Dynamics of Nonlinear Interactions between Electron Temperature Gradient Mode and Ion-scale Fluctuations in Linear Magnetized Plasmas

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1. Background and Purpose
2. Experimental Apparatus
3. Experimental Results
4. Summary
Background and Purpose

Anomalous heat transport — ion scale —

Gradient Driven Instability
- Drift wave (DW) mode\(^1\)
- Flute mode\(^2\)
- Ion temperature gradient (ITG) mode\(^3\)

Anomalous electron heat transport

Electron Temperature Gradient (ETG) Mode\(^4\)

- Open problems on ETG mode
- Excitation mechanism
- Suppression mechanism

Research of ETG mode

ETG mode:
- Electrostatic mode (low $\beta$)
- Electron diamagnetic direction
- $k \parallel \rho_e \leq 1 < k \parallel \rho_i$, $\Omega_i < \omega << \Omega_e$

Theory

- **Excitation**

- **Suppression**

Multi-Scale Interactions

- **Excitation**
    - Observation
    - Identification
    - High $\beta$

- **Suppression**
    - Large $E \times B$ Shear
    - Magnetic Shear

Experiment

Wavelength scales of Fluctuations
### \( \nabla \)-Driven Instability / Turbulence

<table>
<thead>
<tr>
<th>Excitation 'Bare' Instability</th>
<th>Multi-Scale Interaction</th>
<th>Suppression</th>
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</thead>
<tbody>
<tr>
<td>Ion-scale ( \sim \rho_i )</td>
<td></td>
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<tr>
<td>- Drift wave ~ 1960</td>
<td>- Geodesic acoustic mode (Conway, Nagashima)</td>
<td>- ( E \times B ) flow shear</td>
</tr>
<tr>
<td>- Geometrical ~ 1980 (( \nabla B ), ...)</td>
<td>- Zonal flow (Fujisawa, … )</td>
<td>- linear or nonlinear</td>
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<td>- Trapped particle</td>
<td>- Streamer (Yamada, …)</td>
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<tr>
<td>- ITG mode ~ 1990</td>
<td>- *Causal relations “Multi-scale Renormalized Turbulence”</td>
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<thead>
<tr>
<th>Electron-scale ( \sim \rho_e )</th>
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<td>*Experi.</td>
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<tr>
<td>- \checkmark ETG mode</td>
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<tr>
<td>- Observation (E. Mazzucato) ~ 2008</td>
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<tr>
<td>- Identification (X. Wei) ~ 2010</td>
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<tr>
<td>- High ( \beta ) (X. R. Fu) ~ 2012</td>
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<td>*Theories</td>
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<td>- \checkmark ETG mode</td>
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<tr>
<td>(Horton, Dorland, Jenko, ….) ~ 1990</td>
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</table>

### Causal relations

- **ETG mode**
  - Observation (E. Mazzucato) ~ 2008
  - Identification (X. Wei) ~ 2010
  - High \( \beta \) (X. R. Fu) ~ 2012

- **Theories**
  - Elongated toroidal (cascade, Jenko)
  - Nonlinear ion-scale DW (radially elongated, Itoh, Jenko)
  - Streamer (Idomura)
  - Zonal flow (?) feeble (Diamond)

- **Suppression**
  - \( E \times B \) shear (Z. Gao)
  - Large \( E \times B \) shear (D. R. Smith)
  - Magnetic Shear (H. Y. Yuh)
  - Density Gradient (Y. Ren)
Experimental Apparatus for ETG Mode

National Spherical Torus Experiment (NSTX)

- Troidal Device
- Difficult to Control ETG
- Changing Plasma Parameters

Columbia Linear Machine (CLM)

- Linear Device
- Using the Electron Beam
- Forming the Density Gradient

Fluctuations: $\omega/2\pi \sim 1$ MHz

$T_e \approx 2$ keV, $n_e \approx 4 \times 10^{19}$ m$^{-3}$

Fluctuations: $\omega/2\pi \sim 2$ MHz

$T_e = 5$-15 eV, $n_e \approx 2 \times 10^{15}$ m$^{-3}$


Experimental Apparatus

\[ P_\mu = 20 \text{ W} \]
\[ P_{\text{HP}} = 3 \text{ kW} \]
\[ P_{\text{Ar}} = 1 \times 10^{-4} \text{ Torr} \]

10 mesh/inch

30 mesh/inch

\[ B : 0.214 \sim 0.23 \text{ T} \]
\[ \rho_e = \sim 0.04 \text{ mm}, \rho_i = \sim 2.5 \text{ mm} \]
Control of the electron density and temperature profiles from the source to experimental regions by using the mesh grids.

Plasma Parameters

\[ P_\mu = 60 \text{ W}, \quad P_{\text{HP}} = 3 \text{ kW}, \quad P_{\text{Ar}} = 1 \times 10^{-4} \text{ Torr} \]

\[ V_{g1} = 0 \text{ V}, \quad V_{g2} = 0 \text{ V}, \quad V_{ee1} = 0 \text{ V}, \quad V_{ee2} = 0 \text{ V} \]

Electron Temperature

Space Potential

Electron Density

Radial Profiles of a Typical Plasma Parameters
Formation and Control of ETG

\[ P_\mu = 20 \text{ W}, \ V_{ee1} = -4 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ V_{g1} = -10 \text{ V} \]

ETG can be generated easily by controlling the grid bias voltages \( V_{g2} \).
Effect of ETG on Fluctuations

\[ P_\mu = 20 \text{ W}, \quad V_{ee1} = -4 \text{ V} \]
\[ V_{ee2} = -1.5 \text{ V}, \quad V_{g1} = -10 \text{ V} \]
\[ r = -1.5 \text{ cm} \]

A high-frequency fluctuation (~ 0.4 MHz) is excited in situations where large ETG is formed (ETG mode).
The perturbed electron density of the linearized Vlasov equation:
\[ \tau + (k_\perp \lambda_{De})^2 + b \left(1 + \frac{1}{2 \lambda_{Te}}\right) - \frac{1}{2 \lambda_{Te}} + b \frac{\omega_{Te}^*}{\omega} + \frac{\omega_{Te}^*}{4 \lambda_{Te}^2 \omega} (1 + b) = 0, \]
where \( \tau = \frac{T_e}{T_i}, \ k_\perp = k_y, \ \lambda_e = \frac{\omega}{k_v e}, \ b = \frac{(k_\|=\rho_e)^2}{2}, \ \omega_{Te}^* = \frac{k_\perp T_e}{eBLT_e}. \)


The typical experimental plasma parameters
\[ T_{e0} = 3 \text{ eV}, \ T_i = 0.3 \text{ eV}, \ n_e = 1 \times 10^9 \text{ cm}^{-3}, \]
\[ \lambda_{De} = 0.039 \text{ cm}, \ B = 2300 \text{ G}, \ L_{Te} = 1.2 \text{ cm}, \]
\[ k_\|= = 0.06 \text{ cm}^{-1}, \ \rho_e = 0.004 \text{ cm}, \ \rho_i = 0.25 \text{ cm}. \]

The high and low frequency of fluctuations are consistent with the theoretical estimation from the linear dispersion relation of ETG and DW modes.

ETG mode (~ 0.4MHz)

DW mode (~ 7 kHz)
Bispectral Analysis (Nonlinear Coupling)

**Bicoherence**: the degree of nonlinear coupling between the three waves (the value in the range from 0 to 1)

The bicoherence is defined as

\[
b_{xyz}^2(f_1, f_2) = \frac{\left| \langle B_{xyz}(f_1, f_2) \rangle \right|^2}{\left\langle |X(f_1 + f_2)|^2 \right\rangle \left\langle |Y(f_1)Z(f_2)|^2 \right\rangle}
\]

\(<> : \text{Averaged ensembles}\)

- The variance of the bicoherence is \(d b^2 (f_1+f_2) \leq 1/N \approx 0.0014\).

The bispectral analysis can clarify the three-wave nonlinear interactions quantitatively.
Bicoherence of the High & Low Frequency Fluctuations

Squared Bicoherence

\[ \Lambda^2_T = 0.7 \text{ eV/cm} \]

\[ \Lambda^2_T = 1.6 \text{ eV/cm} \]

\[ \Lambda^2_T = 2.4 \text{ eV/cm} \]

The nonlinear coupling between the ETG mode and drift wave (DW) mode become stronger as the magnitude of ETG is increased.

\( P = 20 \text{ W}, \ V_{g1} = -10 \text{ V}, \ V_{ee1} = -4.0 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ r = -1.5 \text{ cm} \)
Bicoherence of the High & Low Frequency Fluctuations

ETG (~0.4 MHz) & DW (~7 kHz) modes

$P_\mu = 20 \text{ W}, V_{g1} = -10 \text{ V}, V_{ee1} = -4.0 \text{ V}, V_{ee2} = -1.5 \text{ V}, r = -1.5 \text{ cm}$

When the magnitude of ETG is increased, the slice of bicoherence between ETG mode and drift wave mode has a noticeable peak.
Nonlinear Energy Transfer (drift wave mode)

\[ P_\mu = 20 \text{ W}, \ V_{g1} = -10 \text{ V}, \ V_{ee1} = -4 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ r = -1.5 \text{ cm} \]

The energy of the ETG mode is transferred to the drift wave mode through the nonlinear interaction for \( \nabla T_e \geq 1.2 \text{ eV/cm} \).

※ High-frequency fluctuations (ETG mode) versus low-frequency fluctuations (Drift wave mode).

Effect of ETG on Fluctuations

$P_\mu = 20 \text{ W}, V_{ee1} = -4 \text{ V}, V_{ee2} = -1.5 \text{ V}, V_{g1} = -10 \text{ V}, r = -0.9 \text{ cm}$

It is observed another low-frequency ($\sim 3.6$ kHz) mode when the large ETG is formed.
Identification of a low-frequency fluctuation

\[ P_{\mu} = 20 \text{ W}, \quad V_{g1} = -10 \text{ V}, \quad V_{g2} = -30 \text{ V}, \quad V_{ee1} = -4.0 \text{ V}, \quad V_{ee2} = -1.5 \text{ V}, \quad r = -1.5 \text{ cm}, \quad \Psi \sim 0^\circ \]

\* Flute mode

\( \mathbf{1} \) \( k || \sim 0 \)

\( \mathbf{2} \) phase shift between \( \tilde{n} \) and \( \tilde{\phi} \) is 180 deg.

Phase shift between \( \tilde{n} \) and \( \tilde{n} \)

Phase shift between \( \tilde{n} \) and \( \tilde{\phi} \)
The slice of bicoherence between the DW mode and the flute mode has a noticeable peak when the magnitude of ETG is increased.
Nonlinear Energy Transfer (flute mode)

\[ P_\mu = 20 \text{ W}, \ V_{g1} = -10 \text{ V}, \ V_{ee1} = -4 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ r = -0.9 \text{ cm} \]

It is considered that the energy of the DW mode is transferred to the flute mode through the nonlinear interaction.

Effect of ETG on the Nonlinear Interplay

References
In order to understand the electron temperature gradient (ETG) mode driven anomalous heat transport, we investigate a multi-scale nonlinear coupling between the electron-scale ETG mode and the ion-scale fluctuations in linear magnetized plasmas.

- The formed ETG is found to excite a high-frequency fluctuation (~0.4 MHz), i.e., ETG mode, furthermore, the drift wave (DW) mode (~7 kHz), which is enhanced by the nonlinear coupling with the ETG mode.
- It is observed another low-frequency (~3.6 kHz) fluctuation associated with the flute mode is enhanced by the nonlinear coupling with the DW mode.
- The ETG mode energy was transferred to the DW mode, and then the energy was ultimately transferred to the flute mode, which was triggered by the disparate scale nonlinear interactions between the ETG and ion-scale low-frequency modes.
Acknowledgements

The authors acknowledge helpful discussions by Prof. S. Inagaki and Prof. S.-I. Itoh. This work was supported by the Grant-in-Aid for JSPS Fellows (23-1638), and a Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology, Japan. This work was partly supported by the Grant-in-Aid for Scientific Research of JSPF, Japan (Nos. 15H02155 and 23244113), by the collaboration programs of the RIAM of Kyushu University and of NIFS (NIFS13KOCT001), and by the Asada Science Foundation.
The Temporal Evolution of Fluctuations

\[ P_\mu = 20 \text{ W}, \ V_{ee1} = -4 \text{ V} \]
\[ V_{ee2} = -1.5 \text{ V}, \ V_{g1} = -10 \text{ V} \]
\[ r = -1.5 \text{ cm}, \ \nabla T_e = 2.4 \text{ eV/cm} \]

The modulation of ETG mode with low-frequency fluctuations is well observed when sufficient ETG is formed.
プラズマの半径方向分布

\[ P_\mu = 20 \text{ W}, \ V_{g1} = -10 \text{ V}, \ V_{g2} = -30 \text{ V}, \ V_{ee1} = -4.0 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ r = -1.5 \text{ cm} \]

PRL2013の図1

POP2015の図2

電子密度勾配が谷のように形成

\[ \left| \frac{n_e}{n_e} \right| = 14.5 \text{ m}^{-1} \]

\[ E_x \times B \text{ 方向はイオン反磁性方向} \]

\[ E_r \approx +0.1 \text{ (V/cm)} \]
位相差測定によるモード数の同定（密度揺動のみ）

\[ P_μ = 20 \text{ W}, \]
\[ V_{g1} = -10 \text{ V}, \]
\[ V_{g2} = -30 \text{ V}, \]
\[ V_{ee1} = -4.0 \text{ V}, \]
\[ V_{ee2} = -1.5 \text{ V}, \]
\[ r = -1.5 \text{ cm} \]

磁場に垂直（θ）方向への伝搬方向とそのモード数

<table>
<thead>
<tr>
<th>mode number</th>
<th>ETG</th>
<th>DW</th>
<th>flute</th>
</tr>
</thead>
<tbody>
<tr>
<td>~40</td>
<td>~8</td>
<td>~2</td>
<td></td>
</tr>
</tbody>
</table>

波長 \( \lambda \) (mm)
| ~2.4 | ~11.5 | ~44.9 |

周波数 \( f \) (kHz)
| 460 | 7 | 3.6 |

伝搬方向
| 電子反磁性 | イオン反磁性（？） | イオン反磁性（？） |

ETG mode (~0.4 MHz)
Drift wave mode (~7 kHz)
Flute mode (~3.6 kHz)
It is found that the fluctuations with $f \approx 1$ kHz is the poloidal wave number $k_\theta = 0$ (m $\sim$ 0).
Cross-Correction with $\tilde{n}$ and $\tilde{n}$
Spatial Structure of ETG mode

![Spatial Structure of ETG mode graph](image-url)
Radial Profiles of ETG driven Fluctuations

\[ P_\mu = 20 \text{ W}, V_{g1} = -10 \text{ V}, V_{g2} = -30 \text{ or } 3 \text{ V}, V_{ee1} = -4 \text{ V}, V_{ee2} = -1.5 \text{ V} \]

ETG mode

Drift wave mode

Flute mode

It is observed another low-frequency (~4 kHz) mode when the electron temperature gradient \( T_e \) exceeds a certain threshold.
Bicoherence of the High & Low Frequency Fluctuations

**Squared Bicoherence**

\[ P_\mu = 20 \text{ W}, \ V_{g1} = -10 \text{ V}, \ V_{ee1} = -4.0 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ r = -0.9 \text{ cm} \]

The nonlinear couplings between the ETG mode and the ion-scale Fluctuations become stronger as the magnitude of ETG is increased.
The temporal evolution of fluctuations

$P_\mu = 20 \text{ W}, \ V_{ee1} = -4 \text{ V}, \ V_{ee2} = -1.5 \text{ V}, \ V_{g1} = -10 \text{ V}, \ r = -1.5 \text{ cm}$

$\nabla T_e = 0.7 \text{ eV/cm}$

$\nabla T_e = 2.4 \text{ eV/cm}$

The modulation of ETG mode with low-frequency fluctuations is well observed when sufficient ETG is formed.
付録1: ETGモードの機構

圧力勾配による力 $F$

$$F = -\nabla p_{e,i}/n_0 = -kT_{e,i}\nabla n_0/n_0$$

$$p = nkT$$

ドリフト不安定性場合

$$\nabla p = T k \nabla n$$

① わずがの波状の外乱.
② 正負電荷のドリフト運動が発生.
③ 荷電分離が生じて電界$E$が誘起.
④ $E \times B$ドリフトによって$x$の正負両方向に波状の変位が助長される.

ETG不安定性場合

$$\nabla p = nk\nabla T$$
プラズマの圧力によって各粒子が受ける力は

\[ F = -\frac{1}{n} \nabla p \]

これは（仮想的な）反磁性ドリフトである。これはイオンと電子に対してドリフトの方向が異なるため電流を生ずる。

\[ V_{de} = -\frac{B \times \nabla p}{enB^2} = \frac{k_bT_e}{eB} \frac{n'}{n} = \frac{k_bT_e}{eB} \frac{T_e'}{T_e} \]

\[ V_{di} = -\frac{B \times \nabla p}{enB^2} = \frac{k_bT_e}{eB} \frac{n'}{n} = \frac{k_bT_e}{eB} \frac{T_e'}{T_e} \]

これから反磁性電流は

\[ j_D = ne(v_{Di} - v_{De}) = k_B(T_i + T_e) \frac{B \times \nabla n}{B^2} \]
We investigate the effects of the radial electric field ($E_r$) on suppression of ETG mode through multiscale nonlinear interactions in linear magnetized plasmas.

- The formed ETG is found to excite a high-frequency fluctuation (~0.4 MHz), i.e., ETG mode, furthermore, the drift wave (DW) mode (~7 kHz), which is enhanced by the nonlinear coupling with the ETG mode.

- It is found that a sufficiently large $E_r$ ($E \times B$ velocity shear) can suppress the ETG mode regardless of its signs.

- The ETG mode amplitude is decreased by the energy transfer of ETG mode to DW mode through the multi-scale non-linear coupling in the slightly negative $E_r$. 
We investigate the effects of the radial electric field \( E_r \) on suppression of ETG mode through multiscale nonlinear interactions in linear magnetized plasmas.

- The formed ETG is found to excite a high-frequency fluctuation (~0.4 MHz), i.e., ETG mode, furthermore, the drift wave (DW) mode (~7 kHz), which is enhanced by the nonlinear coupling with the ETG mode.

- It is found that a sufficiently large \( E_r \) \((E \times B\) velocity shear) can suppress the ETG mode regardless of its signs.

- The ETG mode amplitude is decreased by the energy transfer of ETG mode to DW mode through the multi-scale non-linear coupling in the slightly negative \( E_r \).
Spatial Structure of Low-frequency Fluctuations

It is found that the fluctuations with $f \approx 7$ kHz is parallel wave number $k_\parallel = 0$ that the energy of the DW mode is transferred to the flute mode through the nonlinear interaction.

$p_\mu = 20 \text{ W}, V_{g1} = -10 \text{ V}, V_{ee1} = -4.0 \text{ V}, V_{ee2} = -1.5 \text{ V}, V_{g2} = -30 \text{ V}$ ($\nabla T_e = 2.4 \text{ eV/cm}$).

$P_2 = ~7 \text{ kHz}$
It is considered that the energy of the DW mode is transferred to the flute mode through the nonlinear interaction.
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Effect of $E \times B$ Velocity Shear on ETG mode

\[ E_r = -\frac{\partial \phi(r)}{\partial r} \]

Control of Radial Electric Field ($E_r$)

$P_\mu = 20 \text{ W}$, $V_{g1} = -10 \text{ V}$, $V_{g2} = -30 \text{ V}$, $V_{ee2} = -1.5 \text{ V}$

$V_{ee1} = -3 \text{ V}$

$V_{ee1} = -4 \text{ V}$

$V_{ee1} = -5 \text{ V}$

$E_r \approx 1.1 \text{ (V/cm)}$

$E_r \approx -0.1 \text{ (V/cm)}$

$E_r \approx -1.0 \text{ (V/cm)}$
The effect of the $E_r$ on ETG modes in the large ETG case is certainly different from the small ETG case.
Effect of the $E_r$ on Fluctuations

**Theories**

ETG mode


**Experi.**

DW mode


\[ V_{E'} : E \times B \text{ Shear Strength} \]

\[ \gamma_i \equiv -eE_r/m_i L_\phi \omega_{ce}^2 \]

- Small ETG
  - $\tilde{I}_{es}/\tilde{I}_{es} (\%)$
  - $f \sim 0.4 \text{ MHz}$
  - $f \sim 7 \text{ kHz}$

- Large ETG
  - $\tilde{I}_{es}/\tilde{I}_{es} (\%)$
  - $f \sim 0.4 \text{ MHz}$
  - $f \sim 7 \text{ kHz}$

$\nabla T_e \approx 1.0 \text{ eV/cm}$

$\nabla T_e \approx 2.4 \text{ eV/cm}$

The suppression tendency of ETG mode has a significant difference between the slightly negative $E_r$ and positive $E_r$ in the large ETG case.
Dependence of bicoherence on the $E_r$

$E_r \approx 1.5$ (V/cm)

Positive $E_r$

$E_r \approx -1.0$ (V/cm)

Negative $E_r$

$P_\mu = 20$ W, $V_{g1} = -10$ V, $V_{g2} = -30$ V, $V_{ee2} = -1.5$ V, $r = -0.9$ cm

The nonlinear coupling between the ETG mode and drift wave mode become stronger when $E_r$ becomes the slightly negative value.
Effect of $E_r$ on the Nonlinear Interplay

$P_\mu = 20 \text{ W, } V_{g1} = -10 \text{ V, } V_{g2} = -30 \text{ V, } V_{ee2} = -1.5 \text{ V, } r = -0.9 \text{ cm}$

- (A) Suppression of ETG mode
  $\Rightarrow$ Owing to the strong $E_r$
  Effects of $E \times B$ shear

- (B) Suppression of ETG mode
  $\Rightarrow$ Increasing the Nonlinear Interplay
  $\Rightarrow$ Increasing the DW mode

- (C) Sustainment of ETG mode
  $\Rightarrow$ Decreasing the Nonlinear Interplay
  $\Rightarrow$ No energy transfer to the DW mode

The $E_r$ affects the nonlinear interaction of the ETG and DW modes, which cause the resultant suppression of the ETG mode in the slightly negative $E_r$. 

$\Sigma \hat{b}^2 (f_1, f_2)$

$E_r (\text{V/cm})$

$\hat{I}_{es} / \bar{I}_{es}$ (A)

$f \sim 7 \text{ kHz}$

$\nabla T_e \sim 2.4 \text{ (eV/cm)}$

$f \sim 0.4 \text{ MHz}$

$\hat{I}_{es} / \bar{I}_{es}$ (B)

$\hat{I}_{es} / \bar{I}_{es}$ (C)

$\nabla T_e \sim 2.4 \text{ (eV/cm)}$

$\hat{I}_{es} / \bar{I}_{es}$ (A)

$\Sigma \hat{b}^2 (f_1, f_2)$
Effect of ETG on the Nonlinear Interplay