*, PoP (2015)].




- The plasmoid formation [Loureiro *et al.*, PRL (2005)] plays a crucial role in allowing fast magnetic reconnection.

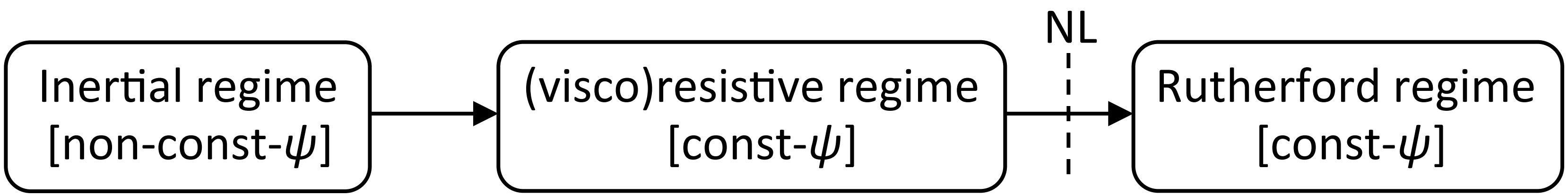
# More Recent Works

- This theory predict the threshold perturbation amplitude required to trigger the new scenario, as well as the analytical expression for the reconnection rate in the plasmoid-dominated regime.



# Possible Scenarios of the Taylor Pb.

Hahm & Kulsrud, PoF (1985)




But what is const-$\psi$ regime?

A const-$\psi$ regime is one in which the outer magnetic flux and the reconnected flux are approximately equal

$$|\psi_{1,out} - \psi_{1,in}| \ll \psi_{1,out}$$

A non-const-$\psi$ regime is the converse of a const-$\psi$ one

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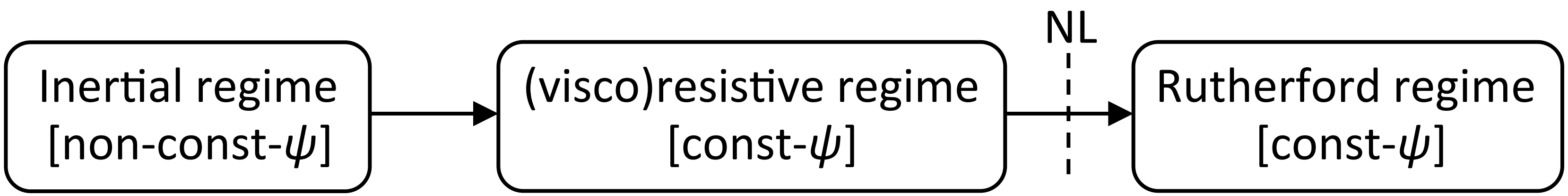
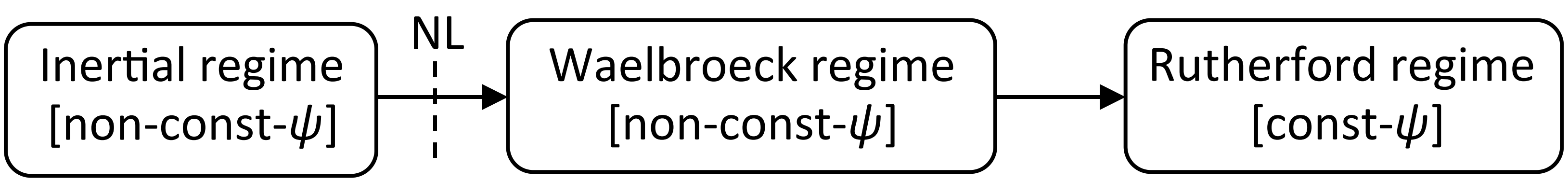



Wang & Bhattacharjee, PoF B (1992)



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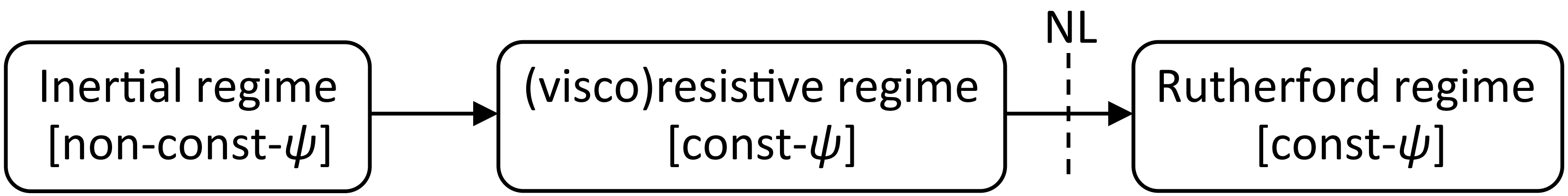


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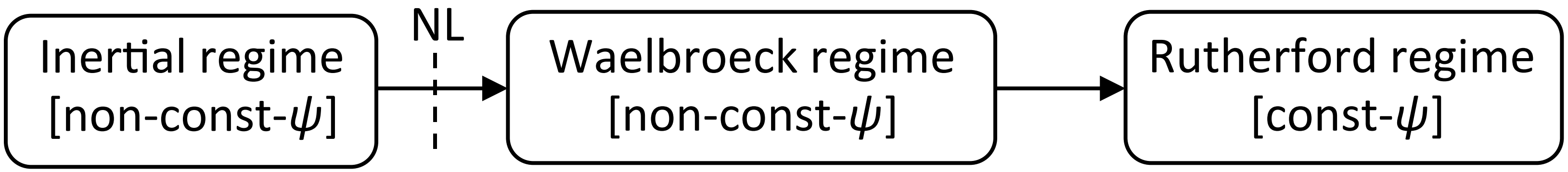
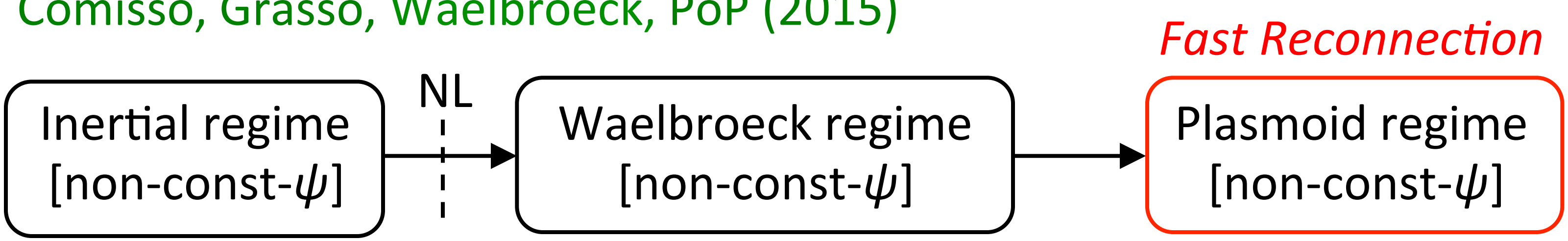



Comisso, Grasso, Waelbroeck, PoP (2015)



# Criteria for the Plasmoid Formation

- From linear theory it is possible to show that the (visco)resistive regime does **not** occur if [Comisso, Grasso, Waelbroeck, PoP (2015) and JPP (2015)]

$$\Psi_0 \gtrsim \Psi_W \, , \qquad \Psi_W = B_0 \underbrace{\frac{1}{\Delta'_s} {\tau_\eta}^{-1/3} \left( 1 + \frac{\tau_\nu}{\tau_\eta} \right)^{-1/6} \left( \frac{\tau_A}{kL} \right)^{1/3}}_{\Xi_W}$$

where

$$\tau_A = \frac{L}{v_A}, \quad \tau_\eta = \frac{L^2}{\eta}, \quad \tau_\nu = \frac{L^2}{\nu} \qquad \left( S = \frac{\tau_\eta}{\tau_A}, \quad P_m = \frac{\tau_\eta}{\tau_\nu} \right)$$

$$\Delta'_s = \frac{2k}{\sinh(kL)} \qquad \left( \left. \frac{d\psi_1}{dx} \right|_{0^-}^{0^+} = \Delta'_0 \psi_1(0) + \Delta'_s \Psi_0 \right)$$

# Criteria for the Plasmoid Formation

- The reconnecting current sheet is sufficiently narrow to undergo the plasmoid instability [Loureiro *et al.*, PoP (2007)] if the amplitude of the perturbation is such that [Comisso *et al.*, PoP (2015) and JPP (2015)]

$$\Psi_0 > \Psi_c \,, \qquad \Psi_c = B_0 \underbrace{C \frac{kL}{\Delta'_s} \frac{\tau_A}{\tau_\eta} \left( 1 + \frac{\tau_\eta}{\tau_\nu} \right)^{1/2}}_{\Xi_c}$$

  where

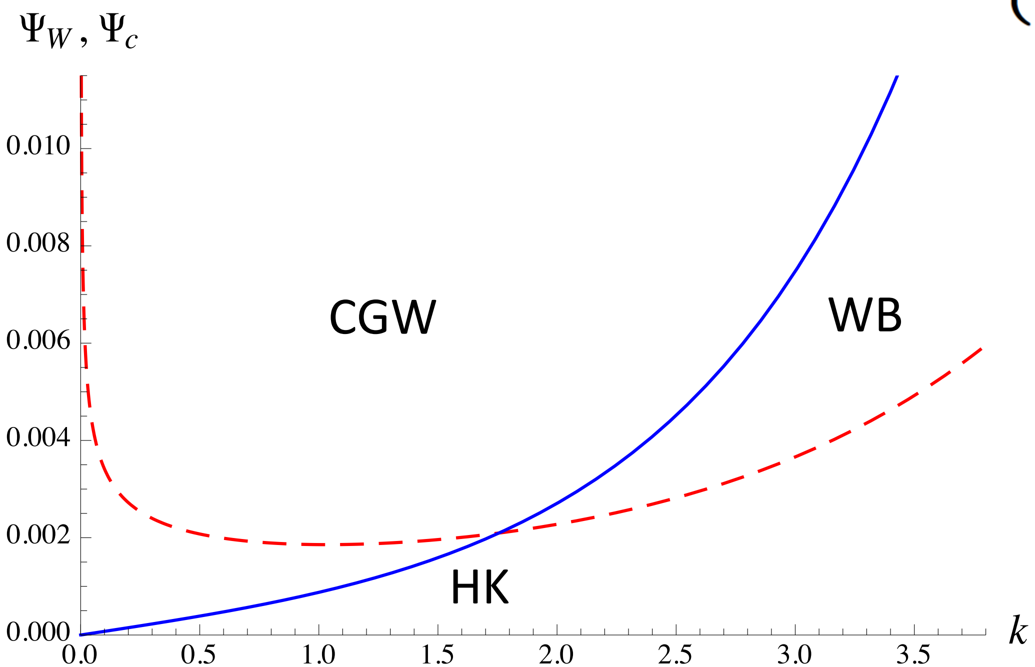
$$C \sim 2\,\epsilon_c^{-2}, \qquad \epsilon_c = \frac{\delta_c}{L_c} \ll 1$$

- Numerical simulations (e.g. [Bhattacharjee *et al.*, PoP (2009)]) suggest $\epsilon_c \sim 10^{-2}$

  (Also heuristic arguments [Loureiro *et al.*, PRE (2013)] suggest the same critical aspect ratio)

# Criteria for the Plasmoid Formation

- The plasmoid formation occur when $\Psi_0 \begin{cases} > \Psi_c\,, & \text{if } \Psi_c \gtrsim \Psi_W \\ \gtrsim \Psi_W\,, & \text{if } \Psi_c < \Psi_W \end{cases}$




[Comisso *et al.*, PoP (2015)]

$\Psi_W$ = red dashed line

$\Psi_c$ = blue solid line

$S = 10^8\,,\;\; P_m = 10$

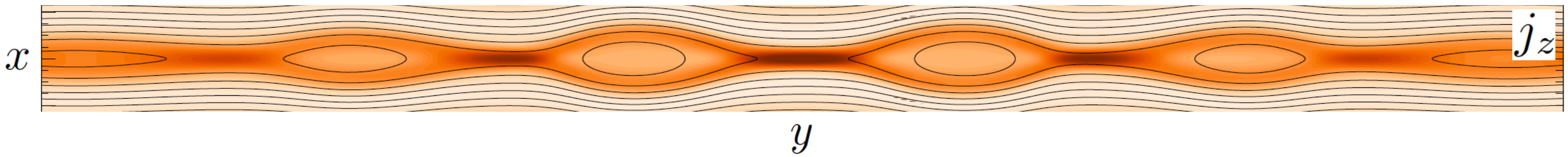
- There exists a critical perturbation wavenumber $k_*$ below which the evolution of the system always leads to the plasmoid-dominated regime.

# Reconnection Rate in the Plasmoid-dominated regime

- The reconnection rate may be evaluated as the rate of change of the flux reconnected at the main *X*-point.

- In the plasmoid-dominated regime the reconnection process is strongly time dependent, with plasmoids constantly being generated, ejected and merging each others.

- We may assume a statistical steady-state with a marginally stable current sheet located at the main X-point.



# Reconnection Rate in the Plasmoid-dominated regime

- In this case, the reconnection rate in statistical steady-state can be evaluated as [Comisso, Grasso, Waelbroeck, PoP (2015)]

$$\partial_t \psi_p \approx \epsilon_c B_0 L (\Delta'_s \Xi_0)^2 \tau_A^{-1} \left( 1 + \frac{\tau_\eta}{\tau_\nu} \right)^{-1/2}$$

- The reconnection rate in the plasmoid-dominated regime depends strongly on the external forcing $\Psi_0 = B_0 \Xi_0$

- The reconnection rate does not depend on $S = \tau_\eta / \tau_A$

- The reconnection rate decreases with increasing $P_m = \tau_\eta / \tau_\nu$

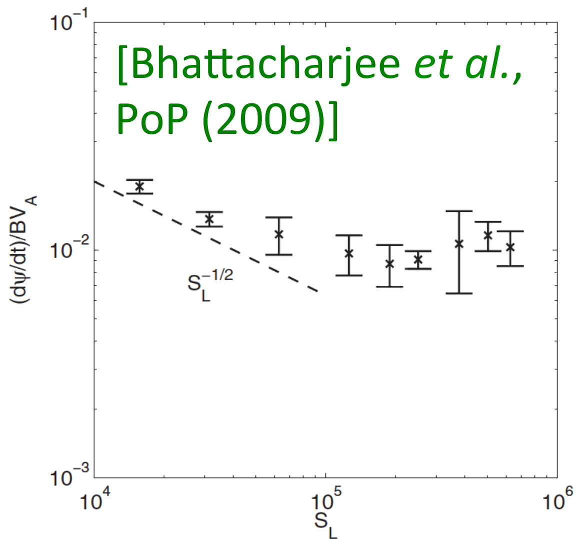
# Reconnection Rate in the Plasmoid-dominated regime

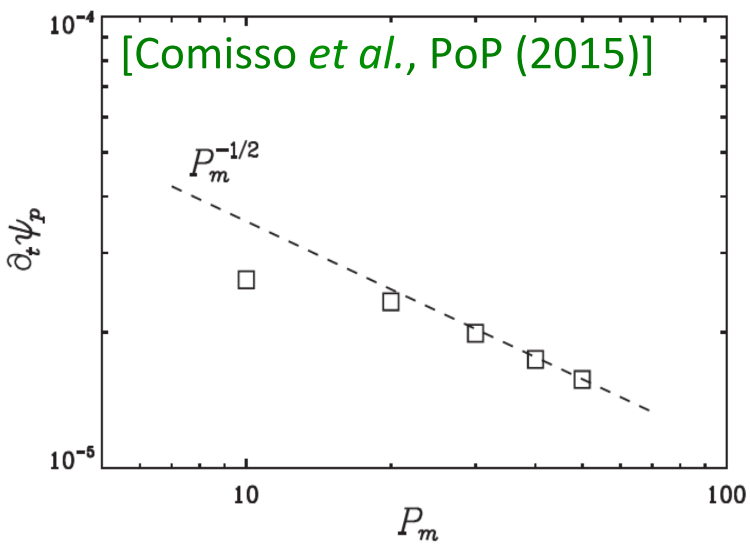
- In the small magnetic-Prandtl number limit

$$P_m \ll 1 \quad \Rightarrow \quad \partial_t \psi_p \approx \epsilon_c B_0 L (\Delta'_s \Xi_0)^2 \tau_A^{-1}$$

- In the large magnetic-Prandtl number limit

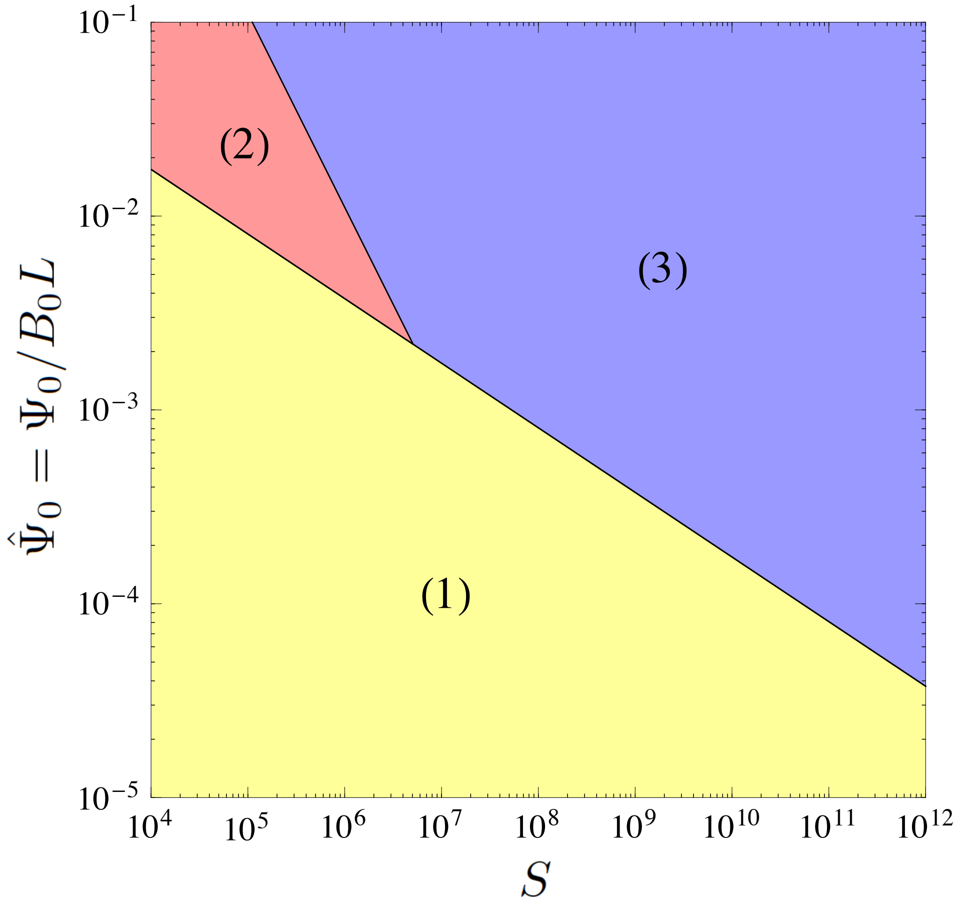
$$P_m \gg 1 \quad \Rightarrow \quad \partial_t \psi_p \approx \epsilon_c B_0 L (\Delta'_s \Xi_0)^2 \tau_A^{-1} \left( \frac{\tau_\eta}{\tau_\nu} \right)^{-1/2}$$



# Parameter Space Diagrams

- Possible evolutions of forced reconnection for $\hat{k} = 1/8,\ P_m = 5$ [Comisso, Grasso, Waelbroeck, JPP (2015)]



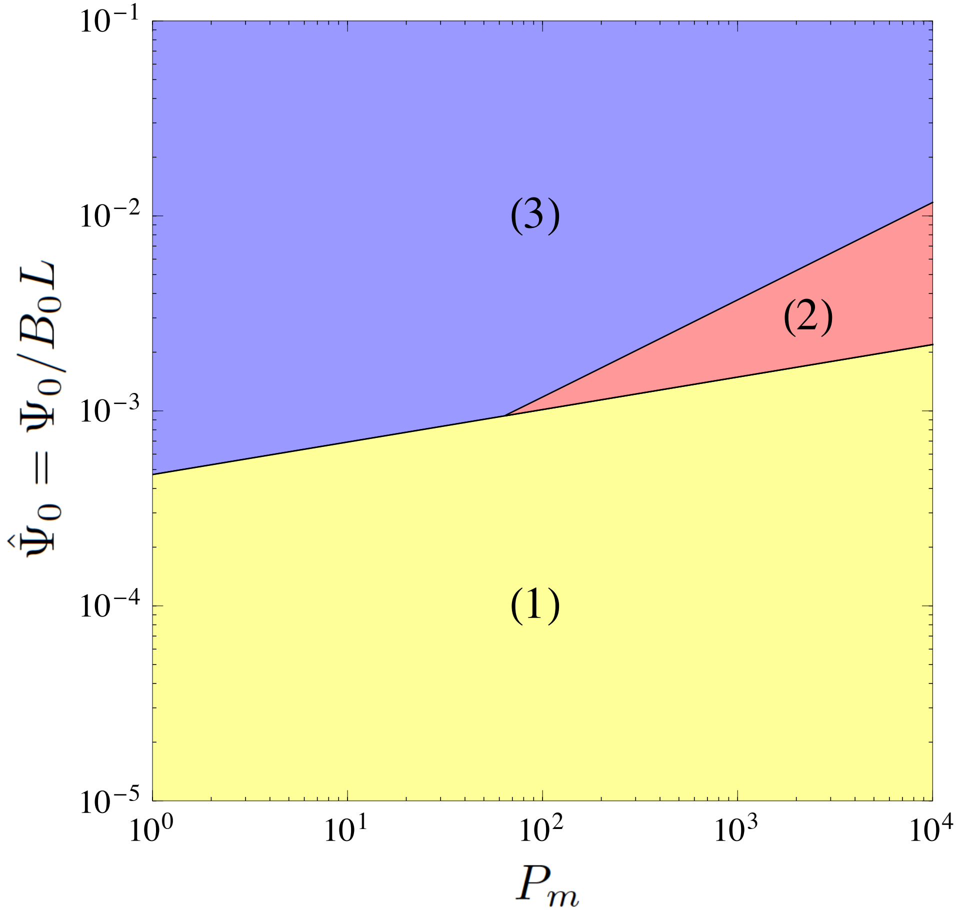

(1) HK scenario [PoF (1985)]
- -inertial regime
- -(visco)resistive regime
- -Rutherford regime

(2) WB scenario [PoF B (1992)]
- -inertial regime
- -Waelbroeck regime
- -Rutherford regime

(3) CGW scenario [PoP (2015)]
- -inertial regime
- -Waelbroeck regime
- -Plasmoid-dominated regime

# Parameter Space Diagrams

- Possible evolutions of forced reconnection for $S = 10^8,\ \hat{k} = 0.5$
  [Comisso, Grasso, Waelbroeck, JPP (2015)]

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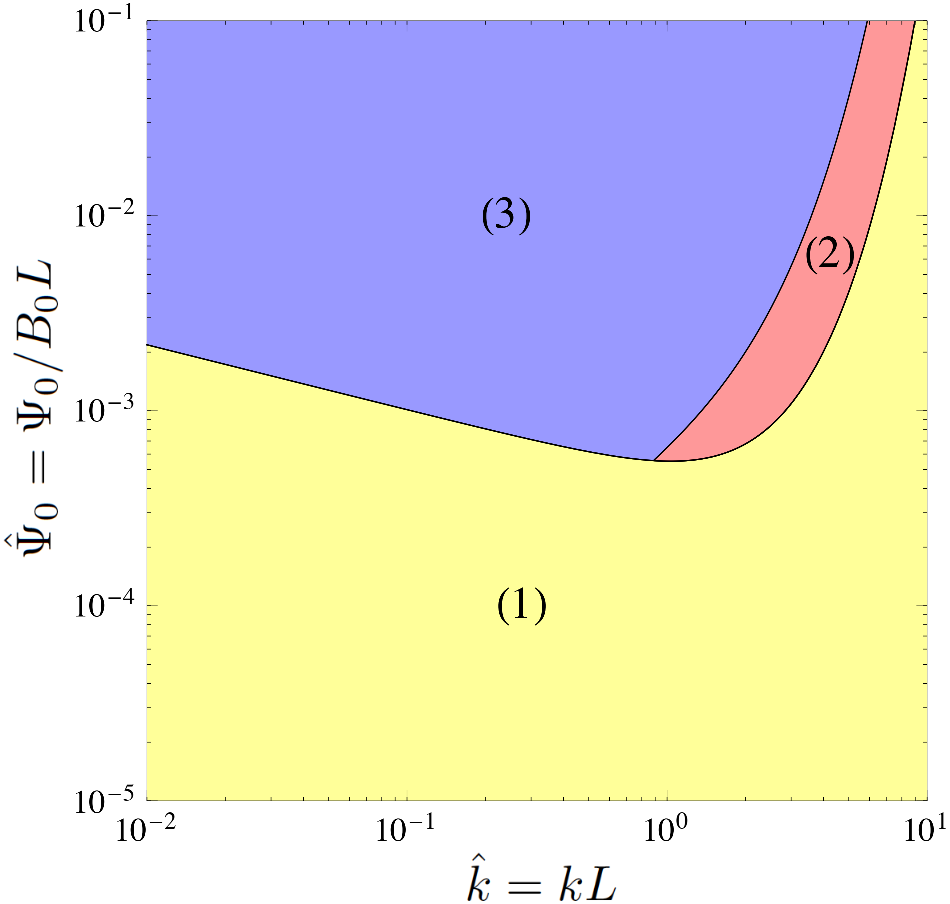


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- -inertial regime
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# Conclusions

- Large magnetic perturbations can give rise to the formation of plasmoids, which are responsible for a substantial speed up of the reconnection process.
- Below a critical perturbation wave-number, there are no stable reconnecting current sheets.
- Since the critical perturbation wave-number increases for decreasing values of the plasma resistivity and viscosity, also modest perturbation amplitudes can lead to plasmoid-dominated reconnection in large tokamaks.
- In the plasmoid-dominated regime the reconnection rate is independent of the Lundquist number, but it depends on the magnetic Prandtl number.
- It is likely that also two-fluid/kinetic effects should be considered in large tokamaks.