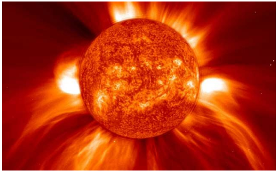

Particle Acceleration in Imbalanced Resistive Turbulence

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[ESA / NASA].

Cross-Helicity in the Solar Wind

- Solar-wind turbulence characteristics are consistent with Alfvénic turbulence
- Positive cross-helicity ($\mathcal{H}^c = \langle \mathbf{u} \circ \mathbf{b} \rangle$) signifies a dominance of outward-propagating Alfvén waves
- with $\mathbf{u} \hat{=}$ bulk velocity, $\mathbf{b} = \mathbf{B}/\sqrt{\mu_0 \rho}$
- However, the magnitude of cross-helicity is not constant throughout the solar wind:
  $\mathcal{H}^c$

[Marsch & Tu, JGpR 1990].

Cross-Helicity in the Solar Wind

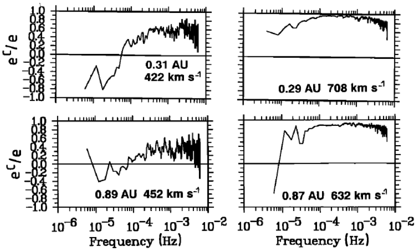

Fig. 4. Normalized cross helicity as a function of heliocentric distance and solar wind flow speed as indicated.

- However, the magnitude of cross-helicity is not constant throughout the solar wind:
  $\mathcal{H}^c$
  - decreases at greater radial distances from the sun
  - is greater in the fast wind than in the slow wind
  - decreases in regions with a high energy cascade rate

[Marsch & Tu, JGpR 1990].

Energy Cascade & Cross-Helicity

- In the Elsasser formulation ($\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}$), the incompressible MHD equations are:

$$\partial_t \mathbf{z}^\pm = -[(\mathbf{z}^\mp \mp \mathbf{B}_0) \circ \nabla]\mathbf{z}^\pm + \frac{\nu + \eta}{2}\nabla^2 \mathbf{z}^\pm + \frac{\nu - \eta}{2}\nabla^2 \mathbf{z}^\mp - \nabla p.$$

- Nonlinear interactions (and hence the energy cascade rate ε) disappear if either of $\mathbf{z}^\pm$ is zero ($\hat{=} \sigma^c = \mathcal{H}^c / \mathcal{E} = \mp 1$)
- Various models, e. g. *Matthaeus et al.*:

$$\frac{d}{dt}\mathcal{E} \propto \left[1 - (\sigma^c)^2\right] \left[(1 + \sigma^c)^{1/2} + (1 - \sigma^c)^{1/2}\right]$$

- The energy cascade slows down as the cross-helicity increases, but details are still debated

Motivation

- If cross-helicity affects the cascade rate, how does it influence the stochastic heating of charged particles?
- Dung & Schlickeiser, A&A (1990): heating goes down;
  Chandran et al., ApJ (2010): heating barely affected;
  Beresnyak et al., ApJ (2011): spatial diffusion unaffected

- First step: compare test-particle acceleration in incompressible 3D MHD simulations for
  - **Balanced** turbulence (zero cross-helicity)
  - **Strongly imbalanced** turbulence (high cross-helicity)

MHD code Turbo

- Pseudospectral code for incompressible resistive MHD

$$\partial_t \mathbf{u} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla]\mathbf{b} + \nu \; \nabla^2 \mathbf{u} + \mathbf{f}^u - \nabla \tilde{p},$$
$$\partial_t \mathbf{b} = -(\mathbf{u} \cdot \nabla)\mathbf{b} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla]\mathbf{u} + \eta \; \nabla^2 \mathbf{b} + \mathbf{f}^b$$

[Teaca et al., 2009].

MHD code Turbo

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$$\partial_t \mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} + [(\mathbf{B}_0+\mathbf{b})\cdot\nabla]\mathbf{b} + \nu\,\nabla^2\mathbf{u} + \mathbf{f}^u - \nabla\tilde{p},$$
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- Tracking of test-particles subject to Lorentz force

$$\ddot{\mathbf{x}}_n = q_n\left(\underbrace{\eta\nabla\times\mathbf{b} - \mathbf{u}\times(\mathbf{B}_0+\mathbf{b})}_{\mathbf{e}} + \dot{\mathbf{x}}_n\times(\mathbf{B}_0+\mathbf{b})\right)$$

[Teaca et al., 2009].

Helical Forcing I

- Helical decomposition of fields into eigenmodes $\mathbf{h}_{R/L}(\mathbf{k})$ of curl operator

$$\tilde{\mathbf{u}}(\mathbf{k}) = \tilde{u}_R(\mathbf{k})\mathbf{h}_R(\mathbf{k}) + \tilde{u}_L(\mathbf{k})\mathbf{h}_L(\mathbf{k})$$

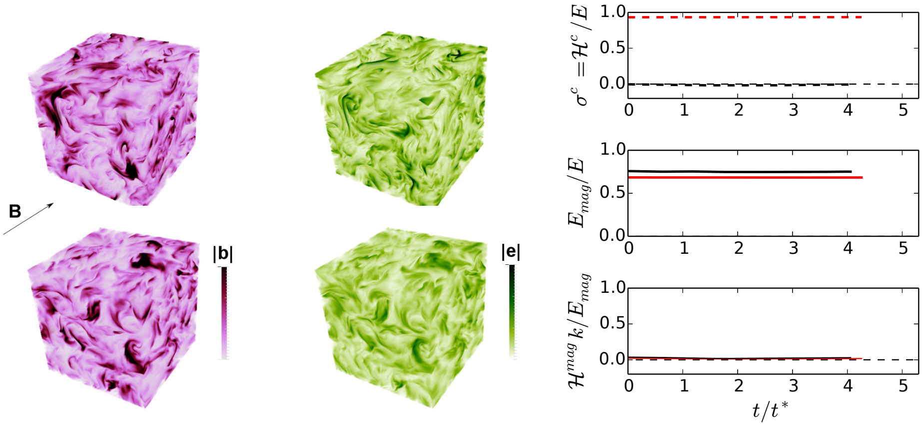
  with $i\mathbf{k} \times \mathbf{h}_{R/L}(\mathbf{k}) = \pm k\mathbf{h}_{R/L}(\mathbf{k})$
- Determine forcing separately for each helical mode, e.g.

$$\mathbf{f}_R^u(\mathbf{k}) = \left[\alpha_R^u(\mathbf{k})\tilde{u}_R(\mathbf{k}) + \beta_R^u(\mathbf{k})\tilde{b}_R(\mathbf{k})\right]\mathbf{h}_R(\mathbf{k})$$

- Allows controlled injection of energy and all three helicities (kinetic, magnetic, cross) via forcing coefficients $\alpha_{R/L}^{u/b}, \beta_{R/L}^{u/b} \in \mathbb{R}$

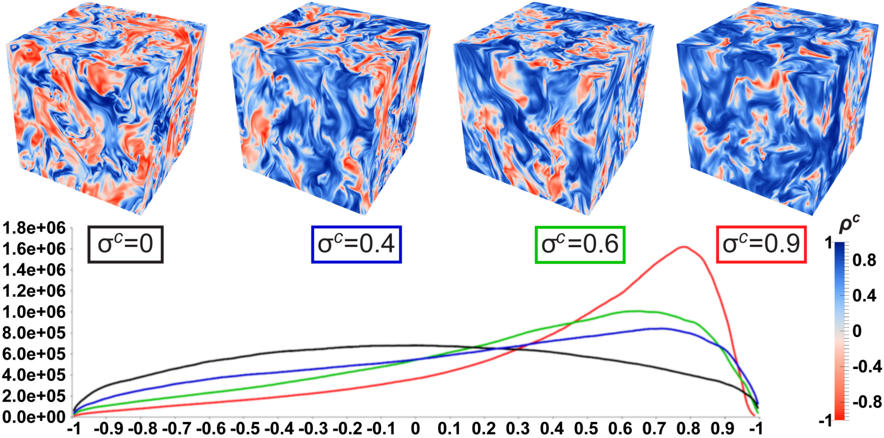
[Teaca et al., 2011].

Balanced vs. imbalanced I



- **Balanced** (top) and **imbalanced** (bottom) MHD turbulence with otherwise identical parameters

Balanced vs. imbalanced II

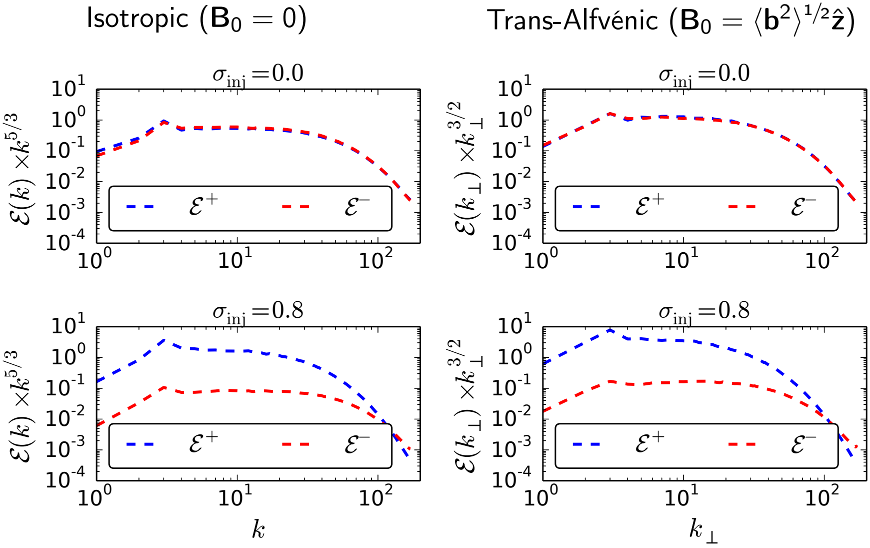


Local Alignment

$$\rho^c(\mathbf{x}) = 2\frac{\mathbf{u} \circ (\mathbf{B}_0 + \mathbf{b})}{|\mathbf{u}|^2 + |\mathbf{B}_0 + \mathbf{b}|^2}$$

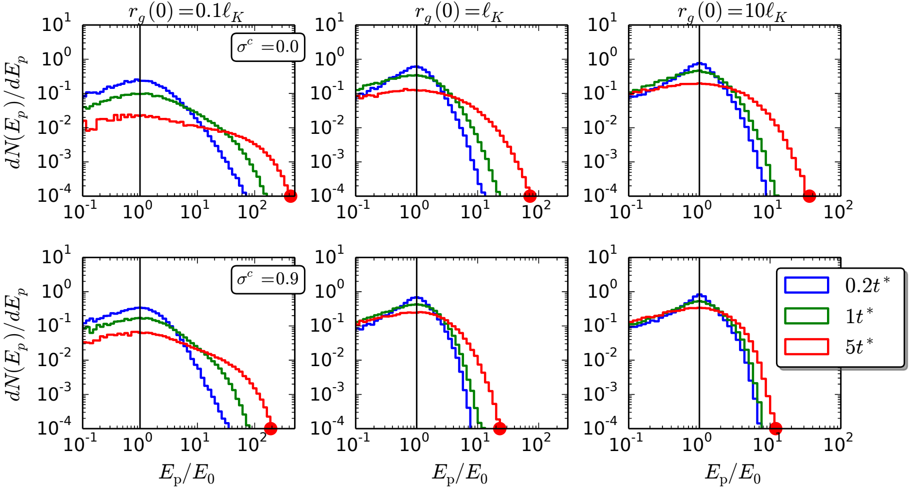
- Even in imbalanced turbulence, ρ^c can be negative in small patches

Comparison of Elsasser spectra



- Steady-state Elsasser energy spectra ($\mathcal{E}^{\pm} = (\mathbf{u} \pm \mathbf{b})^2/4$) both with and without mean-field

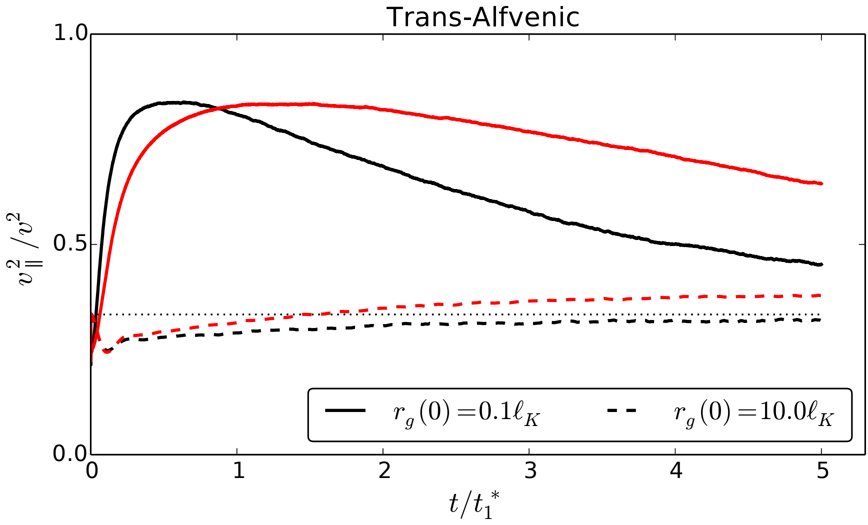
Heating in imbalanced turbulence



- Particle heating is reduced in imbalanced turbulence

[Teaca et al, 2014].

Pitch-angle evolution



- Imbalance reduces perpendicular heating

[Teaca et al, 2014].

Two-stage acceleration

- Particles with small gyroradius experience strong unidirectional acceleration in current sheets ($\mathbf{e}_{\parallel} = \eta \mathbf{j}_{\parallel}$)
- Pitch-angle scattering and energy gain cause the gyroradius to exceed the transverse extent of the current sheet
- Large-gyroradius particles pass through current sheets too quickly to be accelerated, leaving only $\mathbf{e}_{\mathrm{mot}} = -\mathbf{u} \times \mathbf{b}$ as acceleration mechanism

[Dalena et al., ApJ 2014].

Quasi-linear momentum diffusion

Quasi-linear theory

for imbalanced slab turbulence describes particle heating as diffusion in momentum space

$$\frac{\partial}{\partial t} f(p,t) = p^{-2} \frac{\partial}{\partial p} \left[p^2 D_{pp} \frac{\partial}{\partial p} f(p,t) \right]$$

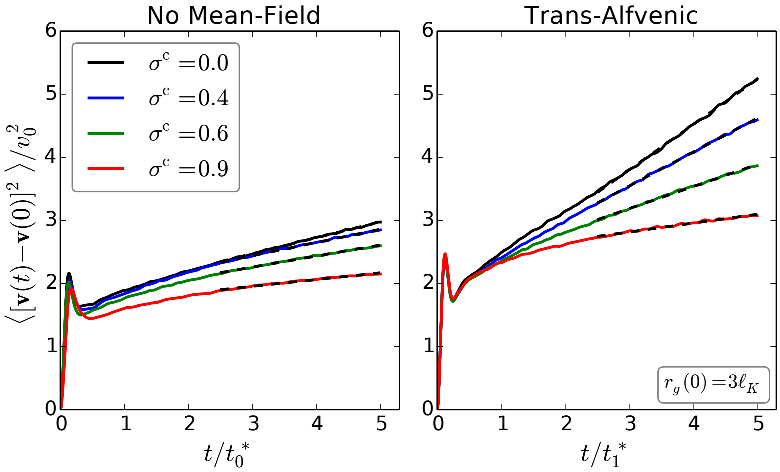
and predicts that momentum diffusion scales as :

$$D_{pp} \sim \frac{p^2}{\tau} \frac{v_A^2}{v^2} \times [1 - (\sigma^c)^2]$$

with the scattering timescale $\tau \propto r_g^{2-s}$ for $v \gtrsim v_A$

[Dung & Schlickeiser, A&A 1990].

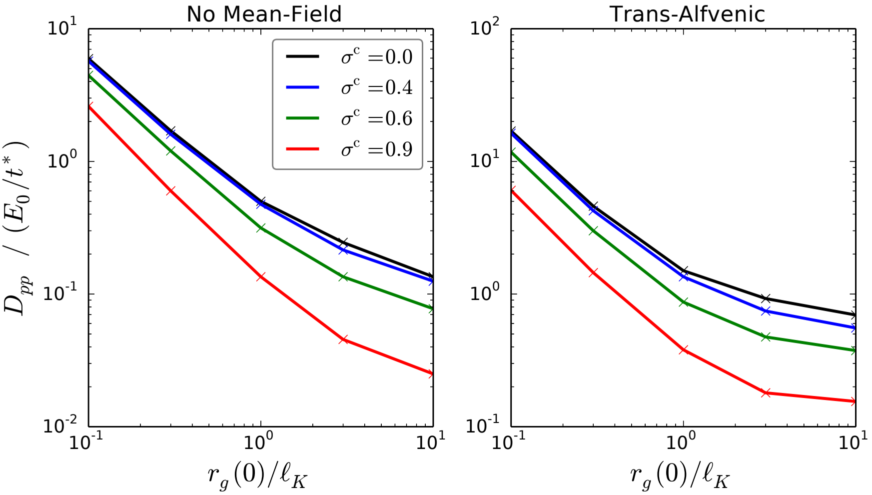
Momentum diffusion



$$D_{pp} = \frac{1}{2}\frac{d}{dt}\langle|\mathbf{v}(t) - \mathbf{v}(0)|^2\rangle$$

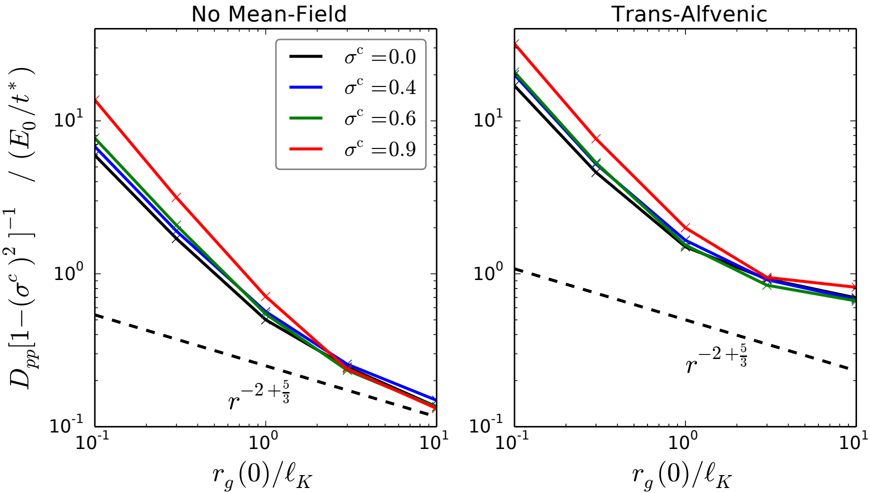
[Teaca et al, 2014].

Heating in imbalanced turbulence



[Teaca et al, 2014].

Heating in imbalanced turbulence



$$D_{pp} \sim [1 - (\sigma^c)^2] r_g^{-2+s}$$

[Teaca et al, 2014].

Summary

- We have compared test-particle acceleration in time-dependent MHD turbulence at various degrees of imbalance, with and without magnetic mean-field
- Strong imbalance (non-zero cross-helicity) inhibits the efficiency of ion heating in MHD turbulence
- At gyroradii in the inertial range, the observed scaling agrees with $D_{pp} \sim [1 - (\sigma^c)^2] r_g^{-2+s}$

Next Steps

- Investigate resonance broadening as a function of cross-helicity
- Examine influence of compressible modes
- Check for self-consistency in PIC simulations

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