Turbulence wave number spectrum reconstruction using radial correlation reflectometry (with applications to JET, and Tore Supra tokamaks)

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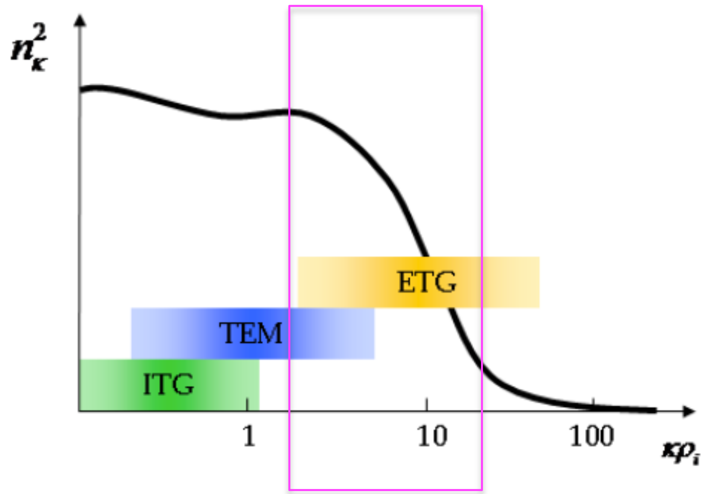
Turbulence in tokamak plasmas

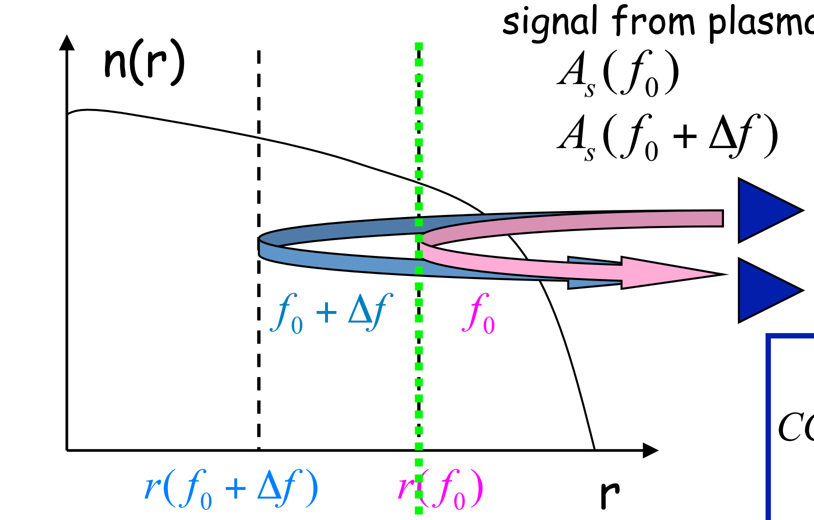
Turbulence → Anomalous transport → Faster loss of heat

Diagnostic of turbulence is important !

Turbulence energy spectrum function S(k)



Principle of radial correlation reflectometry (RCR)



f_0 - reference frequency

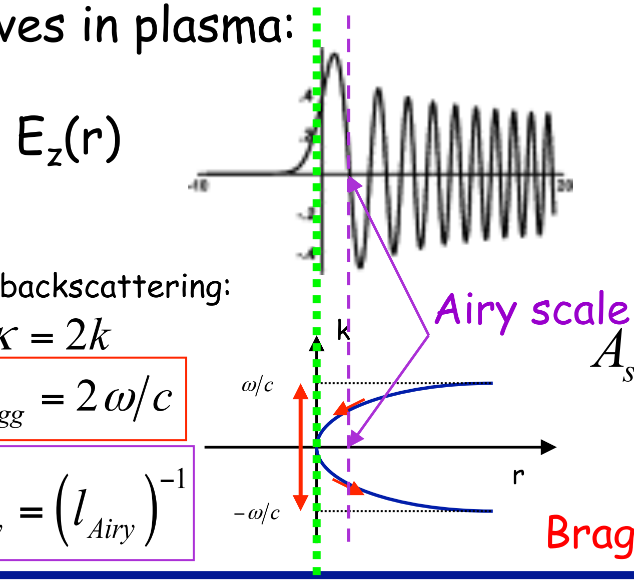
$f_0+\Delta f$ - sweeping frequency

Δr - distance between cut offs

Correlation between RCR signals:

$$CCF(\Delta r)=\frac{\left\langle\left(A_s(f_0)-\langle A_s(f_0)\rangle\right)\left(A_s(f_0+\Delta f)-\langle A_s(f_0+\Delta f)\rangle\right)^*\right\rangle}{\sqrt{\left\langle\left(A_s(f_0)-\langle A_s(f_0)\rangle\right)^2\right\rangle\left\langle\left(A_s(f_0+\Delta f)-\langle A_s(f_0+\Delta f)\rangle\right)^2\right\rangle}}$$

waves in plasma:



Bragg backscattering:

$\kappa=2k$

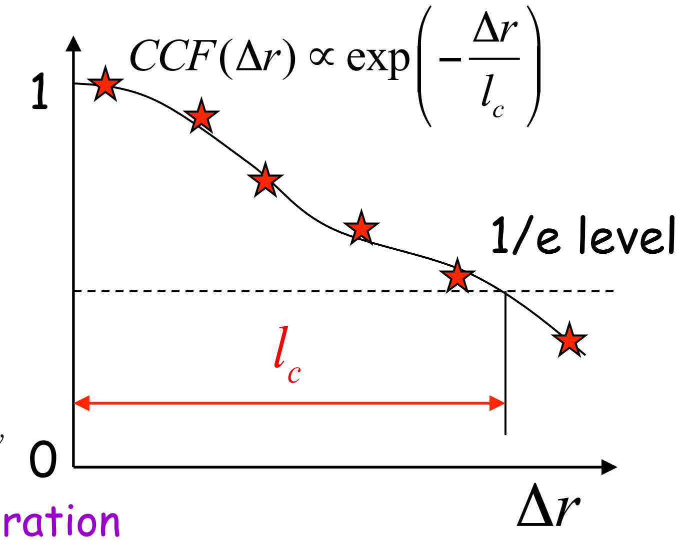
$\kappa_{Bragg}=2\omega/c$

$\kappa_{Airy}=\left(l_{Airy}\right)^{-1}$

measurement interpretation

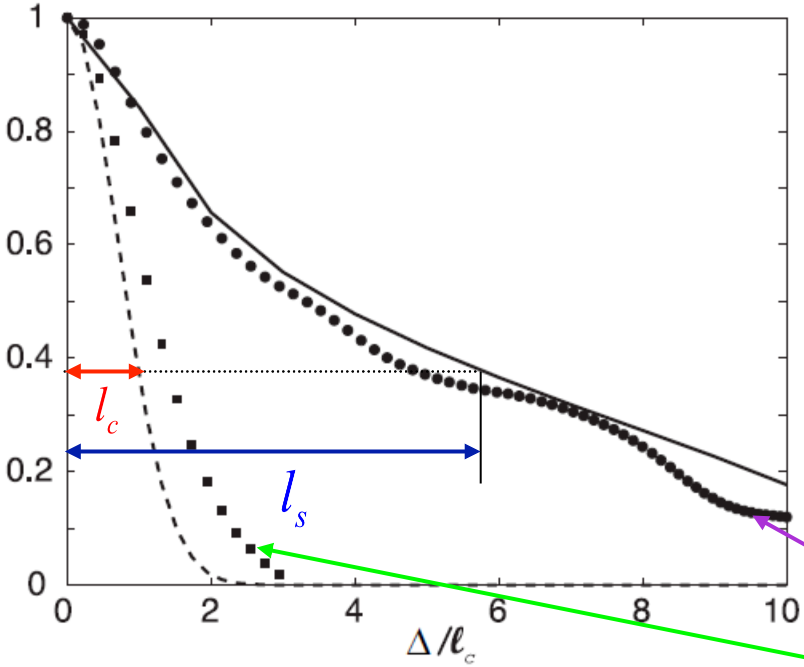
$$A_s\propto\begin{cases}\dfrac{1}{\sqrt{\kappa}}, & |\kappa|>\kappa_{Airy}\\ Const, & |\kappa|\le\kappa_{Airy}\end{cases}$$

Bragg backscattering saturation



■ Problems with RCR

Large difference between RCR CCF and turbulence CCF in 1D Born approximation full wave computations:



l_s – signal correlation length,

l_c - turbulence correlation length

__ numerical integration of Born integral;

- - turbulence space correlation function;

numerical results for 2 values of the lowest wave number in spectrum:

cut at values of the order $0.1(L)^{-1}$

cut at values of the order $(L)^{-1}$

L - density gradient scale length

Relation between RCR CCF and turbulence CCF is needed !

RCR theoretical background (1)

1D model Helmholtz equation in O-mode is solved using perturbation theory in Born approximation.

$$\left\{\frac{d^2}{dr^2} + k^2(r)\right\} E_z(r,\omega) = 0$$

$$k^2(r) = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$ - probing wave number; ω - probing frequency;

$$\omega_p \propto \sqrt{n(r) + \delta n(r)}$$ - plasma frequency; $n(r)$ - plasma density profile;

$$\delta n(r) = \frac{1}{2\pi}\int \delta n_\kappa e^{-i\kappa r} d\kappa$$ - homogeneous density perturbations, κ is a radial wave number;

E_z - total field of the probing wave; S_i – incident wave energy flux density.

Scattering signal from plasma:

$$A_s(\omega) = \frac{i\omega\sqrt{S_i}}{16\pi}\int_0^\infty \frac{\delta n(r)}{n_c} E_0^2(r,\omega)dr$$

$E_0(r,\omega)$ - zero-order solution of the unperturbed Helmholtz equation

RCR theoretical background (2)

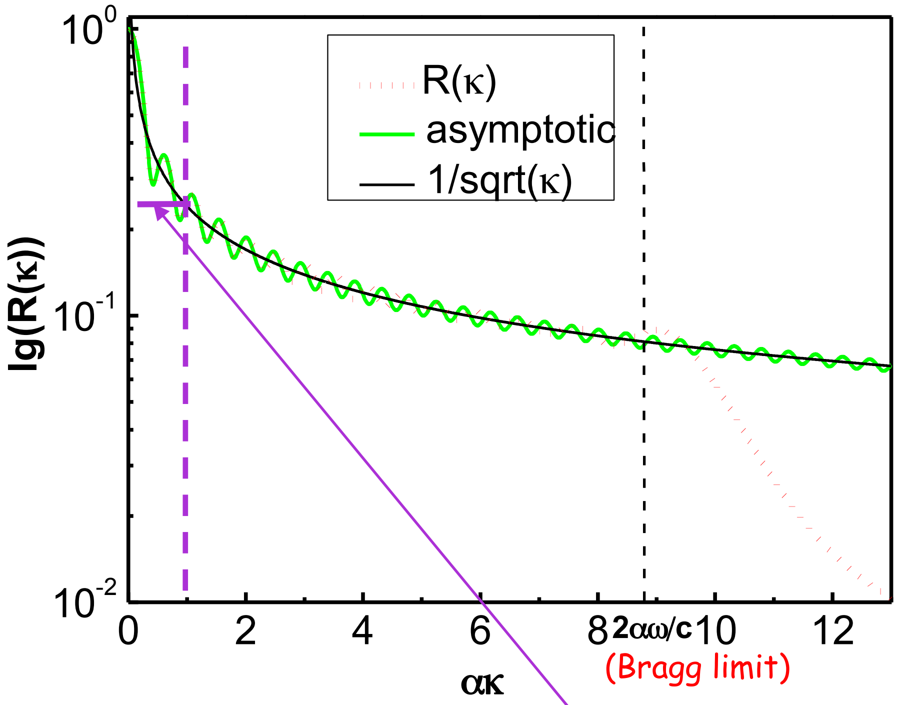
Scattering signal from plasma:

$$A_s(\omega) = \int_{-\infty}^{+\infty} R(\kappa)\frac{\delta n_\kappa}{n_c}\frac{d\kappa}{2\pi}$$

where:

$$\omega = 2\pi(f_0 + \Delta f)$$

$$erf(s) = \int_0^s e^{-\varsigma^2}\, d\varsigma$$



$$R(\kappa) \propto \frac{\omega^2}{c^2} e^{i\kappa L} \begin{cases} 2\pi \dfrac{e^{-i\frac{(l_{Airy}\kappa)^3}{12}}}{\sqrt{l_{Airy}\kappa}}, & \kappa_{Airy} << |\kappa| < \kappa_{Bragg} \\ -4\sqrt{\pi}\dfrac{erf\left(\sqrt{i\kappa L}\right)}{\sqrt{l_{Airy}\kappa}}, & |\kappa| << \kappa_{Airy} \end{cases}$$

Bragg backscattering

$$R(\kappa) \propto \frac{\omega^2}{c^2} e^{i\kappa L} \begin{cases} -\dfrac{2\pi}{\sqrt{l_{Airy}\kappa}}, & 1/L << |\kappa| << \kappa_{Airy} \\ -4\sqrt{\pi}e^{i\frac{\pi}{4}}\sqrt{L/l_{Airy}}, & |\kappa| << 1/L \end{cases}$$

Saturation Arises earlier

Previous approach:

$$A_s \propto \begin{cases} \dfrac{1}{\sqrt{\kappa}}, & |\kappa| > \kappa_{Airy} \\ Const, & |\kappa| \le \kappa_{Airy} \end{cases}$$

(Bragg backscattering)

(saturation)

RCR theoretical background (3)

1) RCR CCF measured in experiment

$$CCF(\Delta r) = \frac{\left\langle \left(A_s(f_0) - \langle A_s(f_0)\rangle\right)\left(A_s(f_0+\Delta f) - \langle A_s(f_0+\Delta f)\rangle\right)^* \right\rangle}{\sqrt{\left\langle \left(A_s(f_0) - \langle A_s(f_0)\rangle\right)^2 \right\rangle \left\langle \left(A_s(f_0+\Delta f) - \langle A_s(f_0+\Delta f)\rangle\right)^2 \right\rangle}}$$

$$CCF(L) \propto \int_{-\infty}^{+\infty} \frac{d\kappa}{|\kappa|} n_\kappa^2 e^{i\kappa\Delta L} erf\left(\sqrt{i\kappa L_0}\right) erf^*\left(\sqrt{i\kappa L}\right)$$

2) Compute turbulence radial wave number spectrum

$$n_\kappa^2 \propto \frac{|\kappa|}{erf^*\sqrt{i\kappa r(f_0)}} \int_{-\infty}^{+\infty} CCF(\Delta r) e^{i\kappa\Delta r} d\Delta r$$

3) Compute turbulence spatial correlation function and correlation length

$$TCCF(\Delta r) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} n_\kappa^2 e^{-i\kappa\Delta r} d\kappa$$

$l_c \propto$ 1/e TCCF level

JET RCR data analysis march 2012

JET reflectometry diagnostic

Probing mid-plane (z=29cm emitter, z=25cm receiver)
JET plasma in X mode, $\vec{E}_{probe} \perp \vec{B} \Rightarrow N(n_e(r), B(r))$

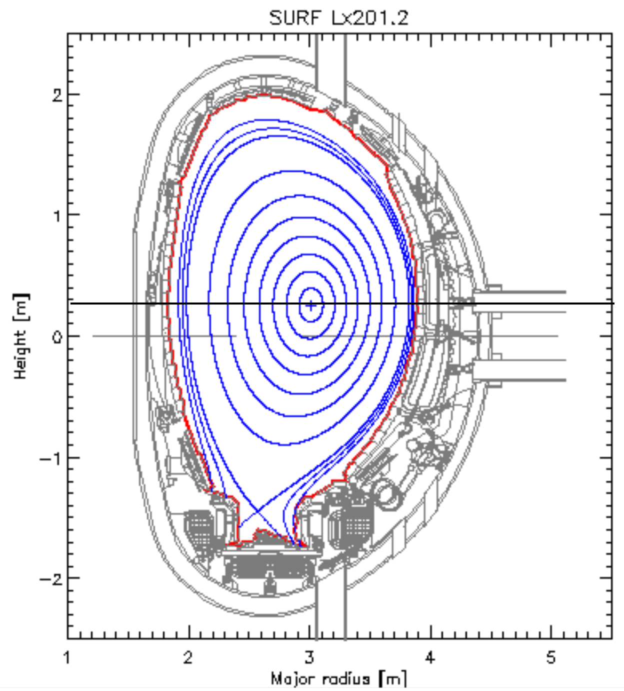
I. KG8B (reference channel)

2 fixed frequency channels: 85GHz and **92GHz**
2 adjustable frequency channels: **85-89GHz** and 92-96GHz

II. KG8C master channel (sweeping channel)
slave channel (not used)
75-110GHz

→ Correlation measurements
at 2 different radial positions

Data acquisition frequency: 2MHz
Time: 39-71s
Duration: 32s (1s before plasma)

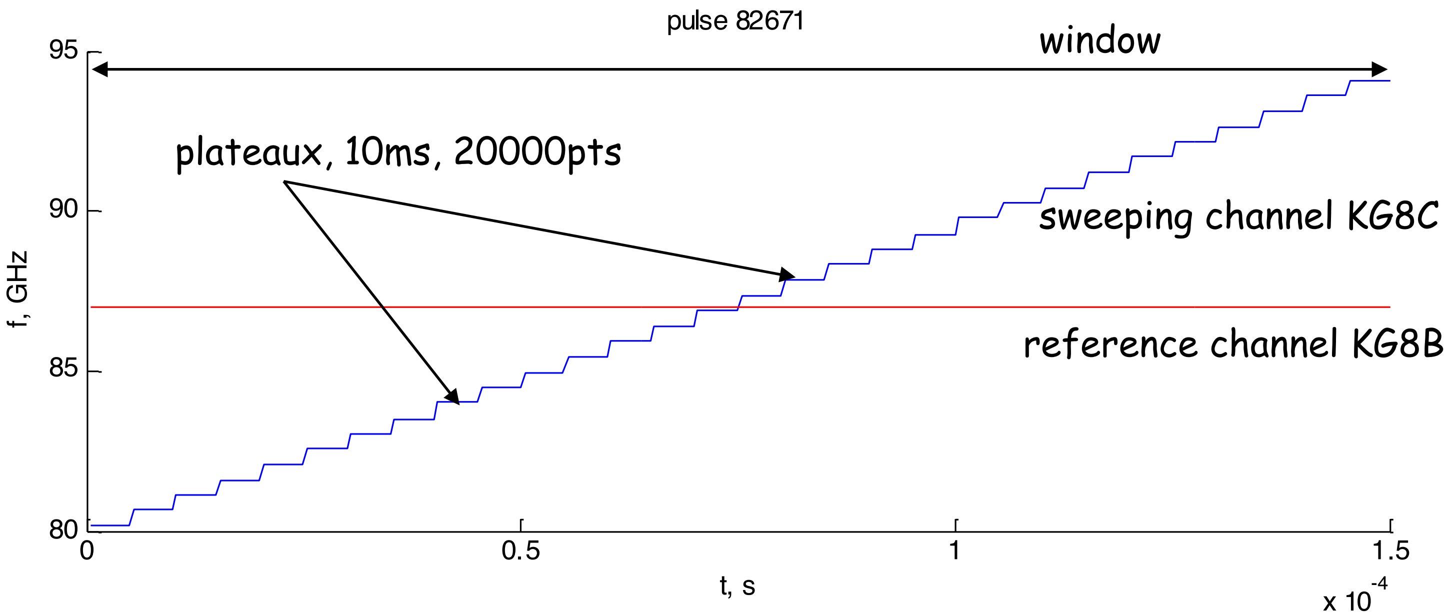


Experimental settings

Example: **shot #82671**, 16/03/2012 (Friday), early shift

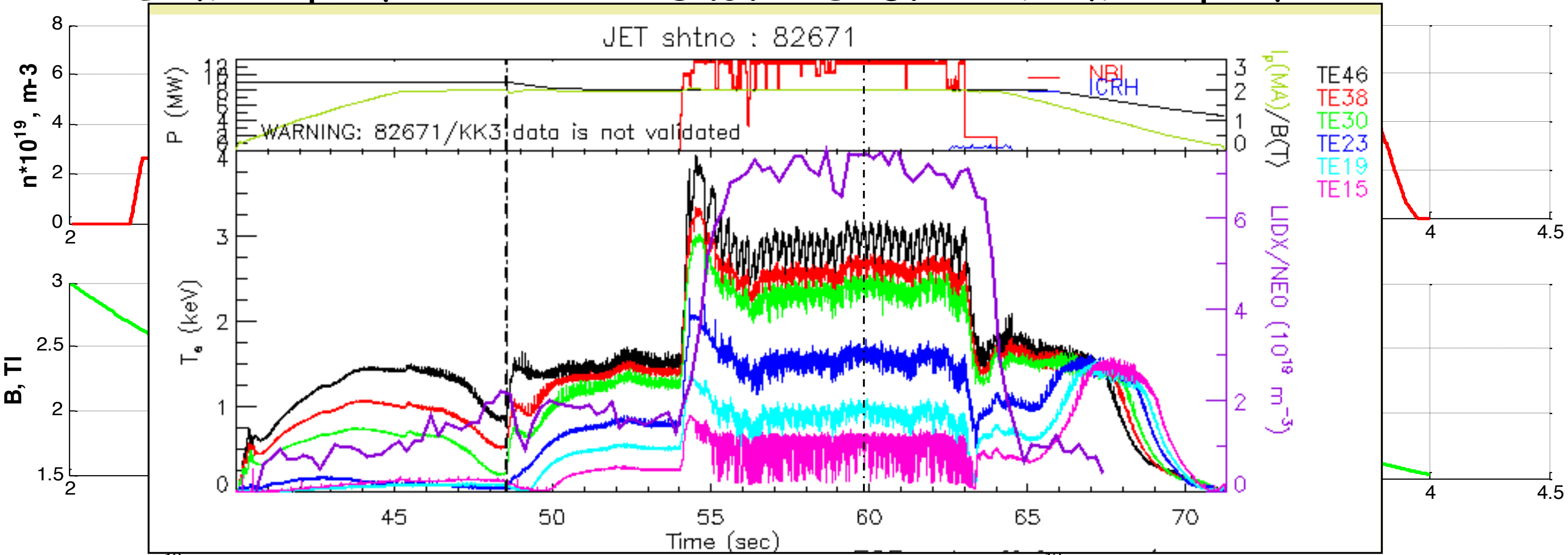
$$dt = 0.5 \cdot 10^{-6} s$$

30 plateaux, plateau duration: 10ms, 20000 pts per plateau
window duration: 0.3s, 600000 pts per window
106,7 windows per shot, 64000000 pts per shot
reference frequency: **87GHz**
sweeping start frequency: **80.16GHz**, frequency step: **480MHz**



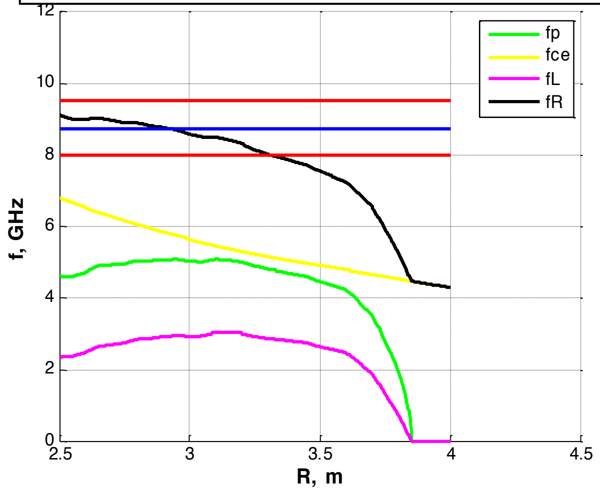
■ Cut off positions

L-mode profiles shot #82671 H-mode profiles



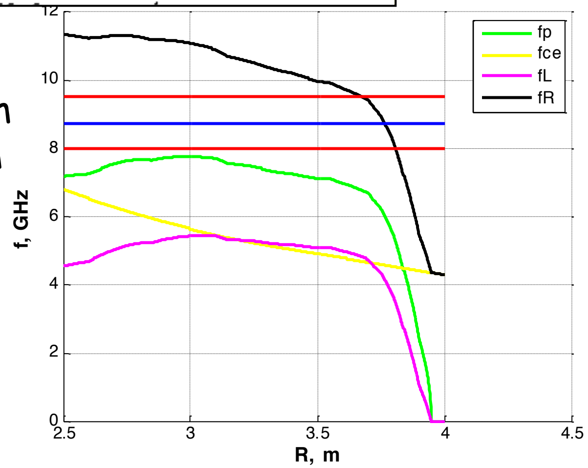
$x_c$min =3.296m
$x_c$max=2.303m
x_c0 =2.939m

t=48.4s



$x_c$min =3.811m
$x_c$max=3.672m
x_c0 =3.761m

t=59.7s



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Data analysis

Signal acquisition (I/Q detection):

$$I(t) = a(t)\cos(\varphi(t))$$

$$Q(t) = a(t)\sin(\varphi(t))$$

$$\varphi(t) = arctg\left(\frac{Q(t)}{I(t)}\right),\ \mathrm{a}(t) = \sqrt{I^2(t) + Q^2(t)}$$

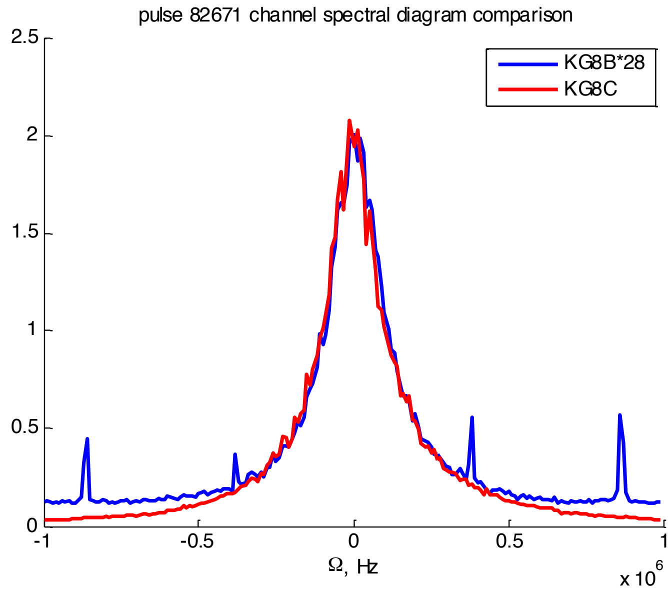
Correlation:

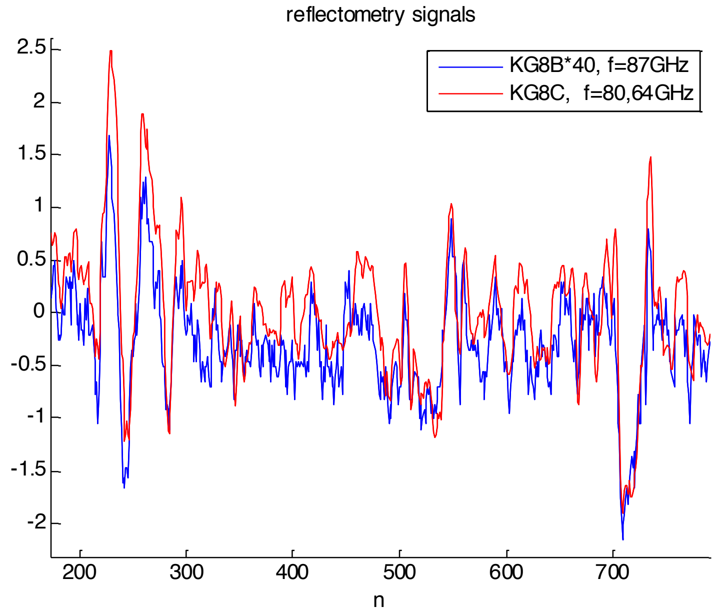
$$A_s(f,t) = I(t) + iQ(t)$$

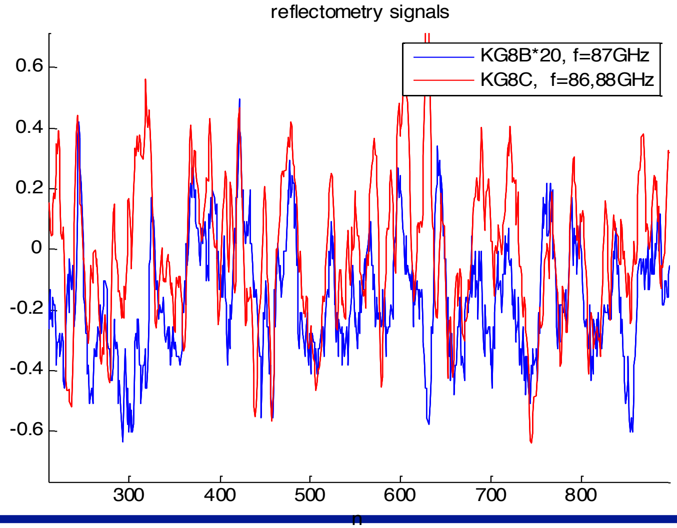
$$A_{s0}(f_0,t) = I_0(t) + iQ_0(t)$$

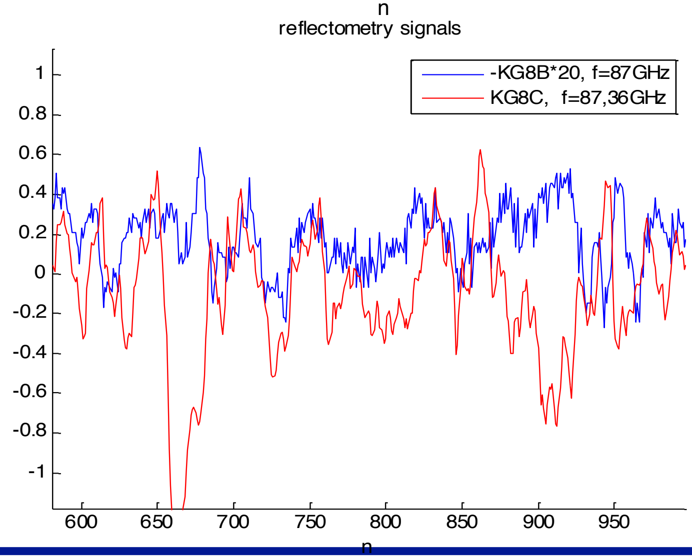
$$CCF(\Delta r) = \frac{\left\langle \left(A_s(f_0) - \left\langle A_s(f_0)\right\rangle_t\right)\left(A_s(f_0+\Delta f) - \left\langle A_s(f_0+\Delta f)\right\rangle\right)^*_t \right\rangle}{\sqrt{\left\langle \left(A_s(f_0) - \left\langle A_s(f_0)\right\rangle_t\right)^2\right\rangle_t \left\langle \left(A_s(f_0+\Delta f) - \left\langle A_s(f_0+\Delta f)\right\rangle_t\right)^2\right\rangle_t}}$$

Problems with experiment (1)









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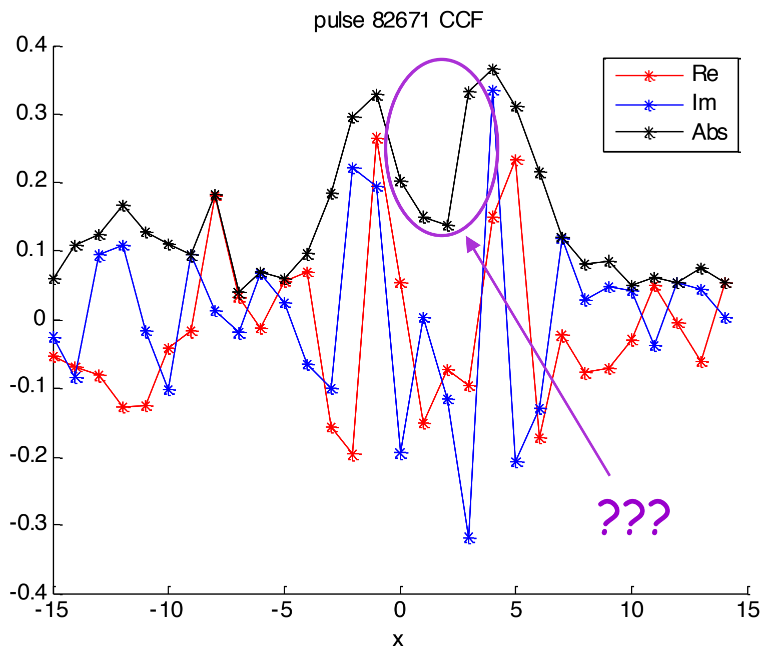
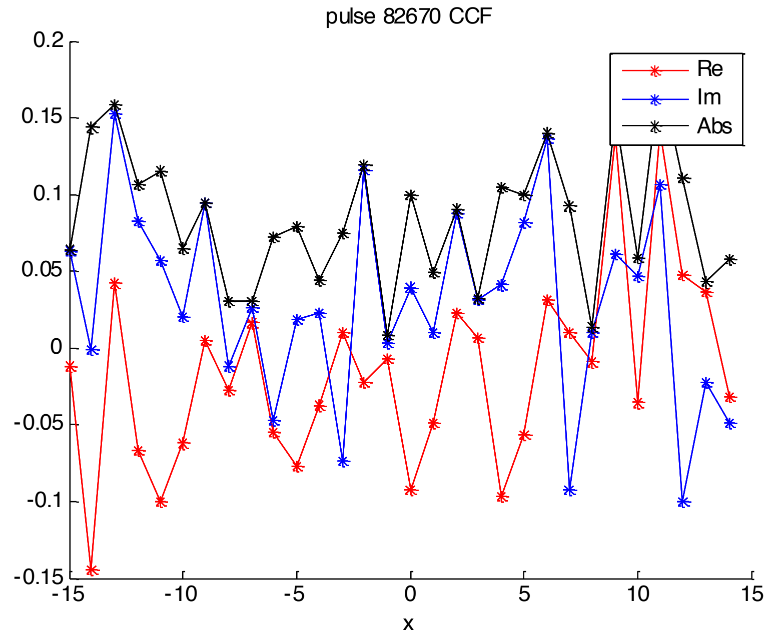
Problems with experiment (2)





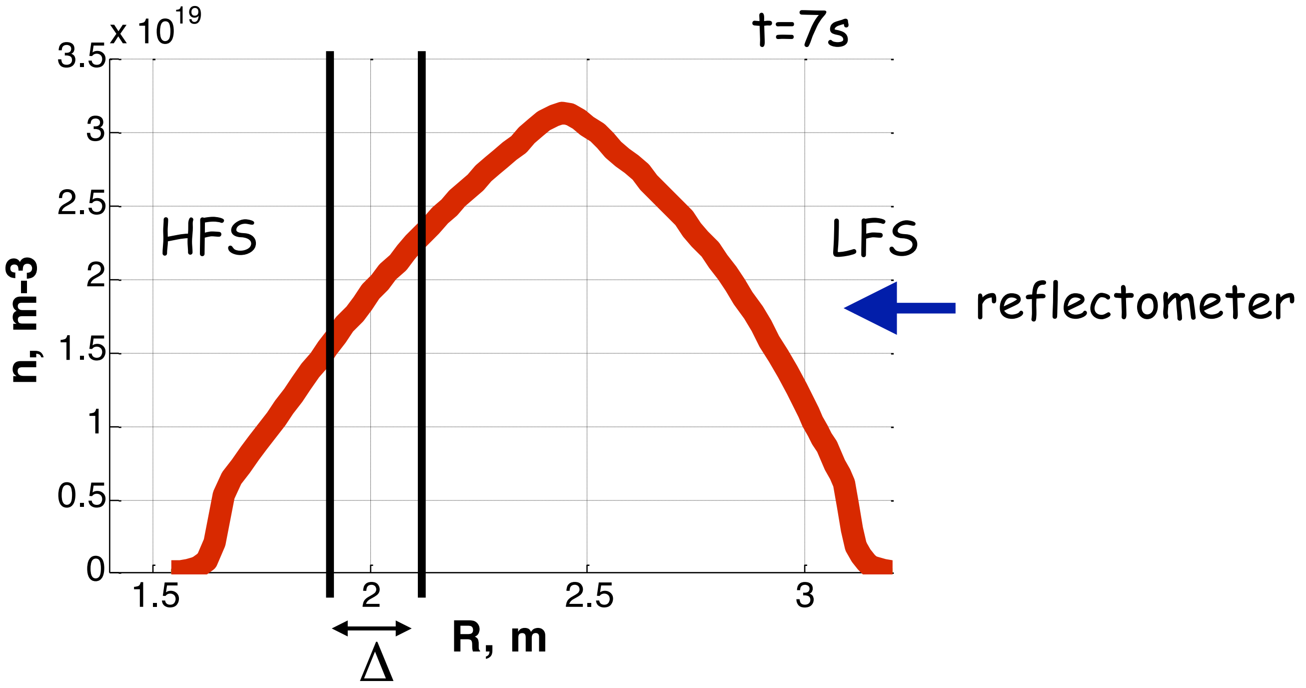
- frequency offset?
- cross talk between channels?
- low power?

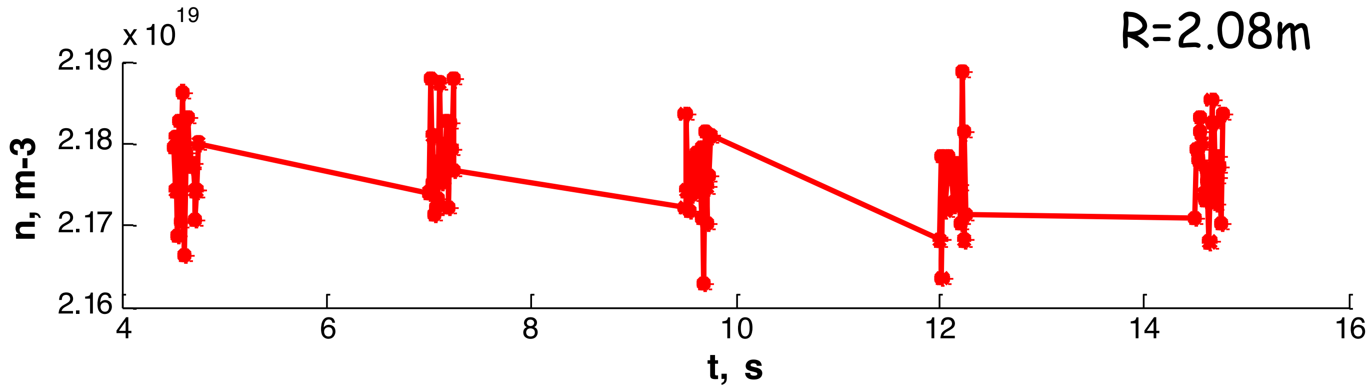
poor level of signals

→ noise...

Tore Supra RCR data analysis

Tore Supra, shot #47669





Experimental data:

5 triggers:
20 plateaux,
2000 points per plateau

duration 18.08s
time step = $1\mu s$

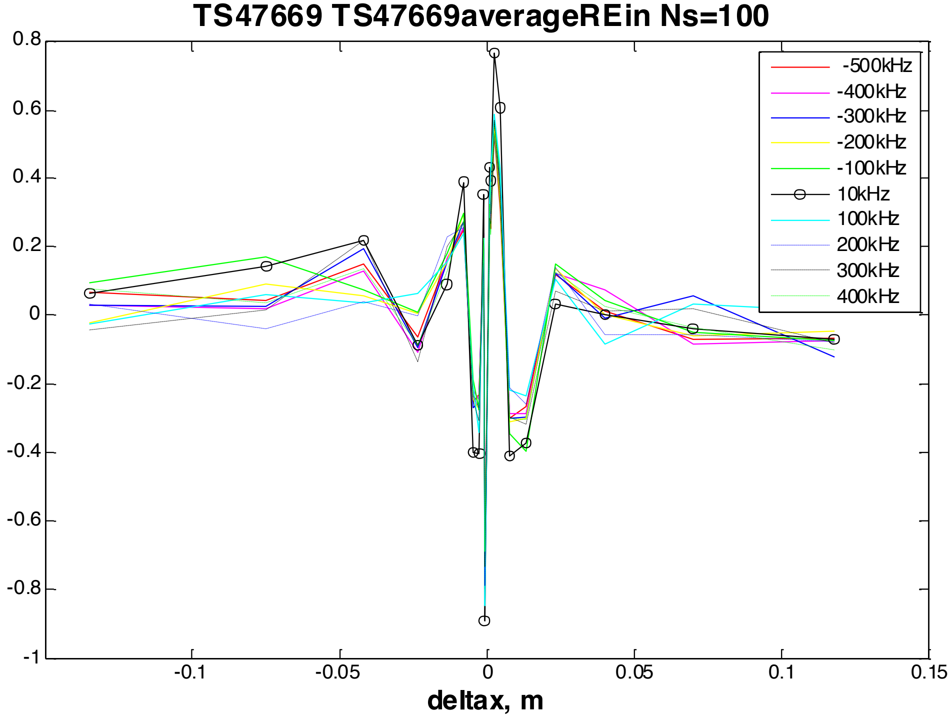
probing interval:
$\Delta = 0.25m$

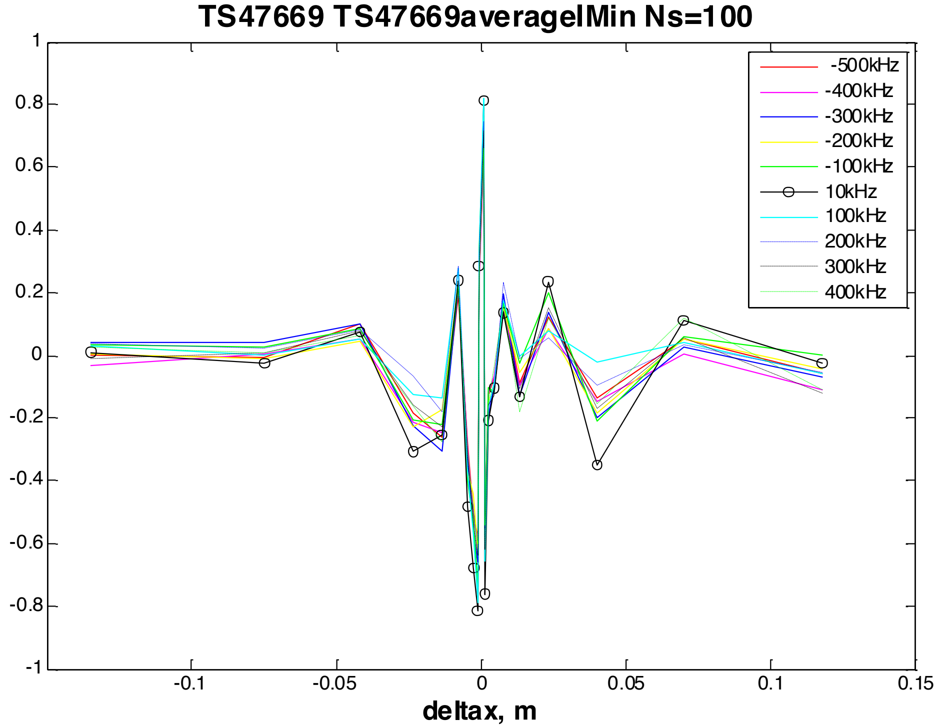
Signal CCF, shot #47669

Input signal, real and imaginary part

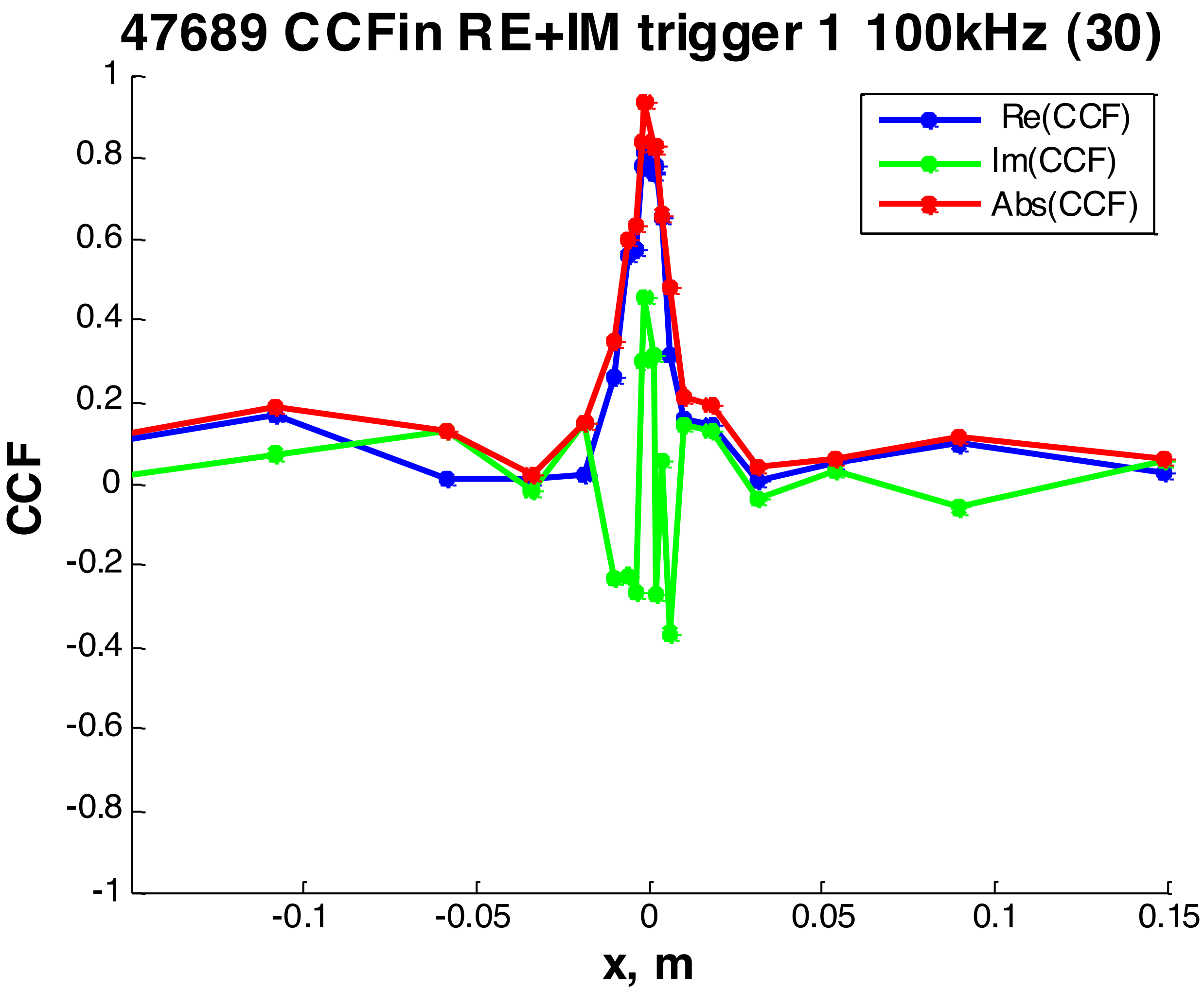
FFT: Ω : -500..500kHz, 100 frequencies

Exponentially growing step





Calibration, shot #47689



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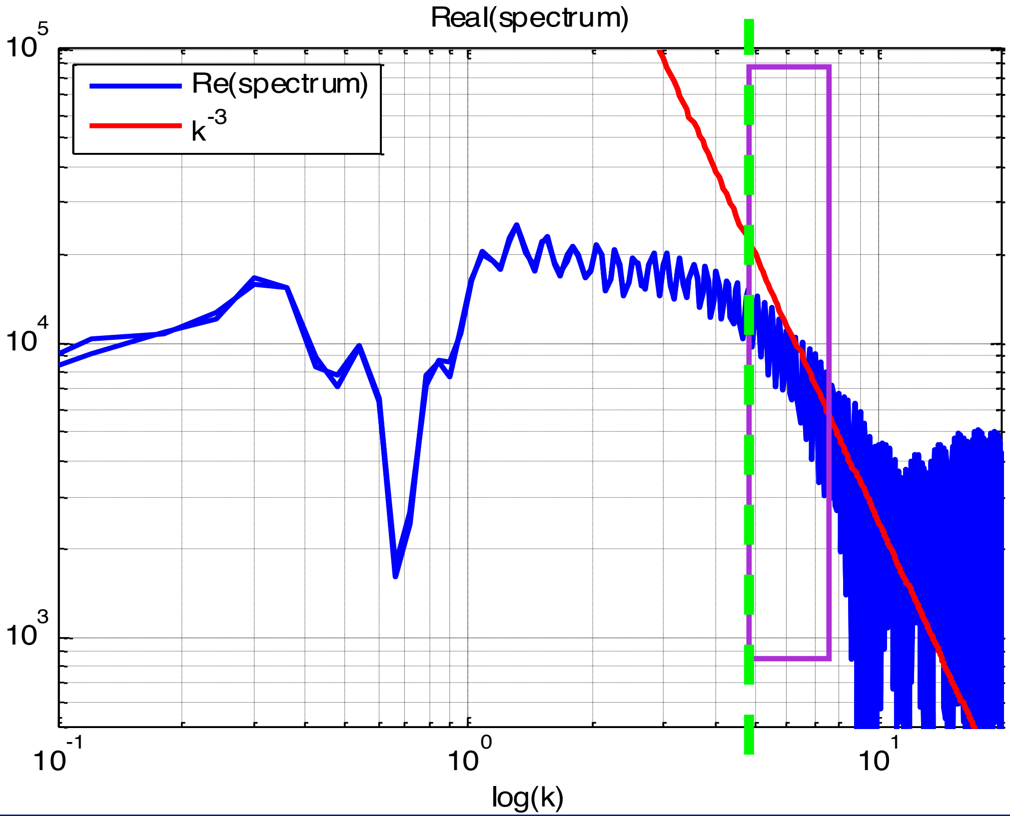
Turbulence spectrum, Tore Supra

Turbulence radial wave number spectrum (real part)

$$n_\kappa^2 \propto const,\ 0cm^{-1} < \kappa < 3cm^{-1}$$

$$n_\kappa^2 \propto \kappa^{-3},\ 5cm^{-1} < \kappa < 10cm^{-1}$$



L. Vermare et al. / C. R. Physique 12 (2011) 115–122

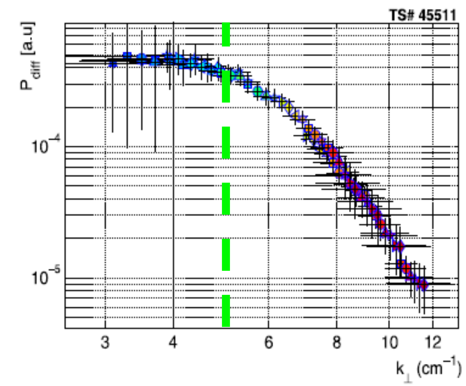
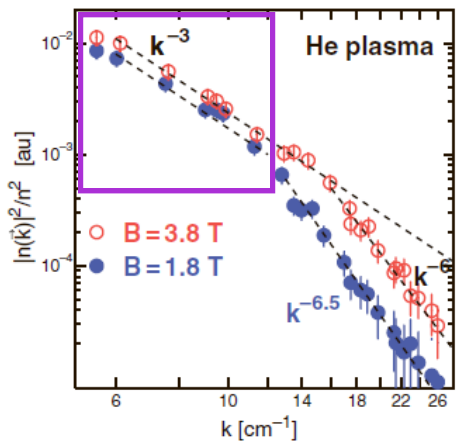


Fig. 4. Example of wavenumber spectrum measured on Tore Supra discharge.

[1]



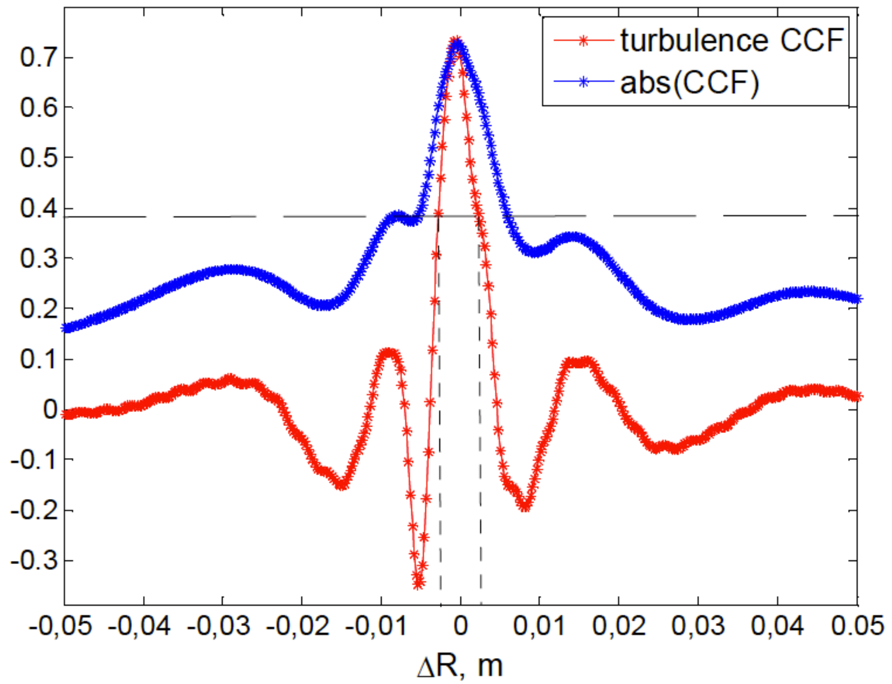
[2]

[1] L Vermare et al C R Physique 2011 12 115-122
[2] P Hennequin et al Plasma Phys. Control. Fusion 2004 46 B121

Turbulence CCF, Tore Supra

Turbulence cross correlation function

Shot #47669



Signal CCF (blue), turbulence CCF (red)

Turbulence correlation length $l_c \approx 2-4\ mm$ (?)

❑ Conclusions & further plans

- ❑ First time the diagnostic has been applied at Tore Supra machines and the experiments have been held successfully;
- ❑ The RCR CCF has been measured and analysed;
- ❑ Information of turbulence correlation length and its spectrum has been extracted;

- ❑ The knowledge obtained during experiments on Tore Supra are being applied at JET tokamak to measure the properties of turbulence.

<u>September 2013 at JET, Natalia Kosolapova's works:</u>

- ❑ Computation of the calibration coefficients (1s before plasma);
- ❑ Measurements…
- ❑ Numerical computations for experimental plasma density profile and various spectra;
- ❑ Data analysis.