Plasma turbulence in the tokamak SOL

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The model to study SOL turbulence

The GBS code and its path towards SOL simulations

Anatomy of SOL turbulence: from linear instabilities to SOL width and intrinsic toroidal rotation

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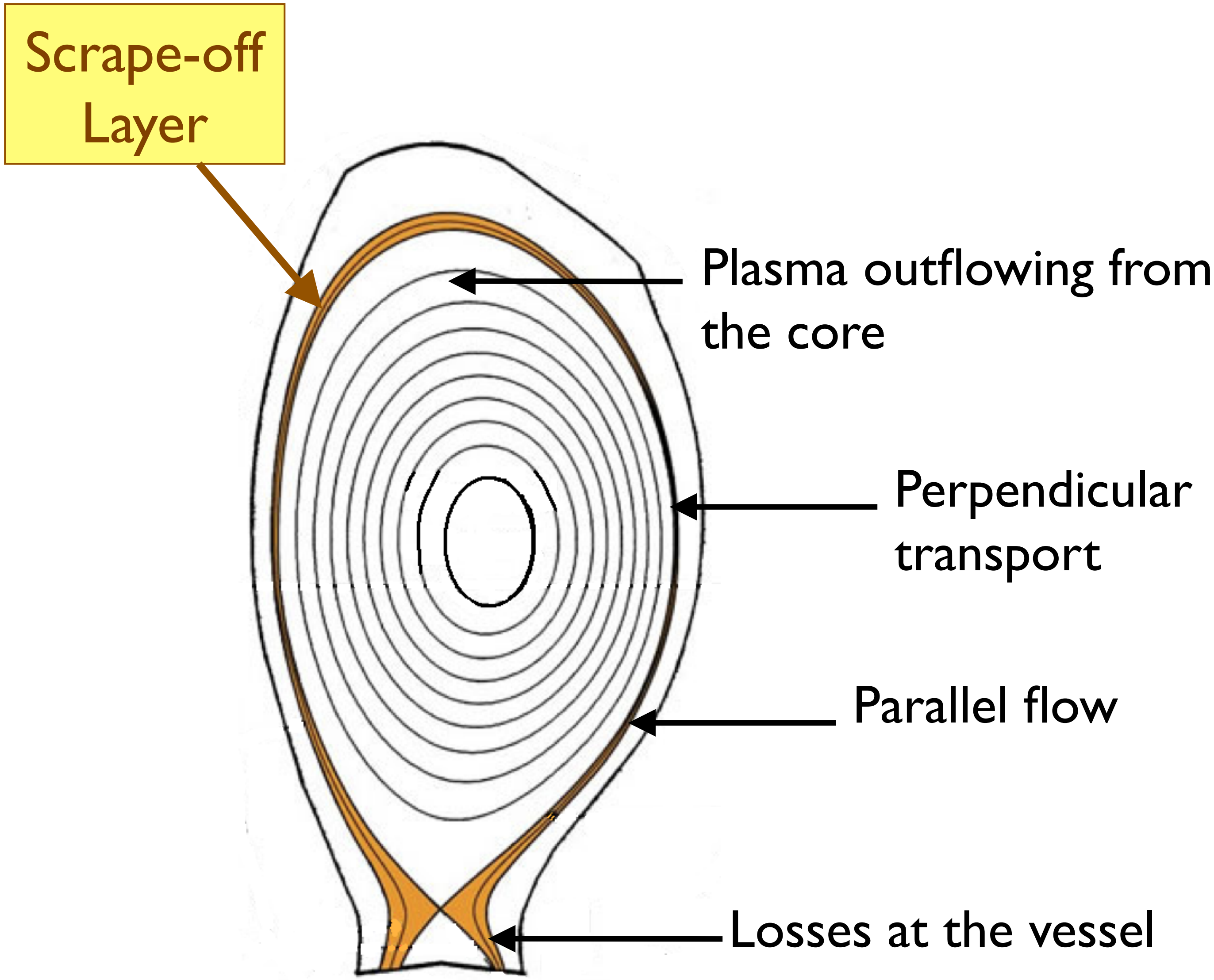
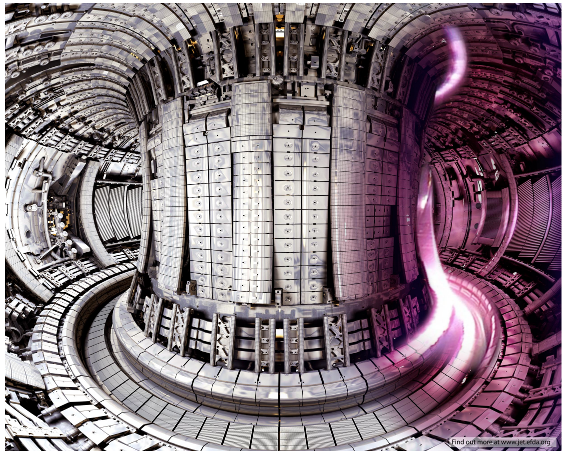
CRPP

SOL channels particles and heat to the wall

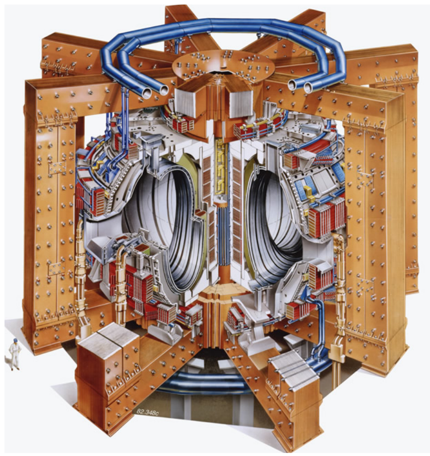


The SOL – a crucial issue for the entire fusion program



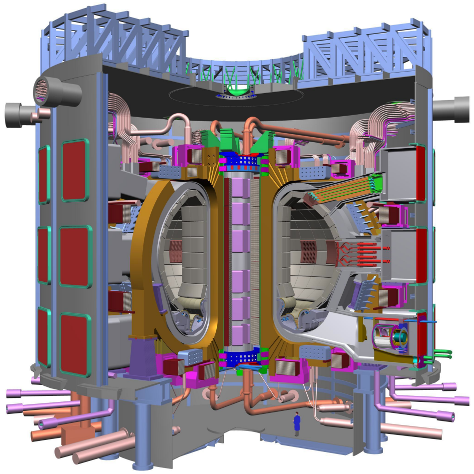
$$P_{\mathrm{wall}} \sim \frac{P_{\mathrm{sep}}}{A_{\mathrm{wet}}} \sim \frac{P_{\mathrm{sep}}}{R L_p} \leq 5 \text{ MW m}^{-2}$$

JET



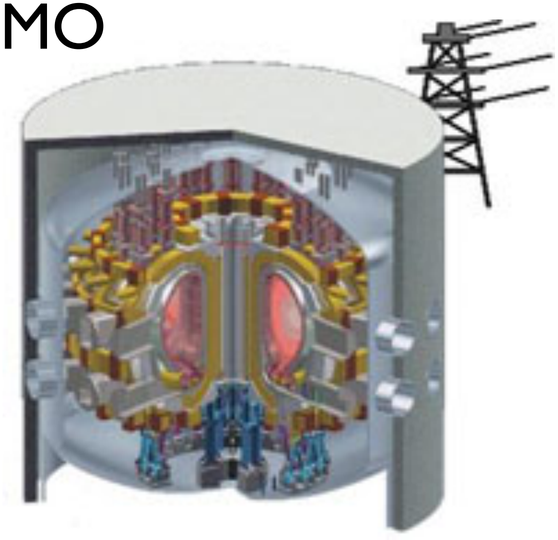
$$\frac{P_{\mathrm{sep}}}{R} = 7$$

ITER



$$\frac{P_{\mathrm{sep}}}{R} = 20$$

DEMO



$$\frac{P_{\mathrm{sep}}}{R} = 80 - 100$$

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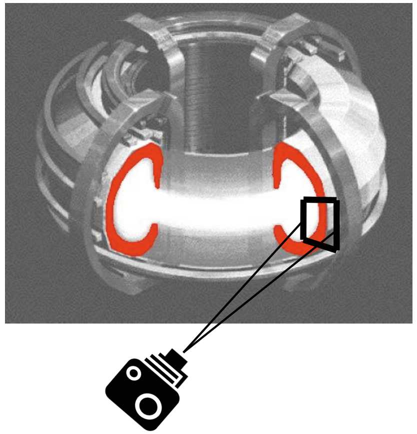
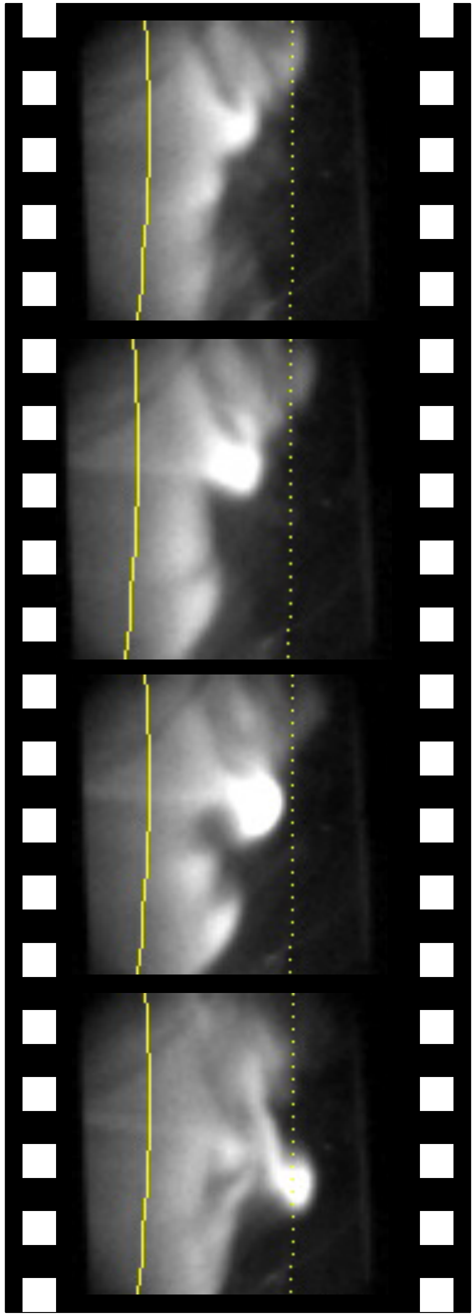
Properties of SOL turbulence





Courtesy of R. Maqueda

- $n_{fluc} \sim n_{eq}$
- $L_{fluc} \sim L_{eq}$
- Fairly cold magnetized plasma

The GBS code, a tool to simulate SOL turbulence

Collisional Plasma → **Braginskii model** → $\rho \ll L, \omega \ll \Omega_{ci}$, $n_e \simeq n_i$ → **Drift-reduced Braginskii equations**

Convection | Magnetic curvature | Parallel dynamics | Source

$$\boxed{\frac{\partial n}{\partial t} + [\phi, n]} = \boxed{\hat{C}(nT_e) - n\hat{C}(\phi)} - \boxed{\nabla_{\parallel}(nV_{\parallel e})} + \boxed{S}$$

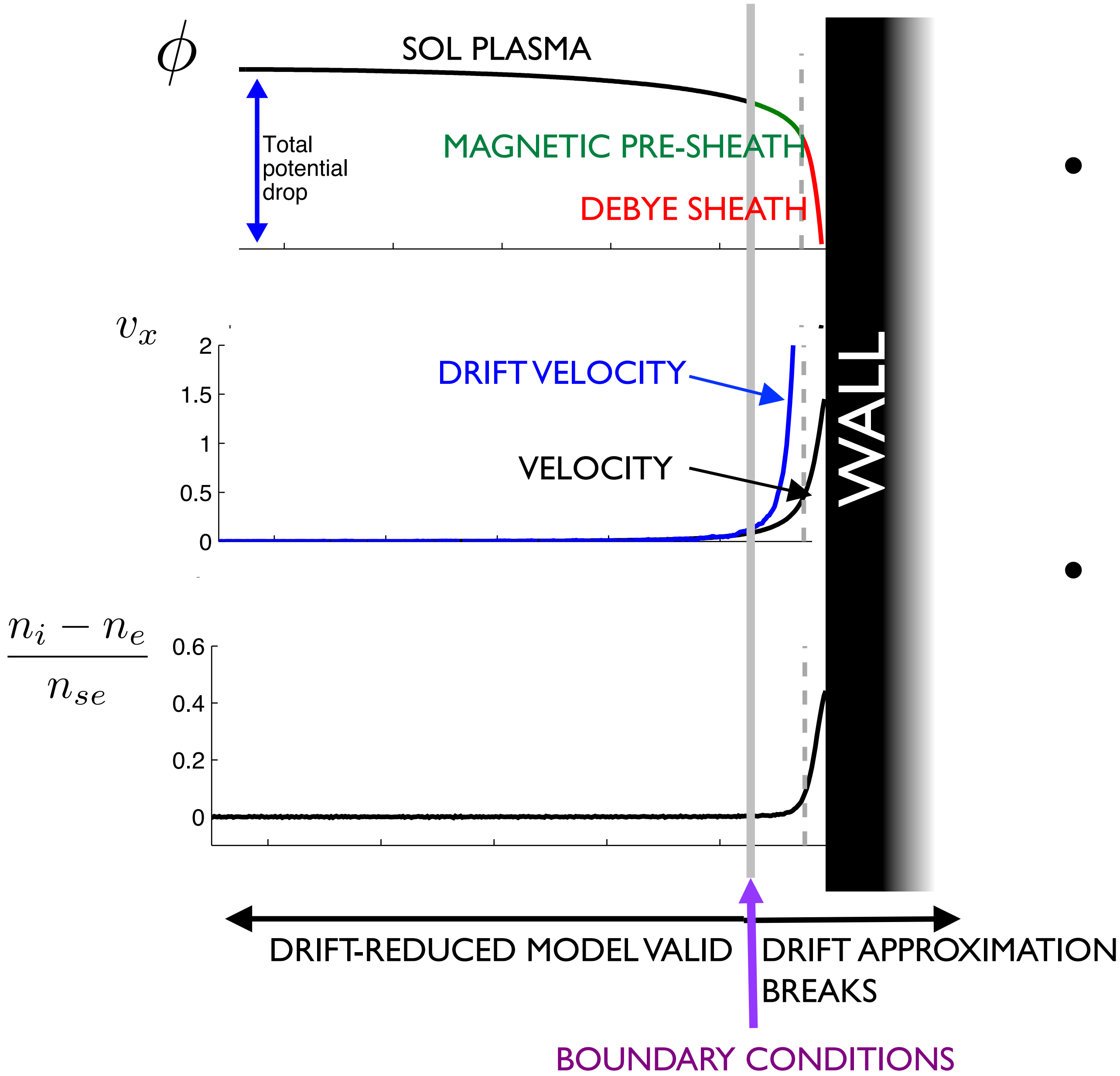
T_e, Ω (vorticity) ➡ similar equations $(T_i \ll T_e)$

$v_{\parallel i}, v_{\parallel e}$ ➡ parallel momentum balance

$$\nabla_{\perp}^2 \phi = \Omega$$

Solved in 3D geometry, taking into account plasma outflow from the core, turbulent transport, and losses at the vessel

Boundary conditions at the plasma-wall interface



- Set of b.c. for all quantities, generalizing Bohm-Chodura
- Checked agreement with PIC kinetic simulations

Code verification, method of manufactured solution

Our model: $A(f) = 0, \quad f$ unknown

We solve $A_n(f_n) = 0,$ but $f_n - f = ?$

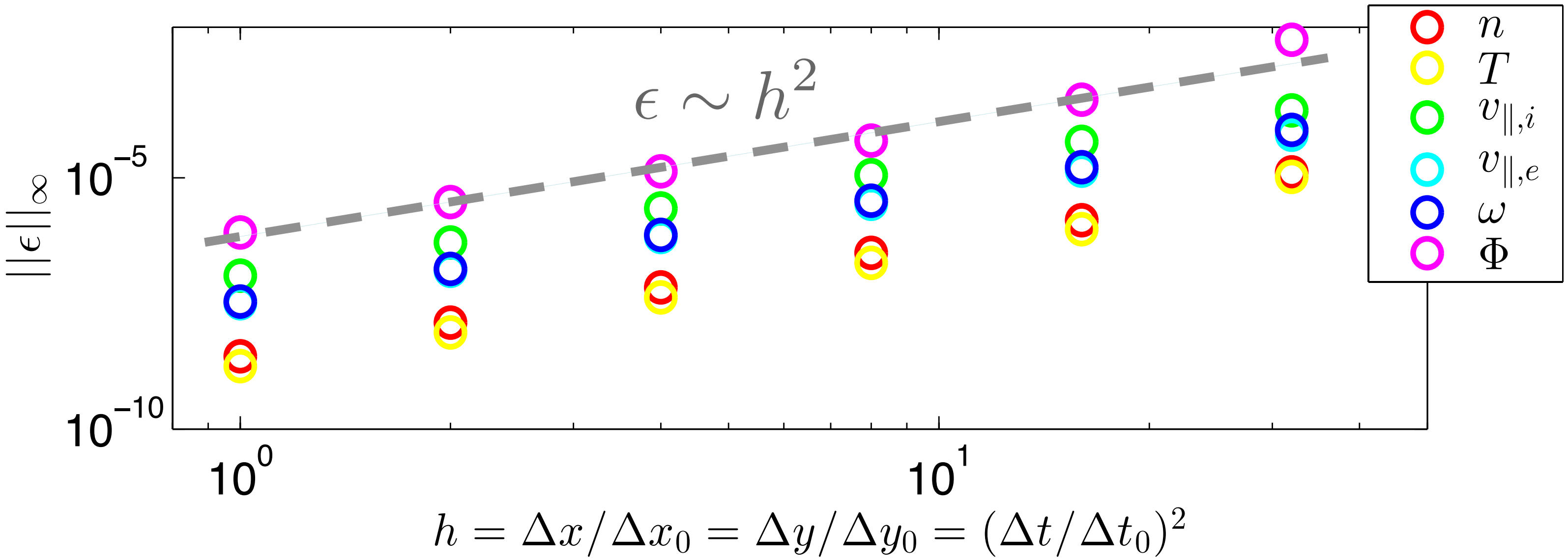
Method of manufactured solution:

1) we choose g, then $S = A(g)$

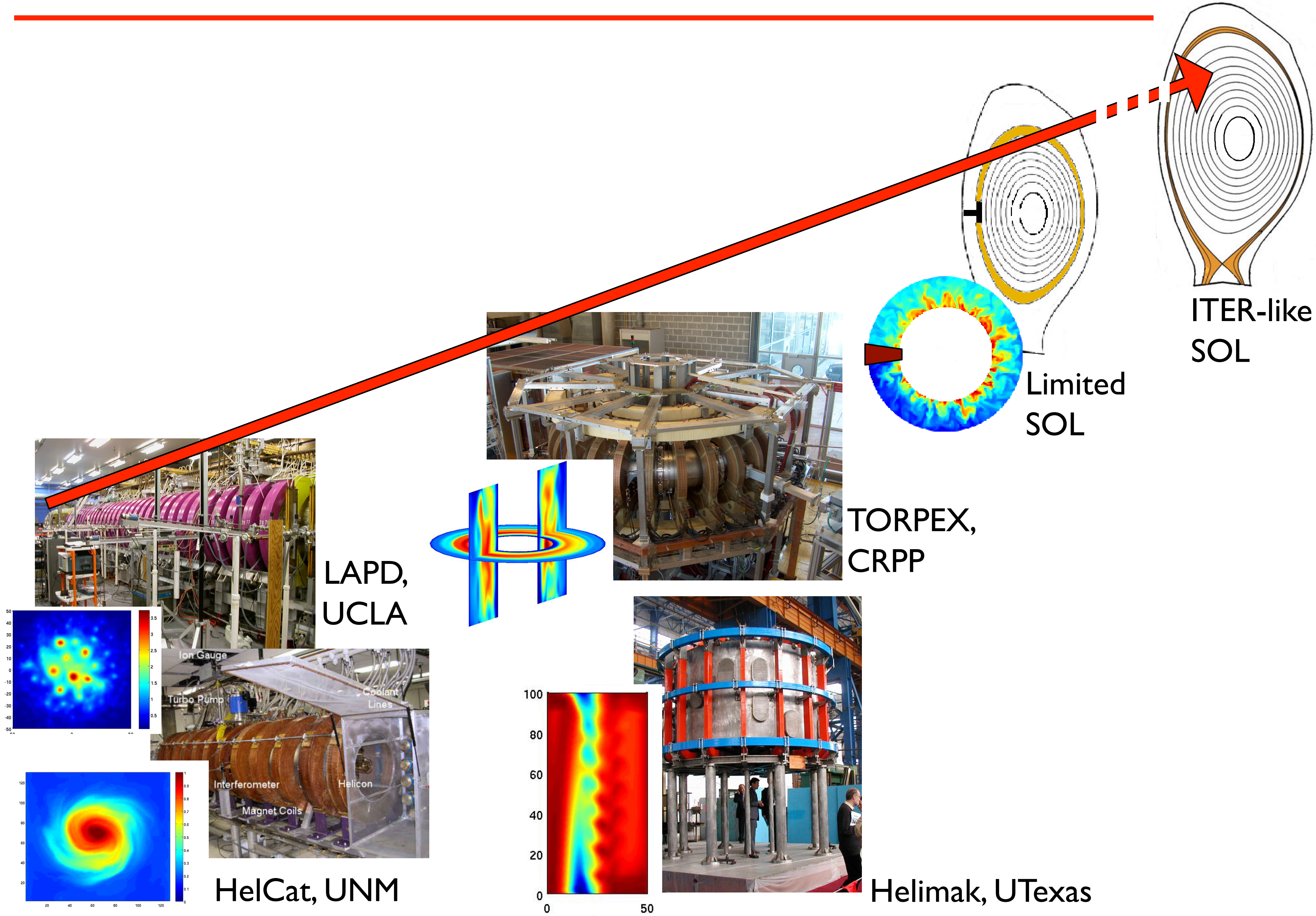
2) we solve: $A_n(g_n) - S = 0$

$\boxed{\epsilon = g_n - g}$

For GBS:

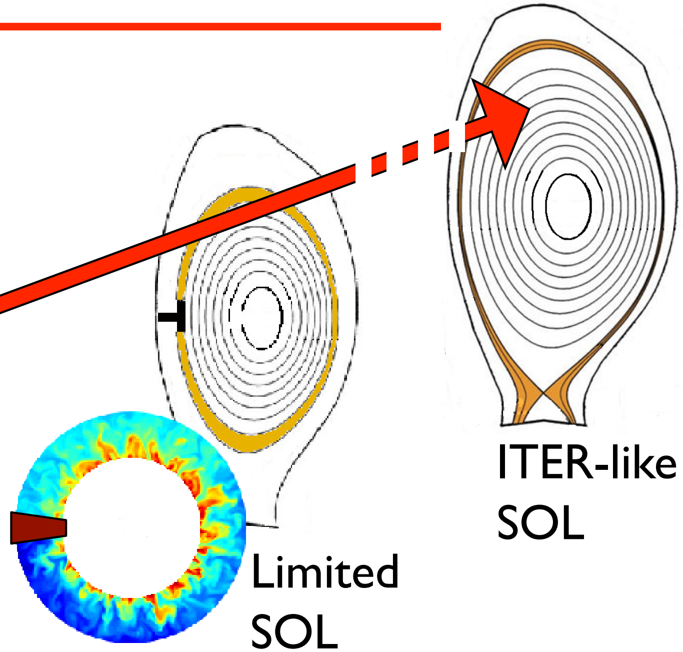


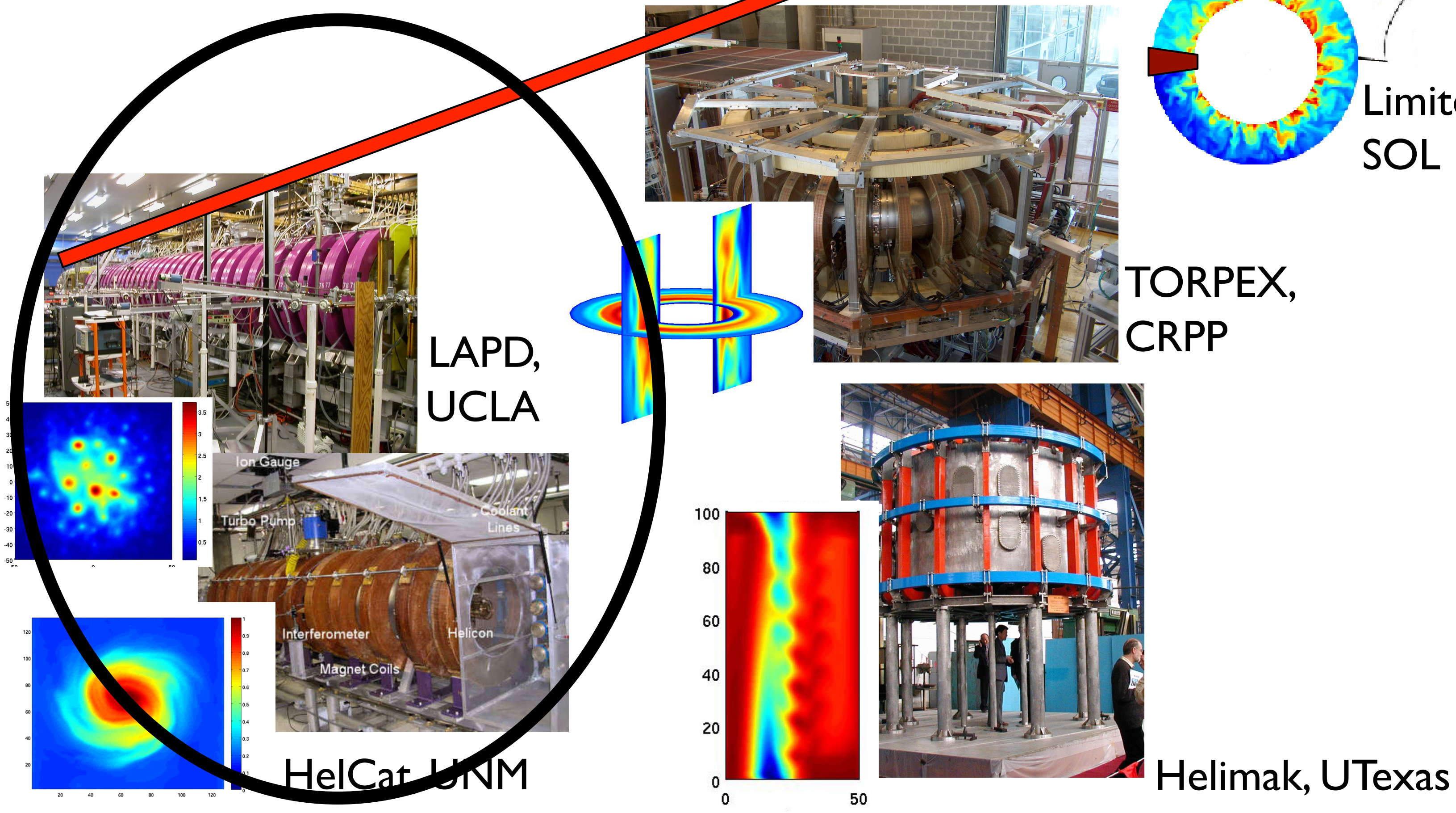
GBS analysis of configurations of increasing complexity



GBS analysis of configurations of increasing complexity

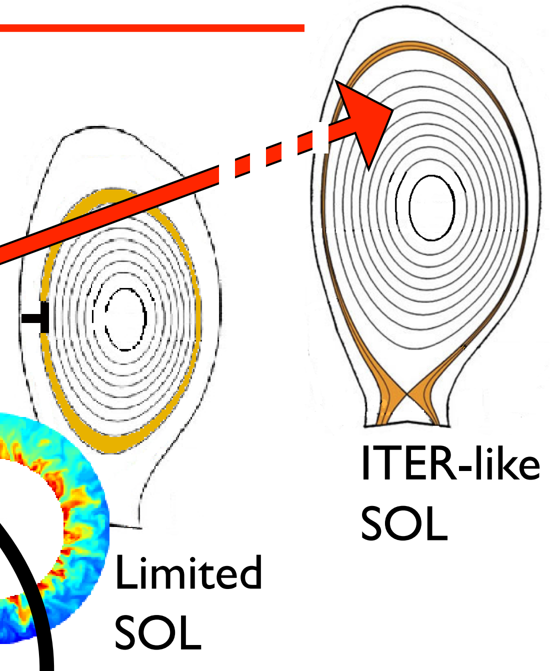
From linear devices…
(role of non-curvature
driven modes, DW vs KH)

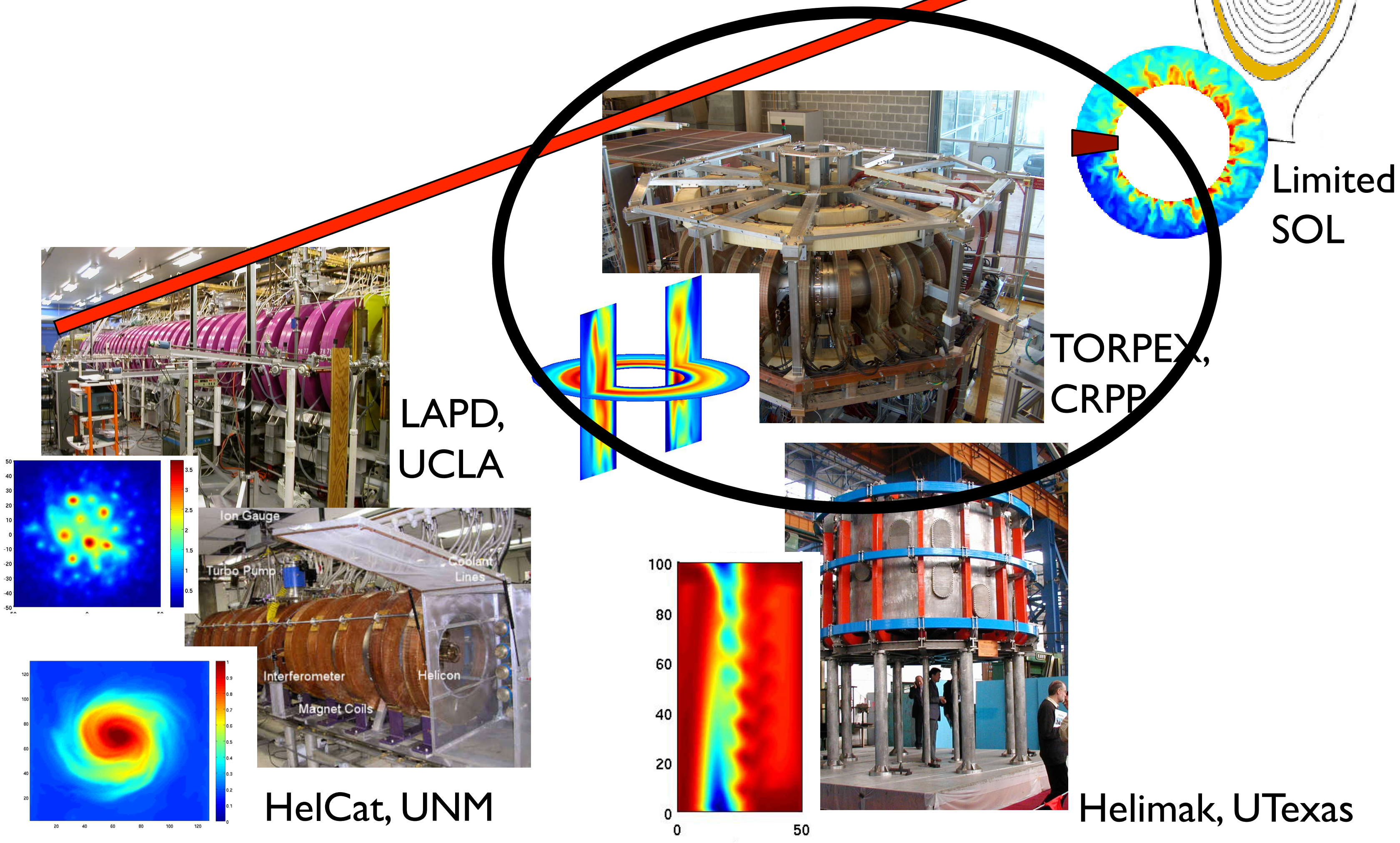




GBS analysis of configurations of increasing complexity

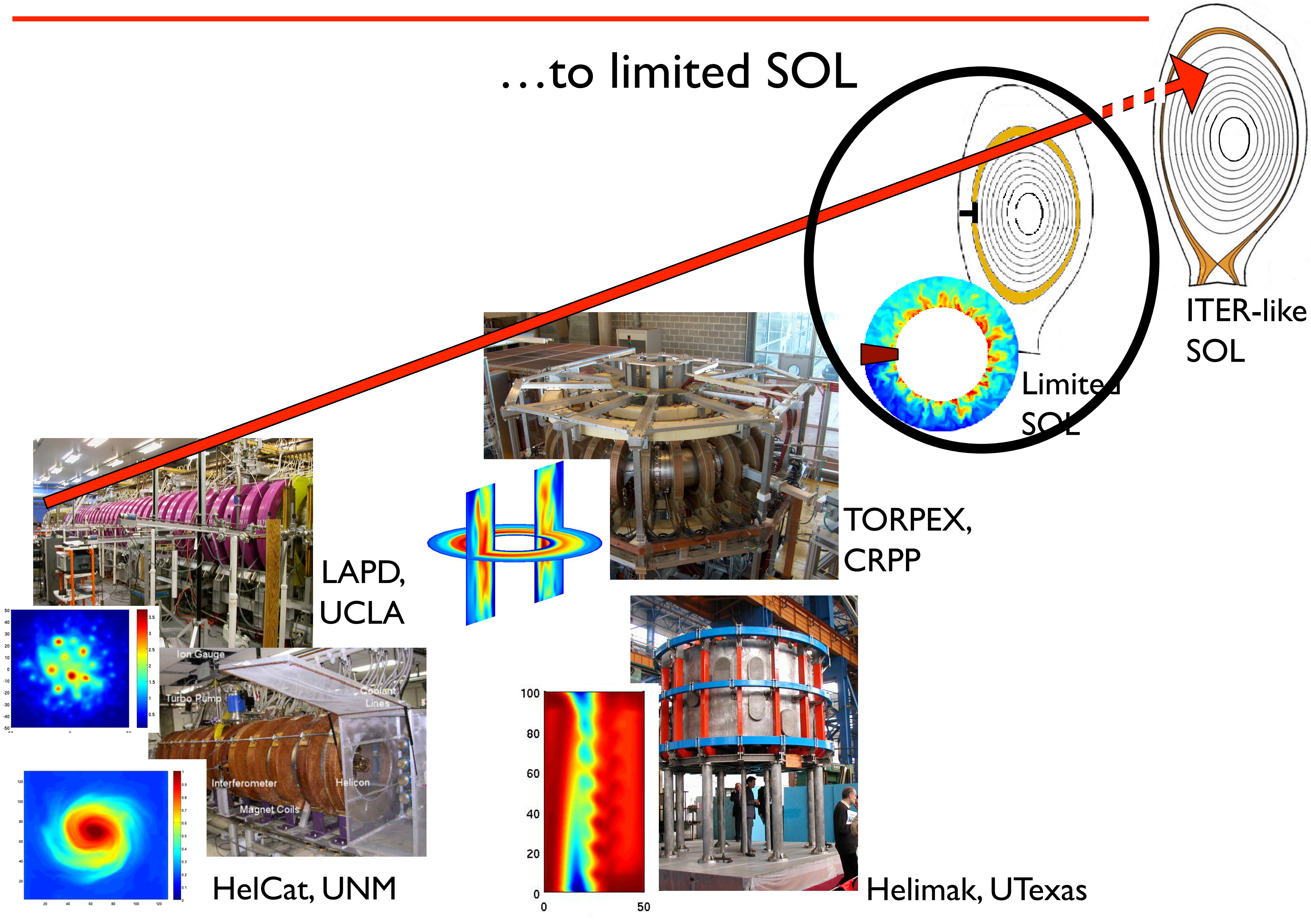
… to the Simple Magnetized Torus…
(role of curvature-driven modes and rigorous code validation)



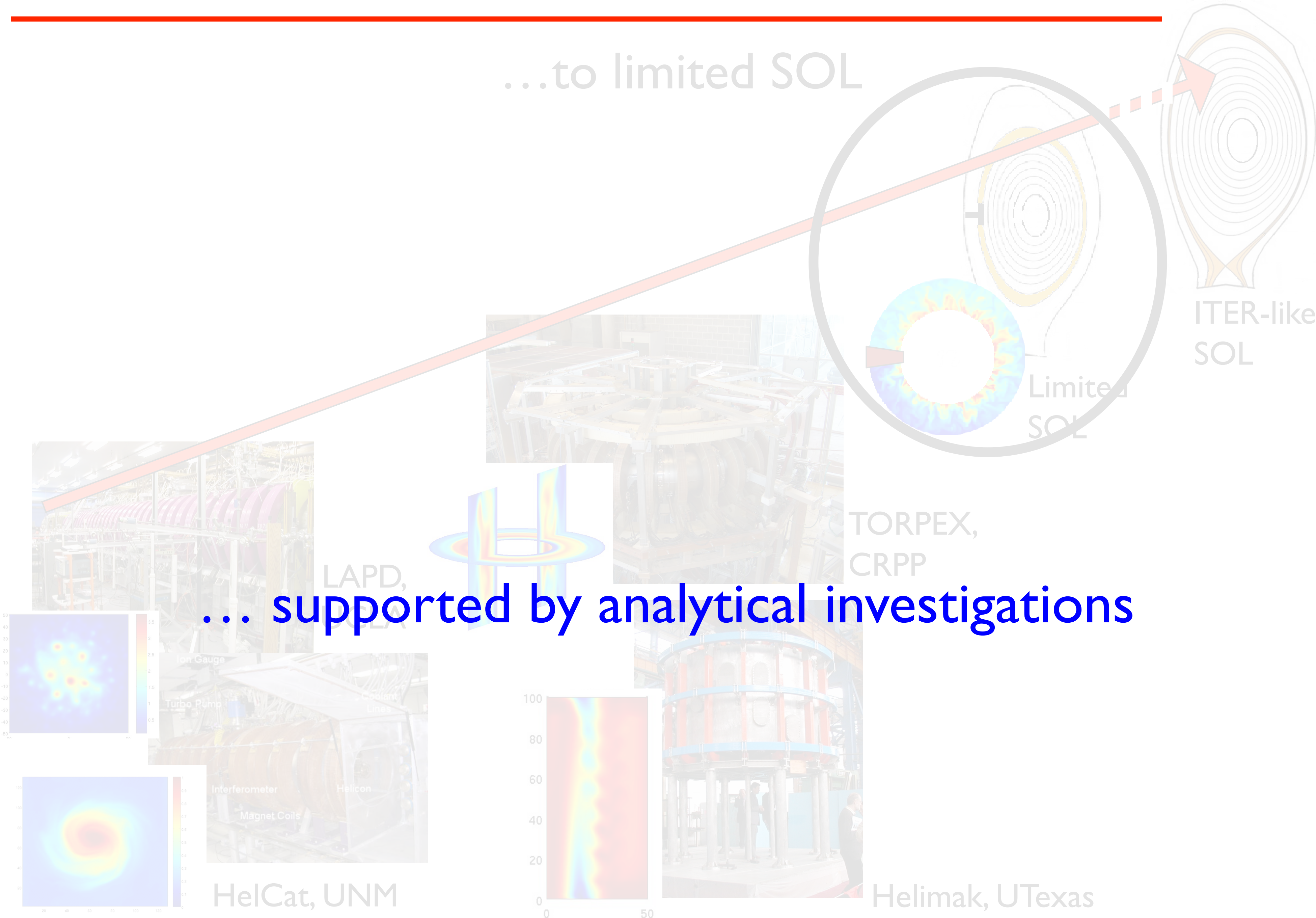


GBS analysis of configurations of increasing complexity

…to limited SOL



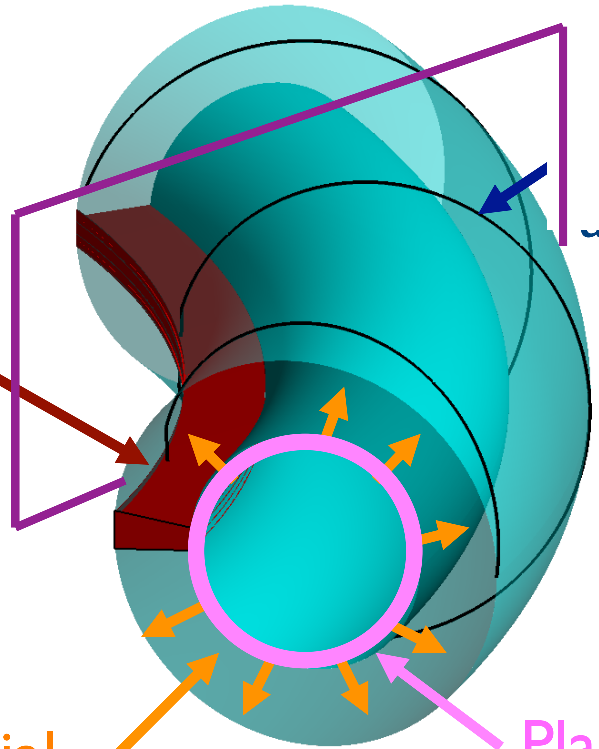
GBS analysis of configurations of increasing complexity



… supported by analytical investigations

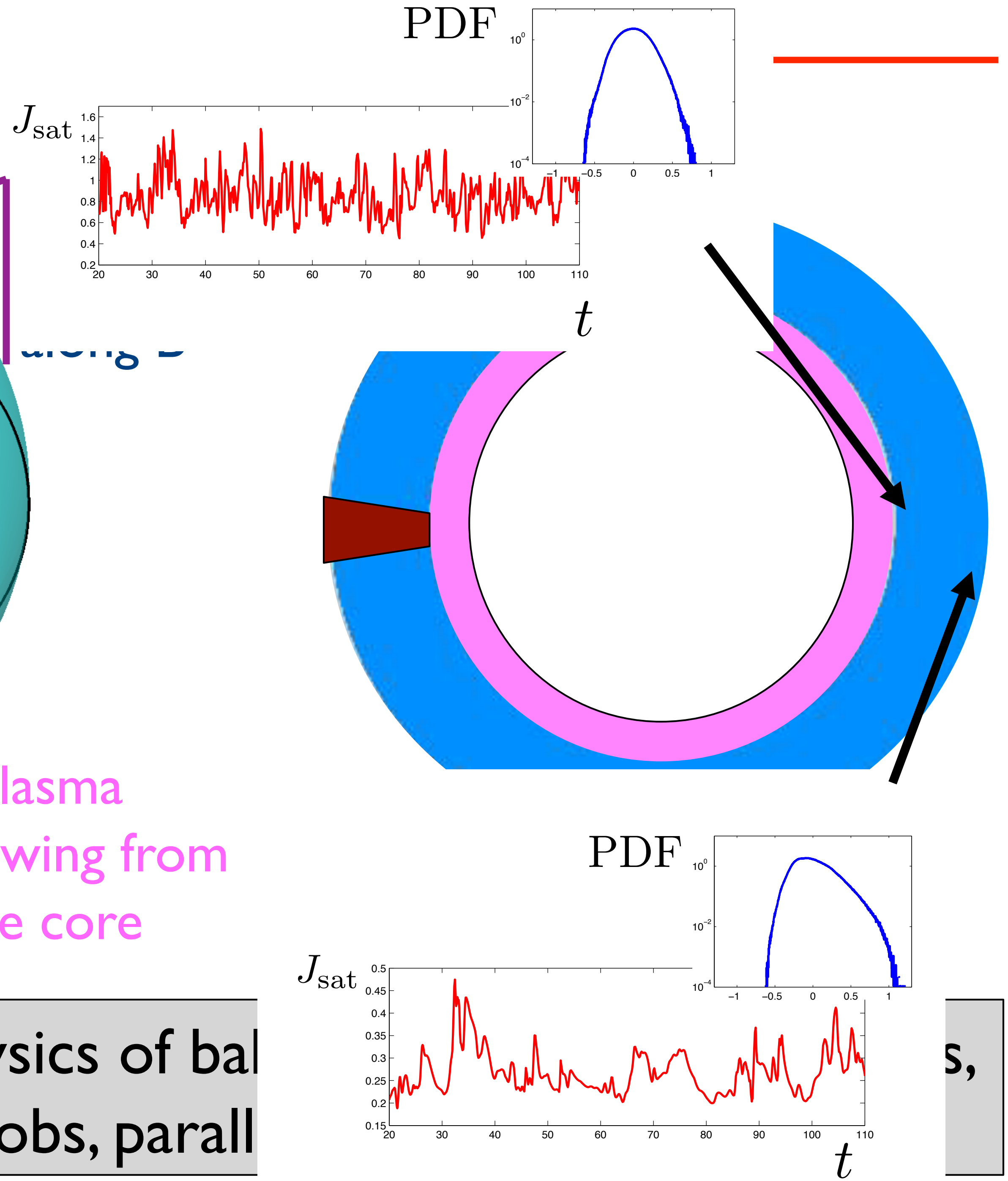
Tokamak

Losses at the limiter



Radial transport

Plasma outflowing from the core



Simulations contain physics of bal… s,
Kelvin-Helmholtz, blobs, parall…

The key questions

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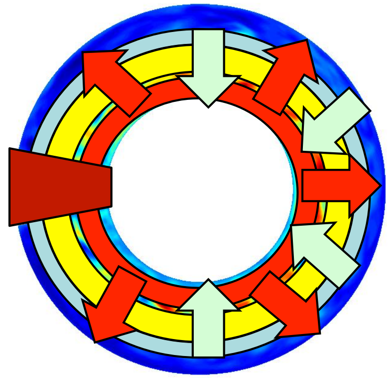
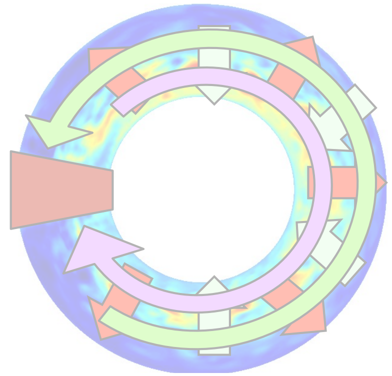
Three possible saturation mechanisms

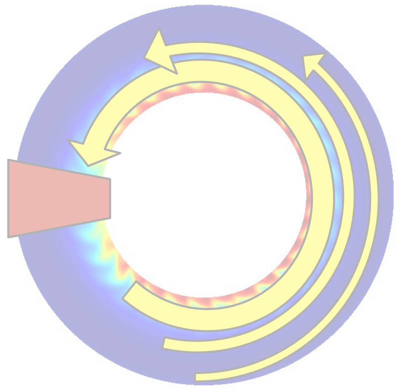
Removal of the turbulence drive (gradient removal):



Kelvin – Helmholtz secondary instability:

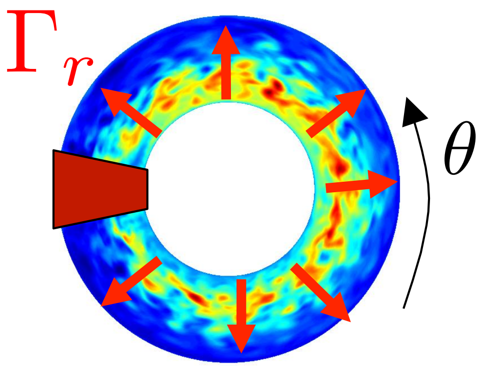


Suppression due to strong shear flow:



Turbulent transport with gradient removal saturation

Turbulence saturates when it removes its drive $\rightarrow$ $\dfrac{\partial \tilde{p}}{\partial r} \sim \dfrac{\partial \overline{p}}{\partial r}$ $\rightarrow$ $k_r \tilde{p} \sim \overline{p}/L_p$



$$\frac{\partial p}{\partial t} \simeq [p, \phi]$$

$$\Gamma_r = \left\langle \tilde{p} \frac{\partial \tilde{\phi}}{\partial \theta} \right\rangle_t$$

GR hypothesis

Nonlocal linear theory

$$\boxed{D_{GR} = \frac{\Gamma_r}{\overline{p}/L_p} \sim \frac{\gamma L_p}{k_\theta}}$$

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SOL width – operational parameter estimate

Balance of perpendicular transport and parallel losses

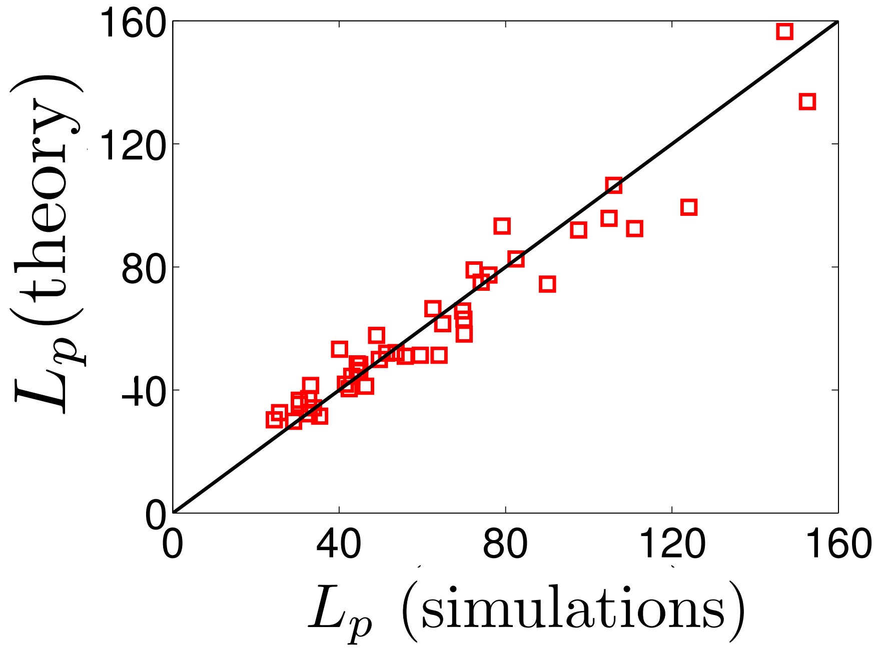
$$\frac{d\Gamma_r}{dr} \sim L_\parallel \sim \frac{n_0 c_s}{qR}$$

Bohm's

$$\boxed{L_p \simeq q\left(\frac{\gamma}{k_\theta}\right)_{\max}}$$

$$\boxed{L_p = L_p(R, q, \hat{s}, \nu)}$$

Simulations show expected scaling



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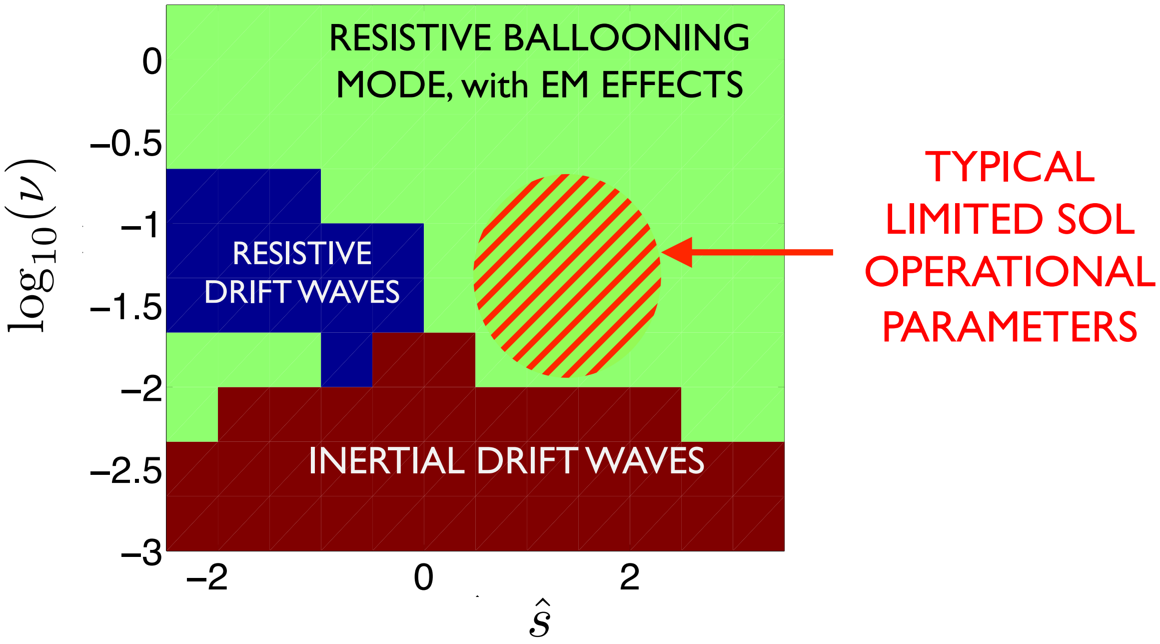
SOL turbulent regimes

Instabilities driving turbulence depends mainly on q, $\nu, \hat{s}$.



LIMITED SOL: $$L_p \simeq q \left(\frac{\gamma}{k_\theta}\right)_{\max}$$

RBM $\gamma \sim \gamma_b = \sqrt{2R/L_p}$

RBM $k_\theta \sim k_b = \sqrt{\dfrac{1-\alpha_{\mathrm{MHD}}}{\nu q^2 \gamma_b}}$

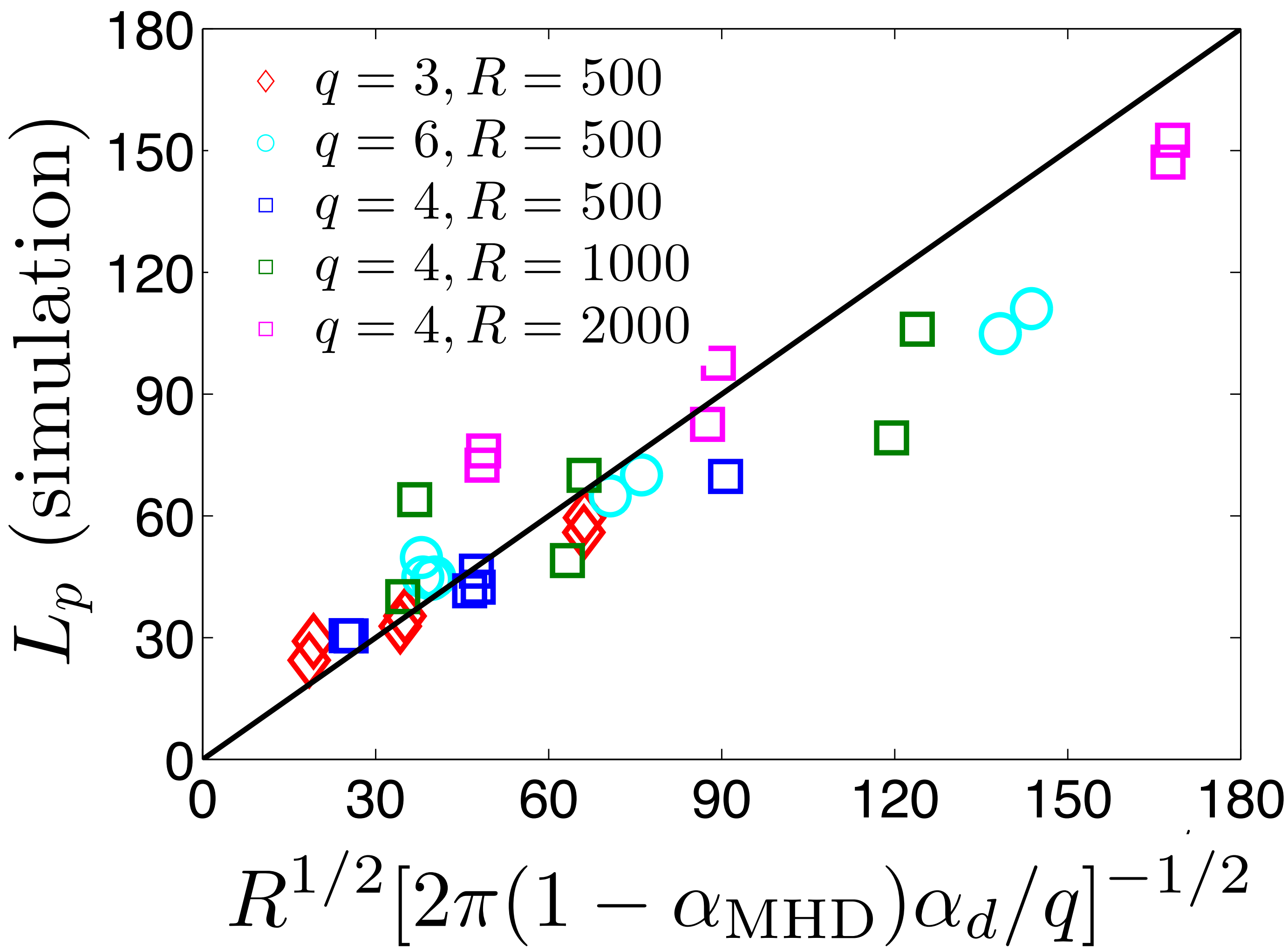
$$\boxed{L_p = R^{1/2}[2\pi(1-\alpha_{\mathrm{MHD}})\alpha_d/q]^{-1/2}}$$

MAJOR RADIUS

$\alpha_{MHD} \sim q^2 \beta R/L_p$

$\alpha_d \sim (R/L_p)^{1/4} \nu^{-1/2}/q$

Simulations agree with ballooning estimates



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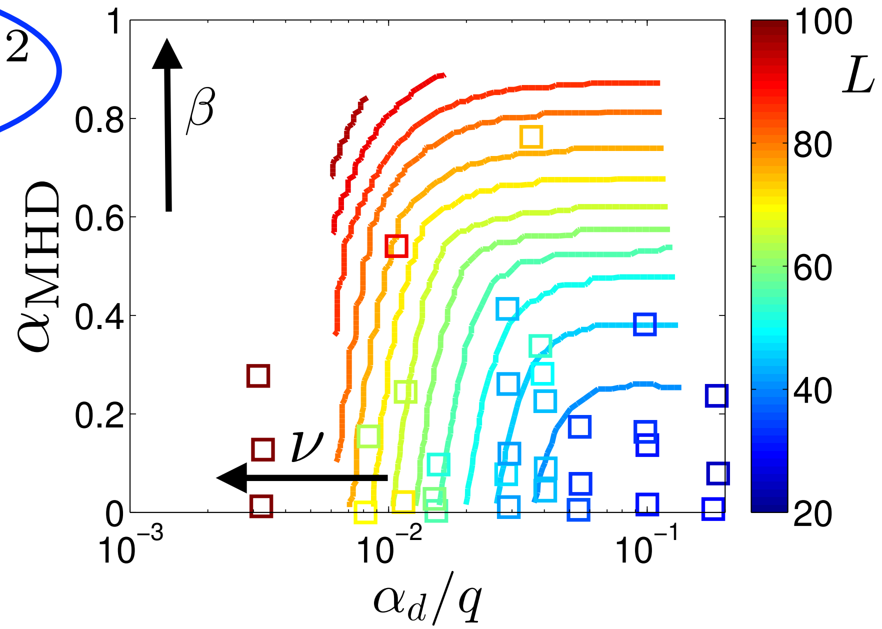
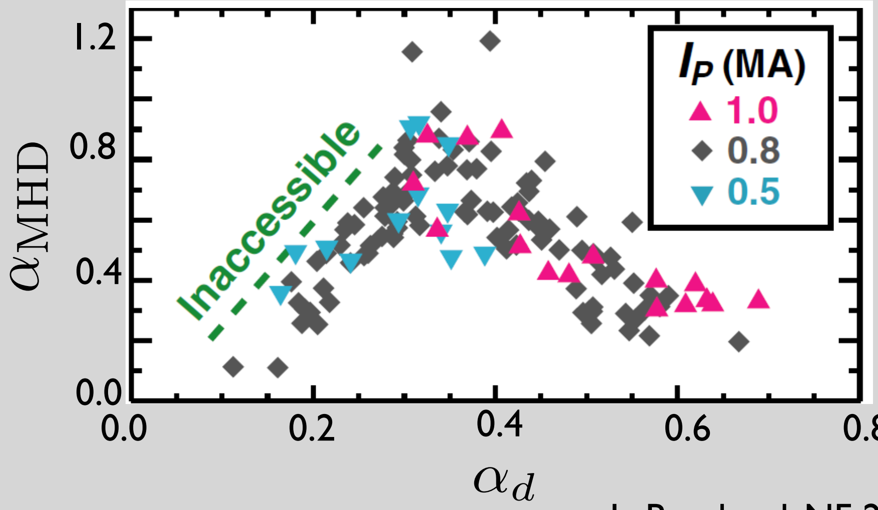
Limited SOL transport increases with β and ν

$$L_p = R^{1/2}[2\pi(1-\alpha_{\mathrm{MHD}})\alpha_d/q]^{-1/2}$$



Maybe related to the density limit?

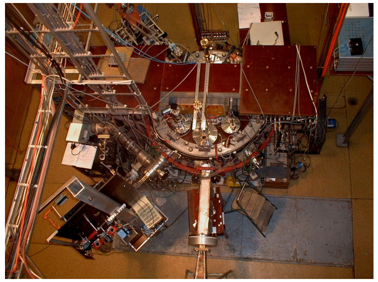
Coupling with core physics needs be addressed…



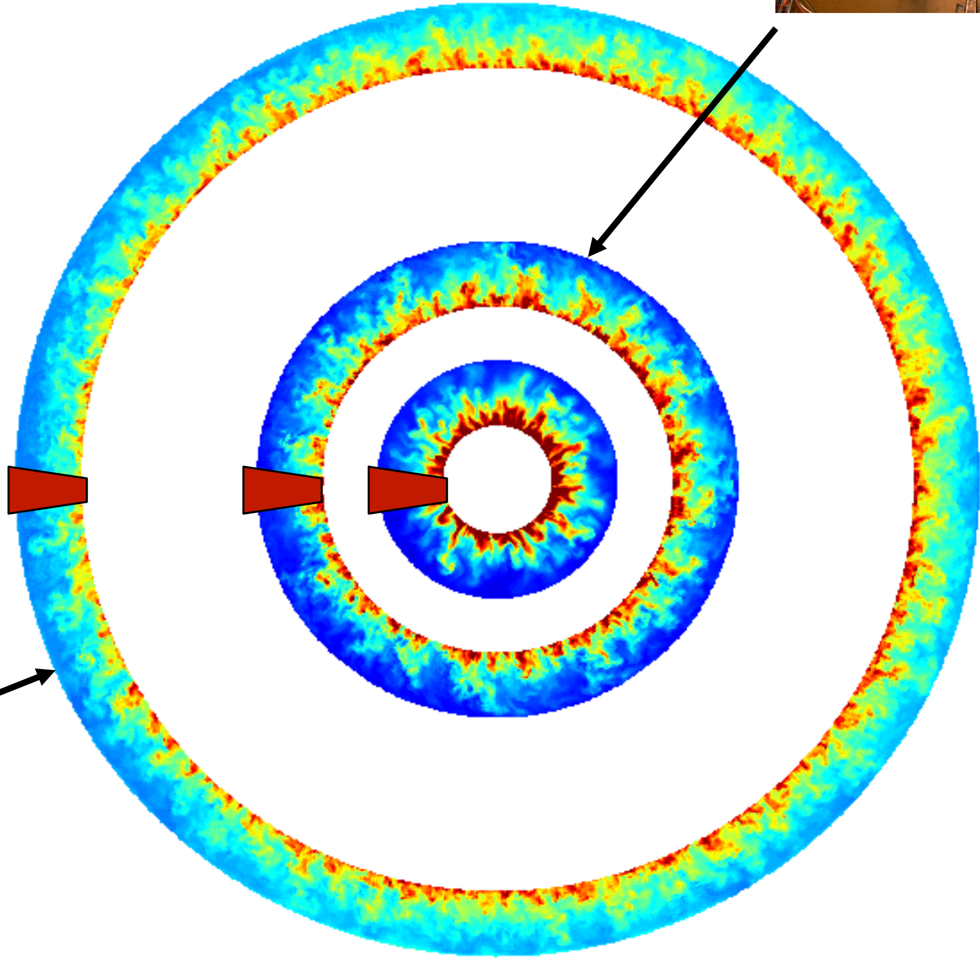
LaBombard, NF 2005

Limited SOL width widens with R

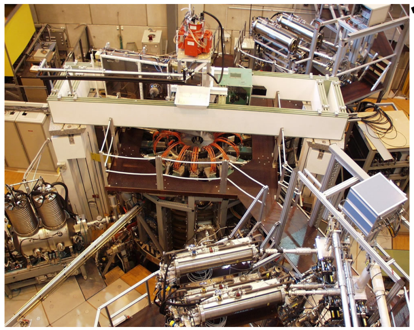
$$L_p = R^{1/2}[2\pi(1-\alpha_{\mathrm{MHD}})\alpha_d/q]^{-1/2}$$



CASTOR



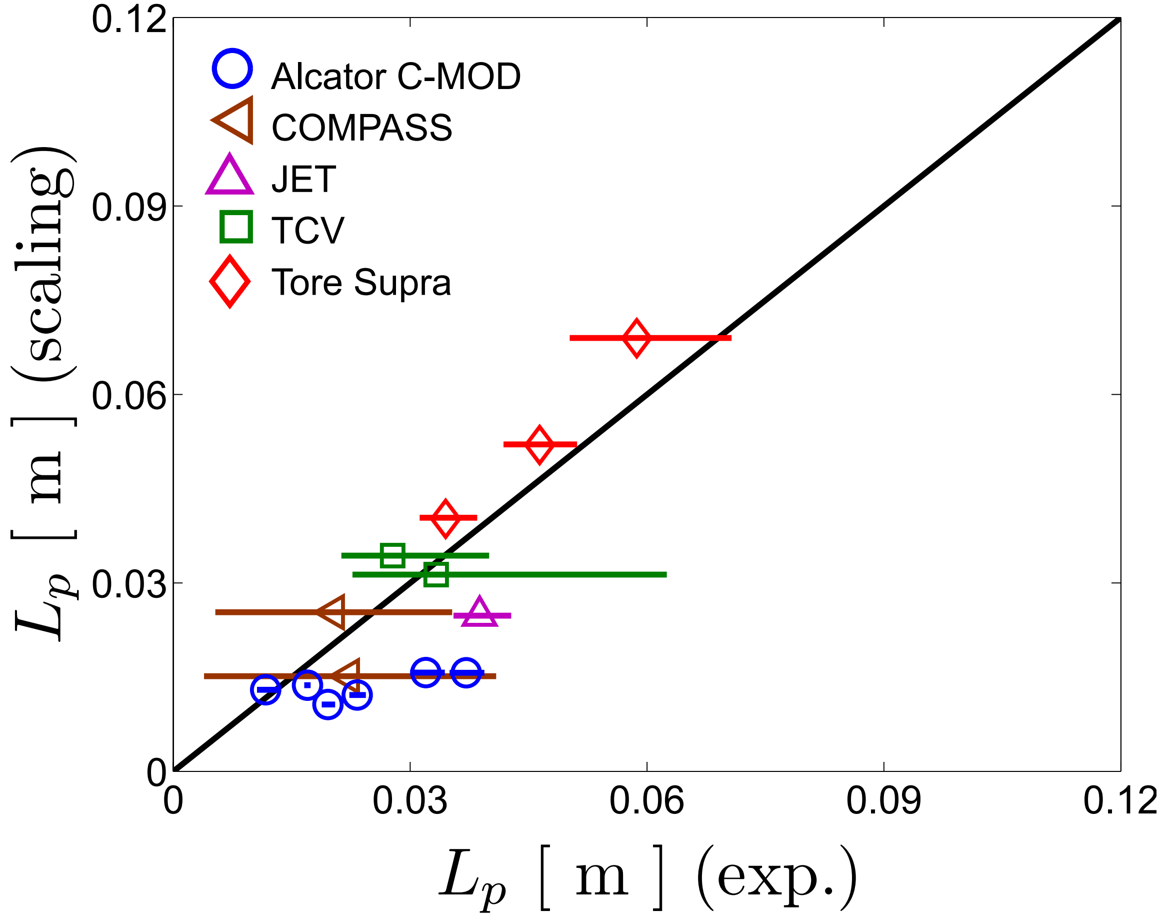
TCV



Good agreement with multi-machine measurements

The ballooning scaling, in SI units:

$$L_p \simeq 7.22 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e,\mathrm{LCFS}}^{-2/7} n_{e,\mathrm{LCFS}}^{2/7} \left(1 + \frac{T_{i,\mathrm{LCFS}}}{T_{e,\mathrm{LCFS}}}\right)^{1/7}$$



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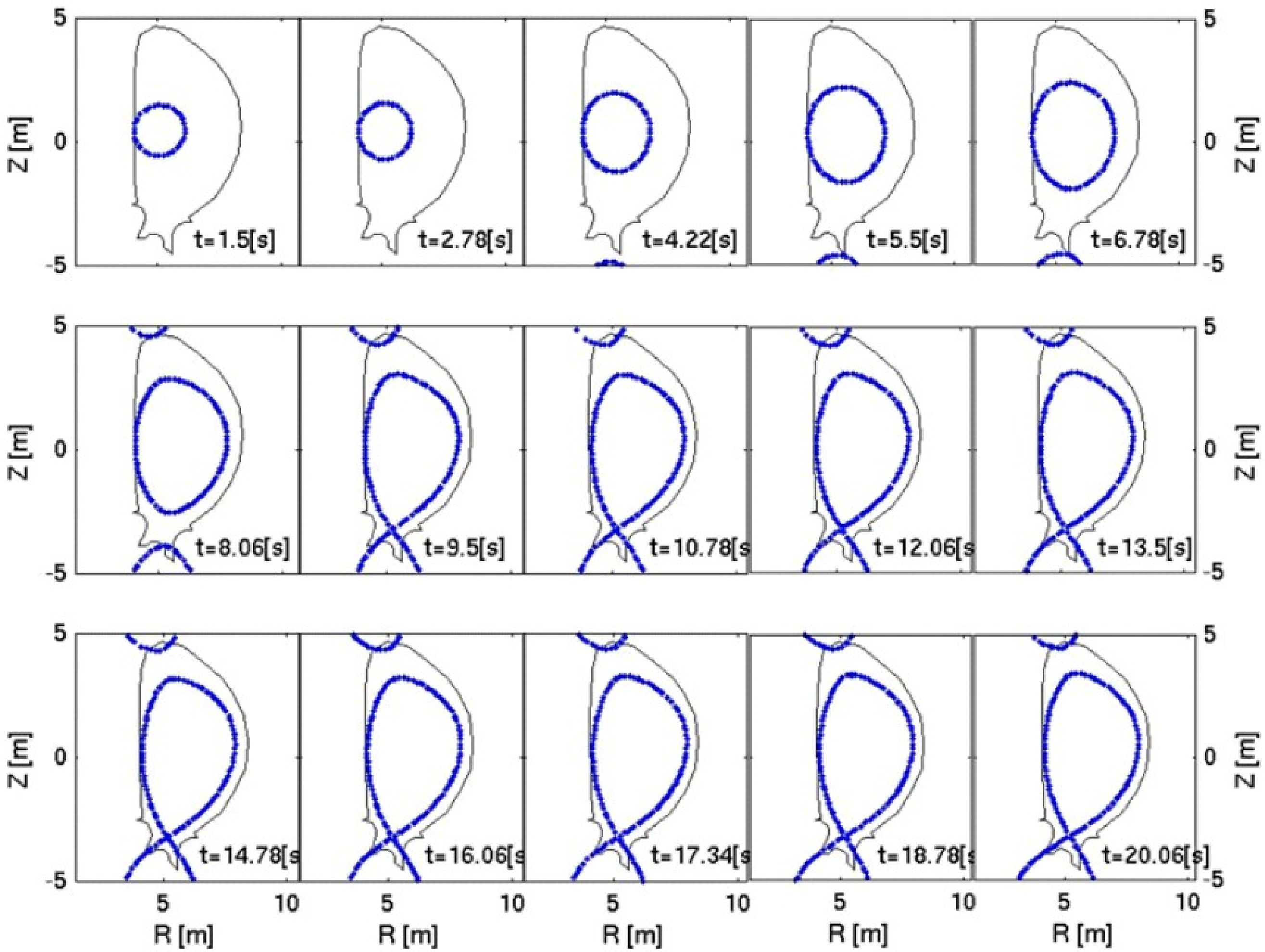
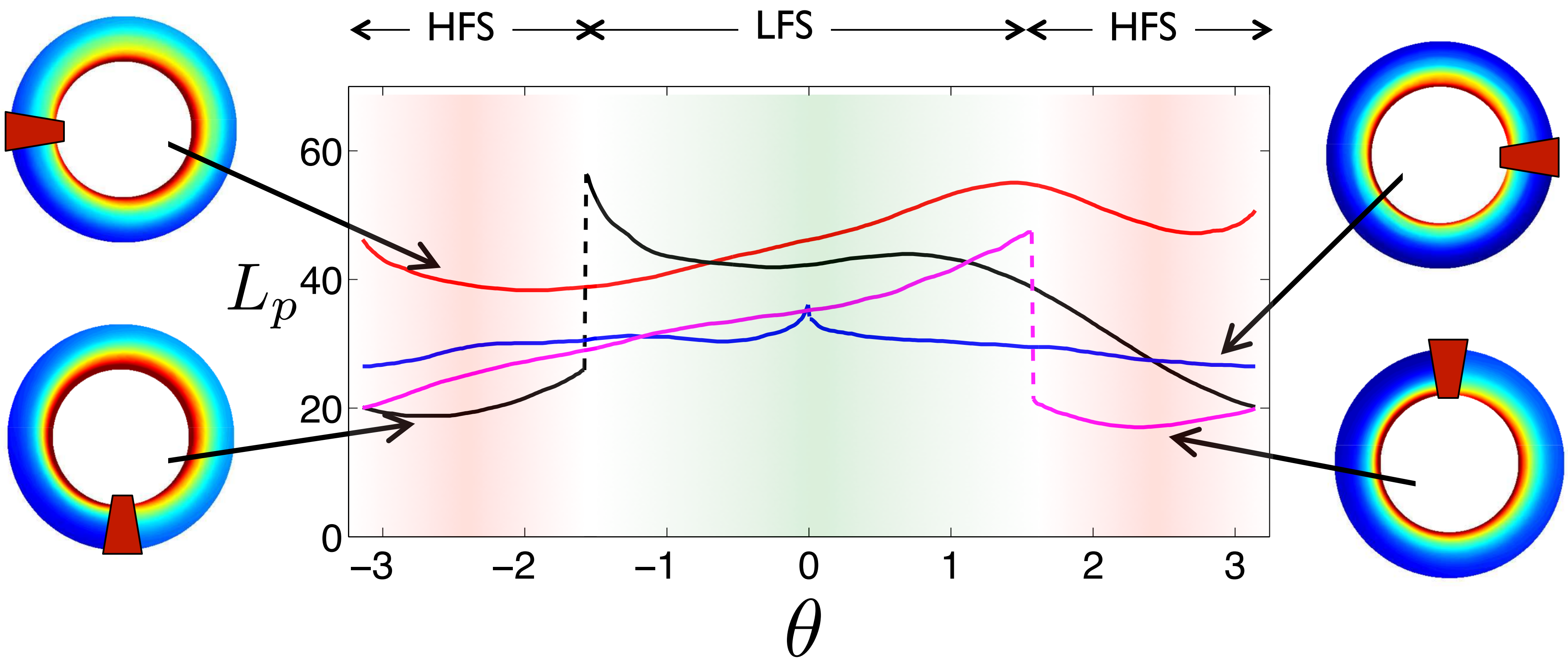
The ITER start-up – minimizing vessel heat load

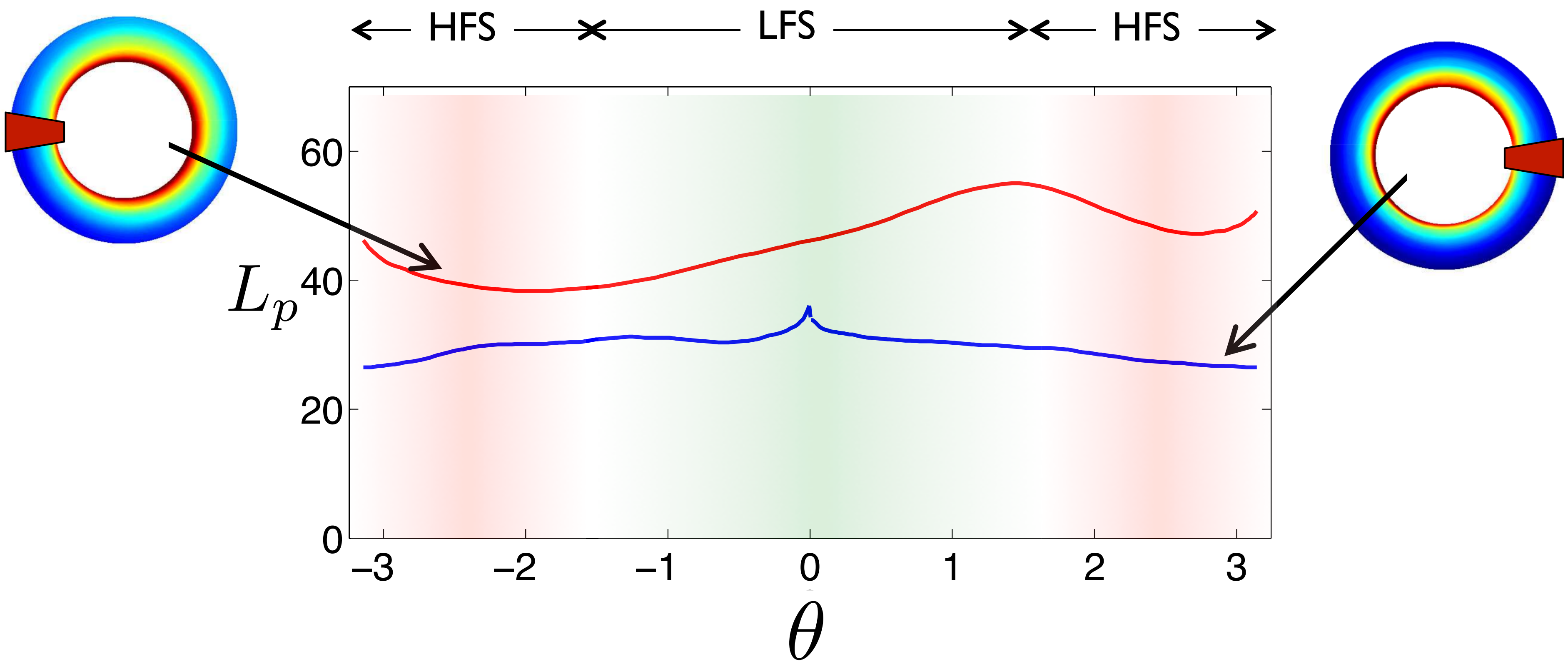


- $P_{\mathrm{wall}} \propto 1/A_{\mathrm{wet}} \propto 1/L_p$
- Is a LFS or HFS limited plasma preferable (L_p larger)?

SOL width larger in HFS limited plasmas



SOL width larger in HFS limited plasmas

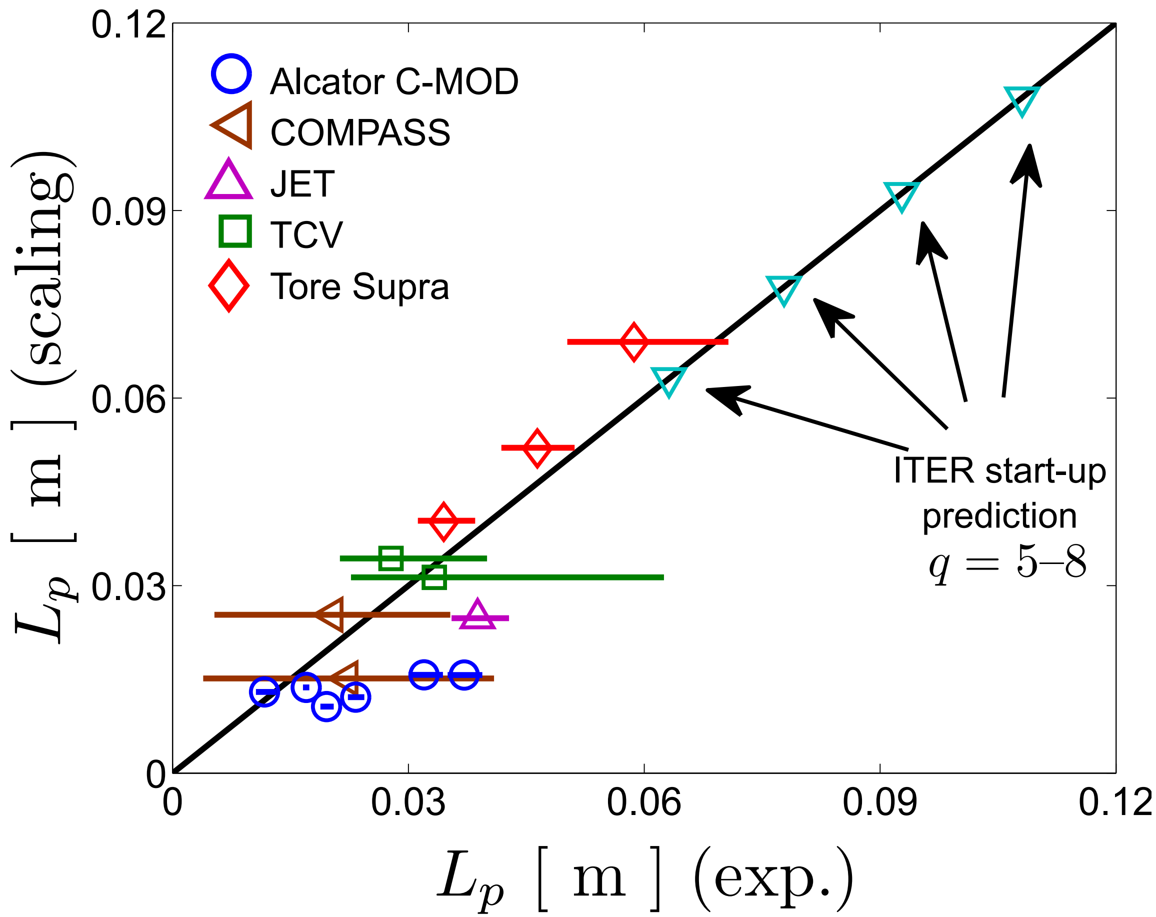


Trends explained by ballooning transport and ExB flow
Confirms experiments, but effects smaller

Prediction of the ITER start-up phase

Obtained from the ballooning scaling:

$$L_p \simeq 7.22 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e,\mathrm{LCFS}}^{-2/7} n_{e,\mathrm{LCFS}}^{2/7} \left(1 + \frac{T_{i,\mathrm{LCFS}}}{T_{e,\mathrm{LCFS}}}\right)^{1/7}$$



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Potential in the SOL set by sheath and electron adiabaticity

Typical estimate: at the sheath

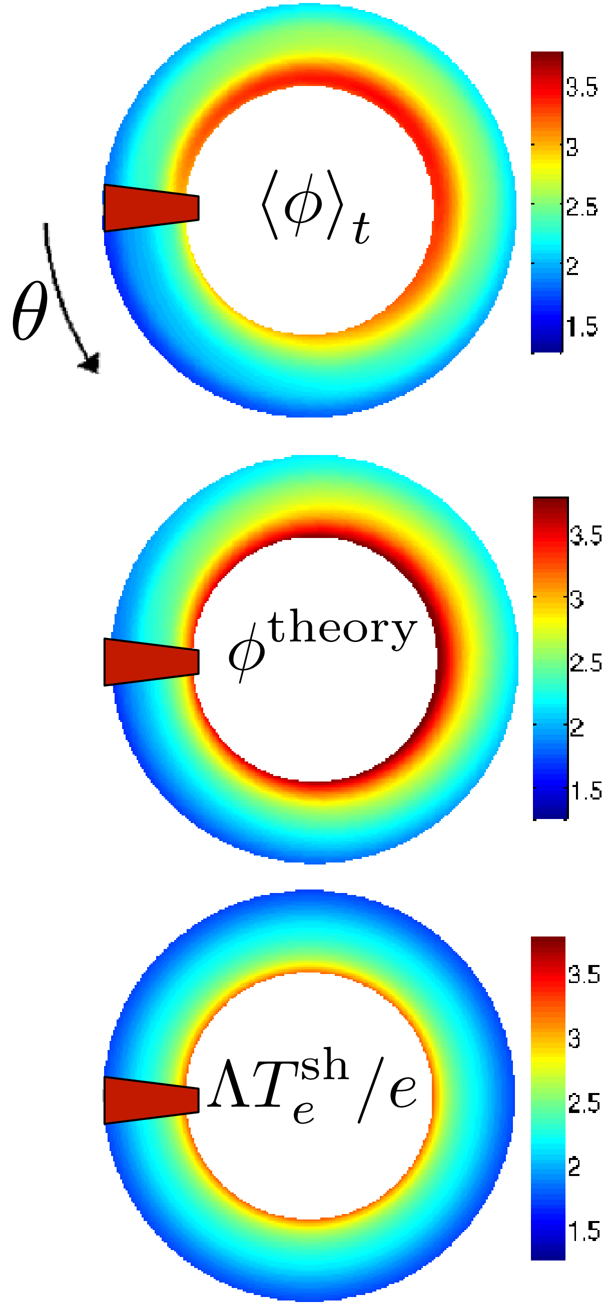
$$v_{\parallel i} = c_s \qquad v_{\parallel e} = c_s \exp(\Lambda - e\phi/T_e^{\mathrm{sh}})$$

to have ambipolar flows, $v_{\parallel i} = v_{\parallel e}$

$$\phi = \Lambda T_e^{\mathrm{sh}}/e \simeq 3T_e^{\mathrm{sh}}/e$$

Our more rigorous treatment, from Ohm's law

$$\phi = \underbrace{\Lambda T_e^{\mathrm{sh}}/e}_{\text{Sheath}} + \underbrace{2.71(T_e - T_e^{\mathrm{sh}})/e}_{\text{Adiabaticity}}$$



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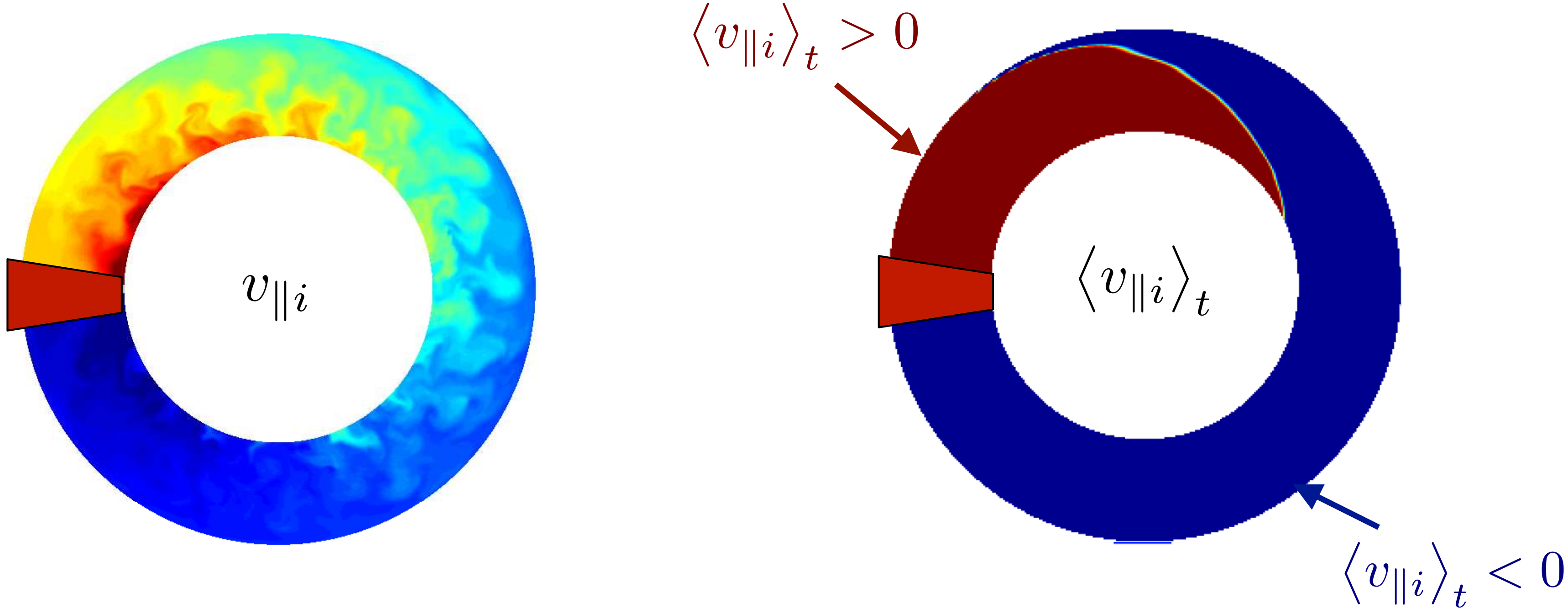
GBS simulations show intrinsic toroidal rotation

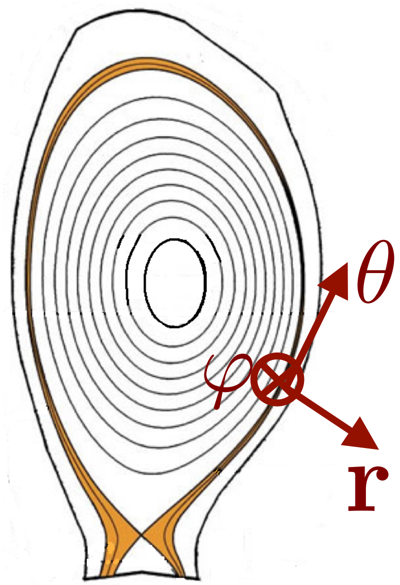


A model for the SOL intrinsic toroidal rotation

Within the drift-reduced Braginskii model:

$$\underbrace{\frac{\partial v_{\parallel i}}{\partial t}}_{\text{Time derivative}} + \underbrace{v_{\parallel i}\nabla_{\parallel} v_{\parallel i}}_{\text{Parallel convection}} + \underbrace{(\mathbf{v}_E \cdot \nabla) v_{\parallel i}}_{\text{ExB transport}} + \underbrace{\frac{1}{m_i n}\nabla_{\parallel} p}_{\text{Pressure gradient}} = 0$$

Time-averaging: $\overline{v}_{\parallel i}\nabla_{\parallel}\overline{v}_{\parallel i} + \nabla \cdot \mathbf{\Gamma}_v + \frac{1}{m_i \overline{n}}\nabla_{\parallel}\overline{p} = 0$



ExB transport:

$$\Gamma_{v,r} \simeq \frac{\sigma_\varphi}{|B_\varphi|}\left\langle \tilde{v}_{\parallel i}\frac{\partial \tilde{\phi}}{\partial \theta}\right\rangle_t$$

$$\Gamma_{v,\theta} \simeq -\frac{\sigma_\varphi}{|B_\varphi|}\overline{v}_{\parallel i}\frac{\partial \overline{\phi}}{\partial r}$$

Gradient-removal estimate of ExB velocity transport

$$\Gamma_{v,r} \sim \left\langle \tilde{v}_{\parallel i} \frac{\partial \tilde{\phi}}{\partial \theta} \right\rangle_t \quad \sim \quad \left\langle \left(\frac{\partial \tilde{\phi}}{\partial \theta} \right)^2 \right\rangle_t \frac{\partial \overline{v}_{\parallel i}}{\partial r}$$

Parallel momentum

$\gamma \tilde{v}_{\parallel i} \sim \partial_r \overline{v}_{\parallel i} \partial_\theta \tilde{\phi}$

$$\sim \quad \langle \tilde{p}^2 \rangle_t \frac{\partial \overline{v}_{\parallel i}}{\partial r} \quad \sim \quad -\frac{\gamma}{k_\theta} L_p \frac{\partial \overline{v}_{\parallel i}}{\partial r}$$

Continuity

$\gamma \tilde{p} \sim \partial_r \overline{p} \partial_\theta \tilde{\phi}$

Grad removal

$\partial_r \tilde{p} \sim \partial_r \overline{p}$

$$\sim \quad -\frac{L_p^2 c_s}{qR} \frac{\partial \overline{v}_{\parallel i}}{\partial r}$$

Lp estimate

$L_p \sim qR\gamma/(c_s k_\theta)$

$$\boxed{\Gamma_{v,r} = -D_{\mathrm{T}} \frac{\partial \overline{v}_{\parallel i}}{\partial r}, \quad D_{\mathrm{T}} = \frac{L_p^2 c_s}{qR}}$$

2D equation for the equilibrium flow

$$-\frac{\partial}{\partial r}\left(D_T \frac{\partial \overline{v}_{\parallel i}}{\partial r}\right) + \frac{\sigma_\varphi}{|B_\varphi|}\frac{\partial \overline{\phi}}{\partial r}\frac{\partial \overline{v}_{\parallel i}}{\partial \theta} + \alpha\sigma_\theta \overline{v}_{\parallel i}\frac{\partial \overline{v}_{\parallel i}}{\partial \theta} + \frac{\alpha\sigma_\theta}{m_i \overline{n}}\frac{\partial \overline{p}}{\partial \theta} = 0$$

Turbulent driven radial transport, gradient-removal estimate

Poloidal convection

Parallel convection

Pressure poloidal asymmetry

Coupling with core physics

with boundary conditions:

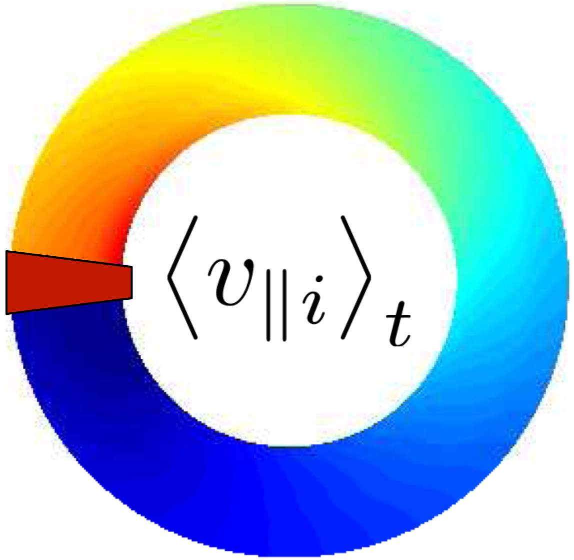
$$\overline{v}_{\parallel i}\Big|_{se} = \pm c_s - \frac{q}{\epsilon}\frac{\partial \overline{\phi}}{\partial r}$$

Bohm's criterion

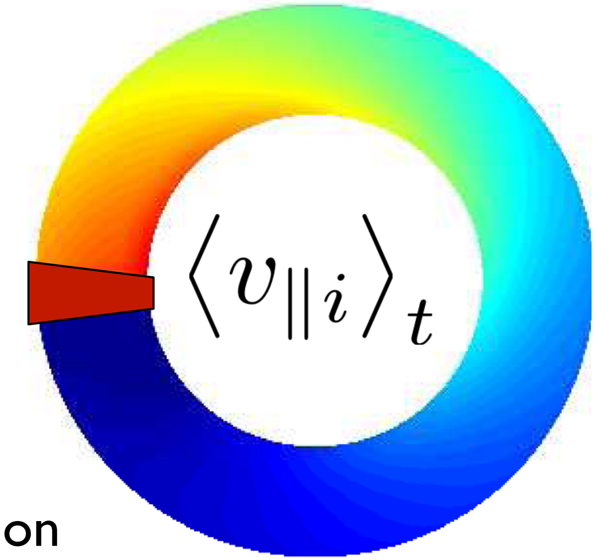
ExB correction

Sources of toroidal rotation

Our model well describes simulation results…



Model



Simulation

… and experimental trends

Analytical solution, far from limiter:

$$M = \boxed{M_s e^{-r/l}} + \left[\boxed{\frac{\Lambda}{2\alpha}\frac{\rho_s}{L_T} e^{-r/L_T}} - \boxed{\frac{\sigma_\varphi}{2}\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right)} \right] \left(1 - e^{-r/l}\right)$$

Core coupling

Sheath contribution, co-current

Pressure poloidal asymmetry at divertor plates, due to ballooning transport, direction: depends

- $M_\parallel \lesssim 1$
- Typically co-current
- Can become counter-current by reversing $\mathbf{B}$ or divertor position

What are we learning from GBS simulations?

- To use a progressive simulation approach to investigate plasma turbulence, supported by analytical theory
- SOL turbulence:
  - Saturation mechanism typically given by gradient removal mechanism
  - Turbulent regimes: in limited plasmas, resistive ballooning modes
  - Good agreement of the scaling of the pressure scale length with multi-machine measurements
  - SOL width larger in HFS limited plasmas
  - Sheath dynamics and electron adiabaticity set the electrostatic potential in the SOL
  - Toroidal rotation generated by sheath dynamics and pressure poloidal asymmetry