Numerical Aspects of Drift Kinetic Turbulence: Illposedness, Regularization and Apriori Estimates of SGS Terms

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Outline

- Introduction and motivation
- A new 4D drift kinetic code
  - *Higher –order numerical method*
- Numerical results
  - *Comparison with different methods*
  - *Ill-posedness and regularization*
- LES for plasma turbulence
  - *Primer on hydrodynamic turbulence*
  - *Quantification of SGS terms*
- Summary and future work

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Introduction

- GOAL: Develop “sub-grid-scale” models for kinetic and gyrokinetic plasma turbulence simulations
  - *To compute plasma turbulence at a coarse-grain resolution without sacrificing physics accuracy*
  - *Fully resolved simulations are prohibitively expensive*

- Motivation for current investigation
  - *CPES (US DOE Scidac Center) is addressing the issue of coupling “coarse-grained” XGC0 simulations with “fine-grained” XGC1 simulations*
  - *Hydrodynamic turbulence simulations frequently employ “large eddy simulation (LES)” methodology to capture “sub-grid-scale (SGS)” physics*

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Introduction – Present Work

- Investigate numerical aspects of plasma turbulence simulations in the context of drift-kinetic turbulence in 4D
  - *5D and 6D fully-resolved simulations are still computationally expensive*
- Developed an Eulerian drift-kinetic code
  - *Investigate a variety of numerical algorithms*
- Perform "Direct Numerical Simulations", i.e., fully-resolved simulations
  - *What does fully-resolved mean?*
  - *Role of collisions (models) and regularization of the equations*
- Quantify the SGS terms
  - *a priori estimates: perform DNS, filter and examine SGS terms*
  - *a posteriori: perform LES and compare with filtered DNS to examine the efficacy of the SGS model*

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Background

- Science question: In gyrokinetic plasma turbulence, what are the mechanisms of energy cascade from large to small scales?
  - *A good exposition of the nonlinear route to dissipation in phase space given by Schekochihin et al. (Plasma Phys. Control. Fusion 2008, Astrophyisical J. Supp. 2009)*
  - *If the collision frequency is small, the distribution function develops small features in velocity space*
  - *Collisionless (Landau) damping redistributes generalized energy: electromagnetic fluctuations are converted to entropy fluctuations*
  - *In order for any heating to occur, the entropy fluctuations must cascade in phase space to collisional scales. Collisions, even if infrequent, are necessary to complete the cascade and satisfy Boltzman H-theorem. Collisions are necessary for the system to converge to a statistical steady-state*
- Other relevant papers
  - *Howes (PRL 2008) , Tatsuno et al. (PRL 2009), Howes (PRL 2011): AstroGK code*
  - *A. Bañón Navarro et al. (PRL 2011): GENE code. Also discuss SGS modeling for plasma turbulence*
  - *Watanabe and Sugama (PoP 2004) and many others by this group.*
  - *Grandgirard et al. (Plasma Phys. & Controlled Fusion, 2008): GYSELA code*
- GYSELA code (Grandgirard et al. , J. Comput. Phys. 2006)
  - *Solves the 4D drift kinetic and 5D gyrokinetic systems*
  - *Semi-Lagrangian approach (splitting - second order)*
  - *Second-order Poisson solver*

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4D Drift-kinetic Vlasov Equation

$$\frac{\partial f}{\partial t} + \vec{v}_{GC}.\vec{\nabla}_\perp f + v_{\|}\frac{\partial f}{\partial z} + \dot{v}_{\|}\frac{\partial f}{\partial v_{\|}} = 0$$

where $\vec{\nabla}_\perp = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}\right)$

- distribution function $f(x,y,z,v_{\|},t)$ in 4D

- drift velocity $\vec{v}_{GC} = \frac{\left(\vec{E}\times\vec{B}\right)}{B^2}$, $\vec{B} = B\vec{e}_z$ in the “toroidal” direction

- $v_{\|}$: velocity along the magnetic field lines

- $\dot{v}_{\|} = \frac{q}{m_i}E_z$, q: ion charge, m_i : ion mass

Adiabatic electrons

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Poisson equation in cylindrical geometry

$$-\nabla_\perp\left[\frac{n_0(r)}{B\Omega_0}\nabla_\perp\phi\right]+\frac{en_0(r)}{T_e(r)}\left(\phi-\langle\phi\rangle\right)=n_i-n_0$$

linearized polarization term

adiabatic response of the electrons

- φ: electric potential;
- $\vec{E}=-\vec{\nabla}\phi$ electric field;
- ion cyclotron frequency $\Omega_0=q_iB_0/m_i$;
- T_e: electron temperature profile;
- n_0: electron density profile;
- ion density profile $n_i(r,\theta,z,t)=\int dv_{\|}f(r,\theta,z,v_{\|},t)$;
- $\langle\cdot\rangle=\frac{1}{L_z}\int\cdot\,dz$, average on the magnetic field lines

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Initial and Boundary Conditions

- Local Maxwellian as the equilibrium part: $f = f_{eq} + \delta f$

$$f_{eq}(r, v_{\|}) = \frac{n_0(r)}{\left(2\pi T_i(r)/m_i\right)^{1/2}} \exp\left(-\frac{m_i v_{\|}^2}{2T_i(r)}\right)$$

- Perturbation: $\delta f = f_{eq} g(r) h(v_{\|}) \delta p(\theta, z)$

  where g(r) and h($v_{\|}$) are exponential functions

$$\delta p(\theta, z) = \sum_{m,n} \varepsilon_{mn} \cos\left(\frac{2\pi n}{L_z} z + m\theta + \phi_{mn}\right)$$

where ε_{mn} and φ_{mn} are the random amplitude and random phase for the mode (m,n)

Periodic boundary condition on θ and z

Boundary condition on r: zero flux of f

Homogeneous Neumann (Dirichlet) boundary condition for potential in r at 0 (r_{max})

Normalization

- Temperature $\hat{T} = \dfrac{T}{T_{e0}}$, where $\dfrac{T_e(r_0)}{T_{e0}} = 1$;
- Time $\hat{t} = \Omega_0 t$, where $\Omega_0 = q_i B_0 / m_i$ - length $\hat{l} = (\Omega_0 / c_s) l$
- Velocity $\hat{v} = \dfrac{v}{c_s}$, normalized to the sound speed $c_s = \sqrt{\dfrac{T_{e0}}{m_i}}$
- Potential & electric field $\hat{\Phi} = (q_i / T_{e0})\Phi$, $\hat{E} = (1/c_s B_0)E$

Normalized equations:

- Vlasov equation: $\dfrac{\partial f}{\partial t} + \vec{v}_{GC} . \vec{\nabla}_\perp f + v_\parallel \dfrac{\partial f}{\partial z} + E_z \dfrac{\partial f}{\partial v_\parallel} = 0$
- Poisson equation:

$$-\nabla_\perp \left[\frac{n_0(r)}{B} \nabla_\perp \phi \right] + \frac{n_0(r)}{T_e(r)} \left(\phi - \langle \phi \rangle \right) = n_i - n_0$$

- Initial condition: $f_{eq}(r, v_\parallel) = \dfrac{n_0(r)}{\left(2\pi T_i(r)\right)^{1/2}} \exp\left(- \dfrac{v_\parallel^2}{2T_i(r)} \right)$

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Numerical Method

In the uniform field case, the Liouville theorem is applicable

$$\vec{\nabla}_\perp \cdot \vec{v}_{GC} + \frac{\partial v_\parallel}{\partial z} + \frac{\partial \dot{v}_\parallel}{\partial v_\parallel} = 0$$

Hence the Vlasov equation can be written in <u>conservative</u> form:

$$\boxed{\frac{\partial f}{\partial t} + \vec{\nabla}_\perp \cdot \left(\vec{v}_{GC} f\right) + \frac{\partial}{\partial z}\left(v_\parallel f\right) + \frac{\partial}{\partial v_\parallel}\left(\dot{v}_\parallel f\right) = 0}$$

Time evolution for $\frac{\partial f}{\partial t} = -divf$

2nd and 3rd order TVD Runge-Kutta. 2nd order scheme written below:

$$f^{n+1} = \frac{(f_1 + f_2)}{2}$$

$$f_1 = f^n + \Delta t \cdot f'^n$$

$$f_2 = f^n + \Delta t \cdot f_1'$$

Numerical Method

For numerical stability, it was empirically found that the following form of the equations leads to a much more robust and stable numerical simulations and requires no filtering (δf numerics on full-f equation)

$$f' = f - f_{eq}$$

$$\frac{\partial f'}{\partial t} + \nabla_\perp \cdot \boldsymbol{v}_\perp f' + \frac{\partial v_\parallel f'}{\partial z} + \frac{\partial \dot{v}_\parallel f'}{\partial v_\parallel} + S = 0$$

$$S = E_\theta f_{eq} \frac{T_i'(r)}{T_i(r)} \left[\frac{v_\parallel^2}{2T_i(r)^2} + \frac{n_0'(r)}{n_0(r)} - \frac{1}{2} \right] - E_z f_{eq} \frac{v_\parallel}{T_i(r)}$$

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Numerical Method Cont.

- Finite volume discretization:

$$c\frac{\partial f}{\partial x} = \frac{\partial F_x}{\partial x} = \frac{F_x\left(i+\frac{1}{2},j\right) - F_x\left(i-\frac{1}{2},j\right)}{\Delta x}$$

-High order upwind discretization

1st, 3rd, 5th and 7th order options in the code

-Central finite difference discretization

2^{nd}, 4^{th} order, 2^{nd} and 4^{th} order tuned finite difference options in the code

- Example: 5^{th} order upwinding flux calculation:

$$F\left(i+\frac{1}{2}\right) = \frac{1}{30}F(i-2) - \frac{13}{60}F(i-1) + \frac{47}{60}F(i) + \frac{9}{20}F(i+1) - \frac{1}{20}F(i+2) \quad \text{if } c>0$$

$$\frac{1}{30}F(i+3) - \frac{13}{60}F(i+2) + \frac{47}{60}F(i+1) + \frac{9}{20}F(i) - \frac{1}{20}F(i-1) \quad \text{if } c<0$$

References: 1) Hill and Pullin, J. Comput. Phys. 2004, for tuned finite differences

2) Pirozzoli, J. Comput. Phys. 2002. for upwind-biased fluxes

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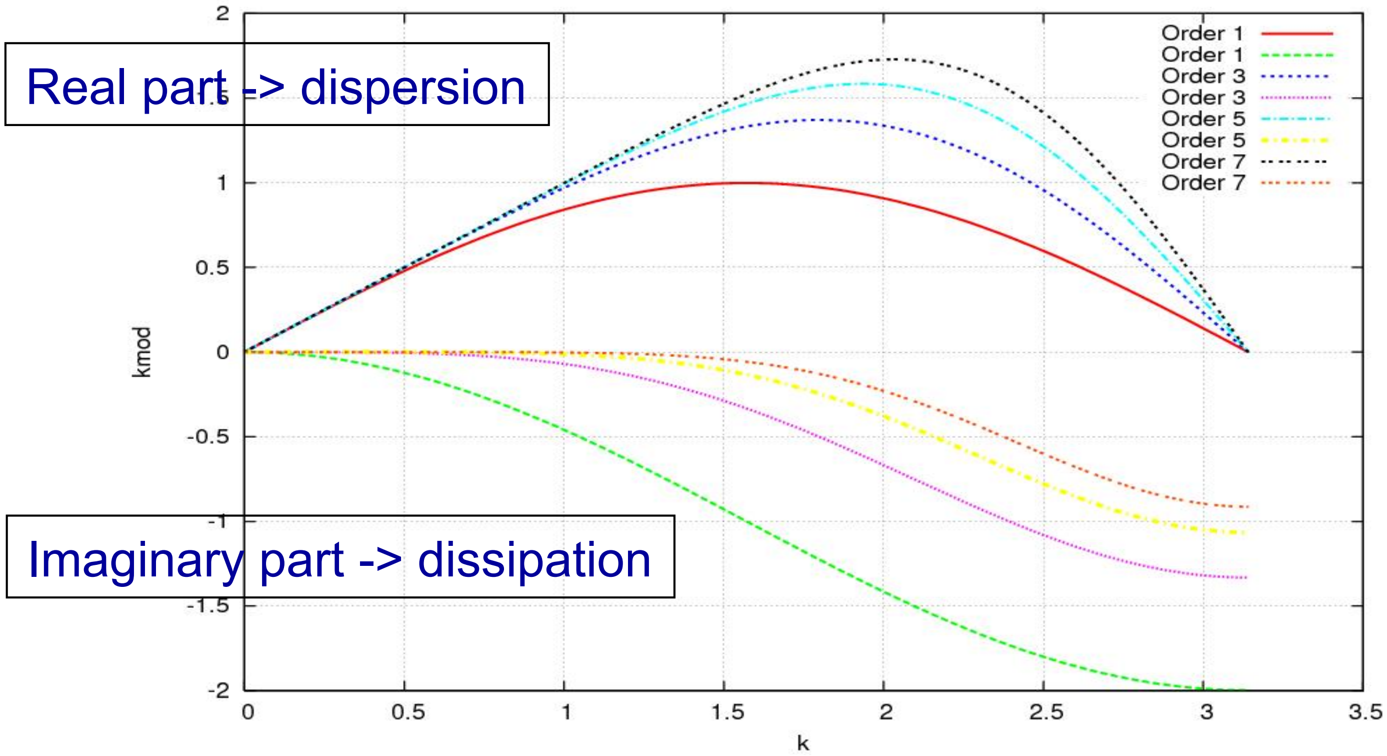
Numerical Method - Dispersion & Dissipation

- Drift-kinetic PDEs have no natural dissipation
  - *Unless collision operator is included*
- In the absence of physical dissipation (except Landau damping), we may resort to "numerical" dissipation to provide high-frequency cut-off (Note that the centered finite differences have no numerical dissipation)
- For 1st, 3rd, 5th and 7th order upwind methods, the dissipation and dispersion characteristics are shown in the figure below



Poisson Solver

Average on z and subtracted from original Poisson equation

$$-\nabla_{\perp}\left[\frac{n_0(r)}{B}\nabla_{\perp} < \phi >\right] = < R >$$

$$-\nabla_{\perp}\left[\frac{n_0(r)}{B}\nabla_{\perp}\delta\phi\right] + \frac{n_0(r)}{T_e(r)}\delta\phi = R - < R >$$

$$R = n_i - n_0$$

$$\delta\phi = \phi - < \phi >$$

Poisson: $-\nabla_{\perp}\left[a\nabla_{\perp}\phi\right] + b\left(\phi - \langle\phi\rangle\right) = n_i - n_0$, where

$$a = \frac{n_0(r)}{B}$$

$$b = \frac{n_0(r)}{T_e(r)}$$

Fourier expansion:

$$\phi(r,\theta,z) = \sum_m \sum_n \phi^{m,n}(r)\exp(im\theta)\exp(inz)$$

Results in a penta-diagonal solve:

$$A\hat{\phi}(i-2) + B\hat{\phi}(i-1) + C\hat{\phi}(i) + D\hat{\phi}(i+1) + E\hat{\phi}(i+2) = \hat{R}$$

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Poisson Solver Cont.

$$A = \frac{a(r)}{12\Delta r^2} - \frac{1}{12\Delta r}\left(\frac{da(r)}{dr} + \frac{a(r)}{r}\right) \qquad B = -\frac{4a(r)}{3\Delta r^2} + \frac{2}{3\Delta r}\left(\frac{da(r)}{dr} + \frac{a(r)}{r}\right),$$

$$C = \frac{5a(r)}{2\Delta r^2} + \frac{n^2 a(r)}{r} + b(r) \qquad D = -\frac{4a(r)}{3\Delta r^2} - \frac{2}{3\Delta r}\left(\frac{da(r)}{dr} + \frac{a(r)}{r}\right),$$

$$E = \frac{a(r)}{12\Delta r^2} + \frac{1}{12\Delta r}\left(\frac{da(r)}{dr} + \frac{a(r)}{r}\right)$$

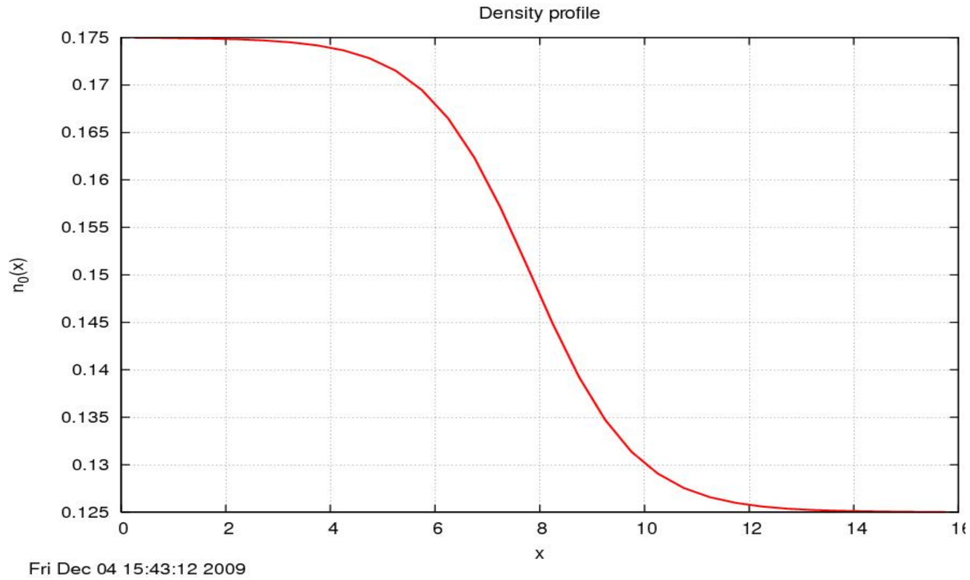
Together with Neumann and Dirichlet boundary condition:

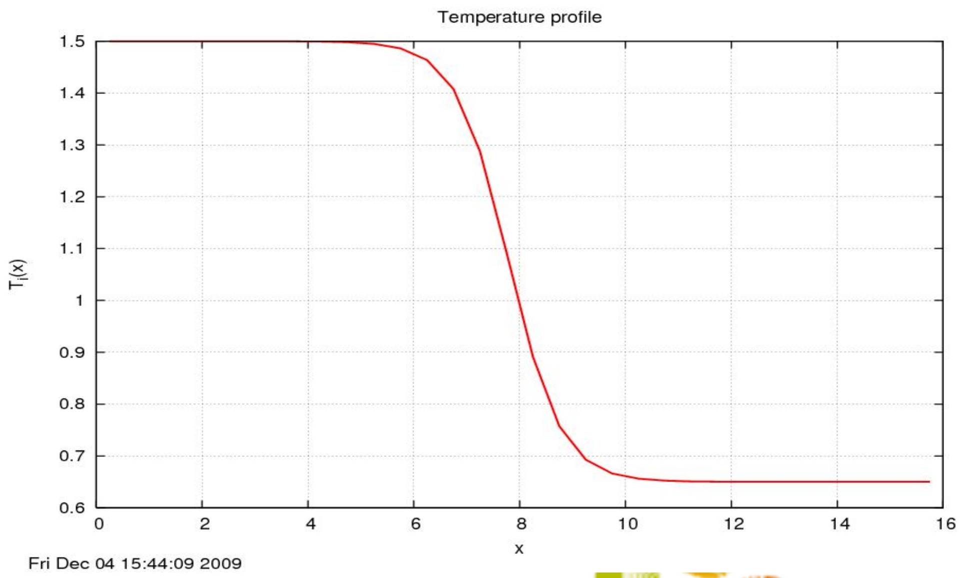
$$\begin{pmatrix} C+B & D+A & E & & & & & \\ B-A & C & D & E & & & & \\ A & B & C & D & E & & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ & & & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & A & B & C & D & E \\ & & & & & A & B & C & D-E \\ & & & & & & A & B-E & C-D \end{pmatrix} \hat{\phi}_i = \hat{R}$$

use 4th order finite difference scheme to solve 1D linear system for the Fourier modes with a fast penta-diagonal solver

Numerical Results

- Domain: [0:15] x [0:2π] x [0:1500] x [-8:8]
- Mesh resolution: 256x256x32x256 (512 procs Shaheen)
- Time step dt=0.1
- 4th Order tuned centered finite difference
- Density and temperature profiles





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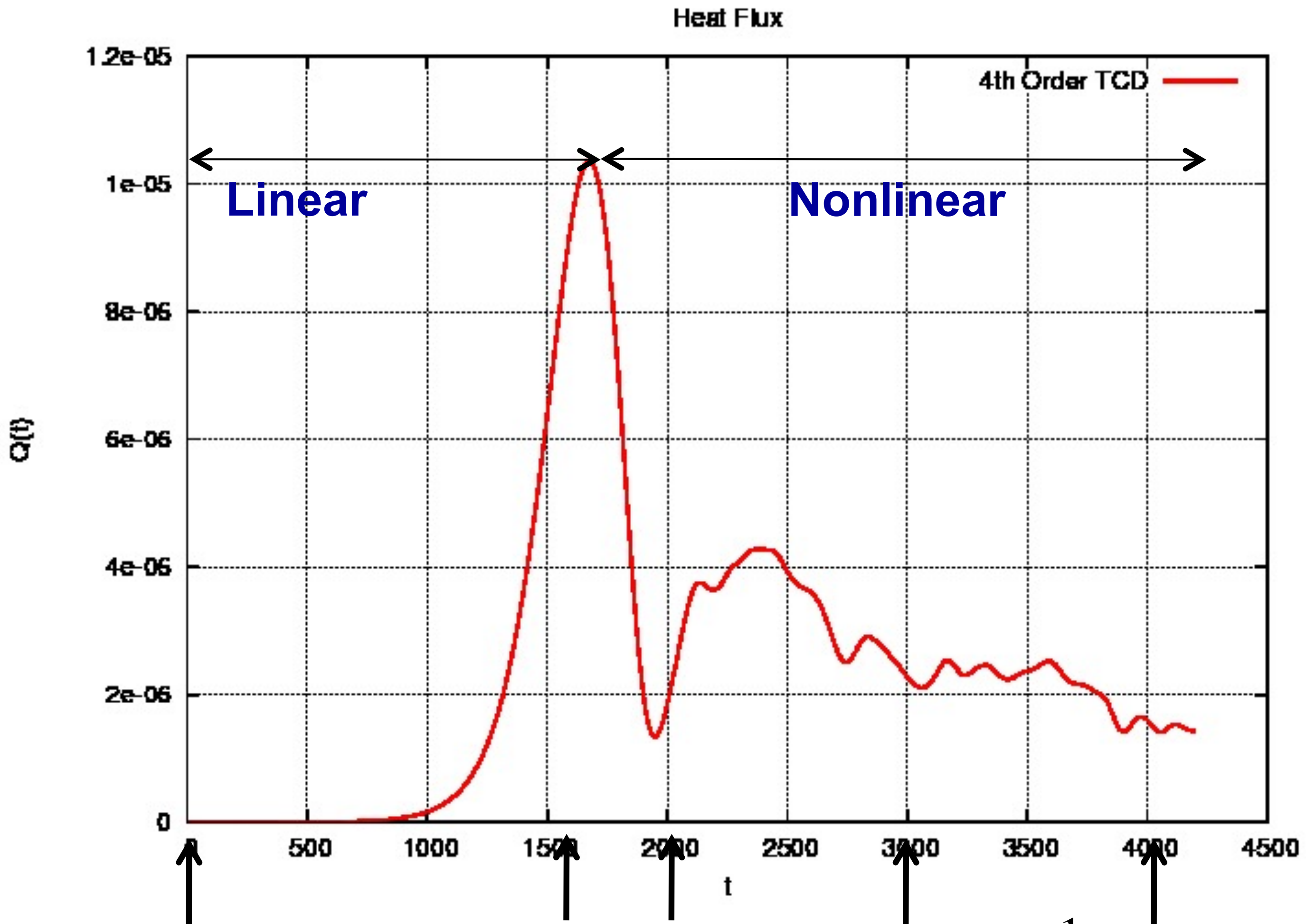
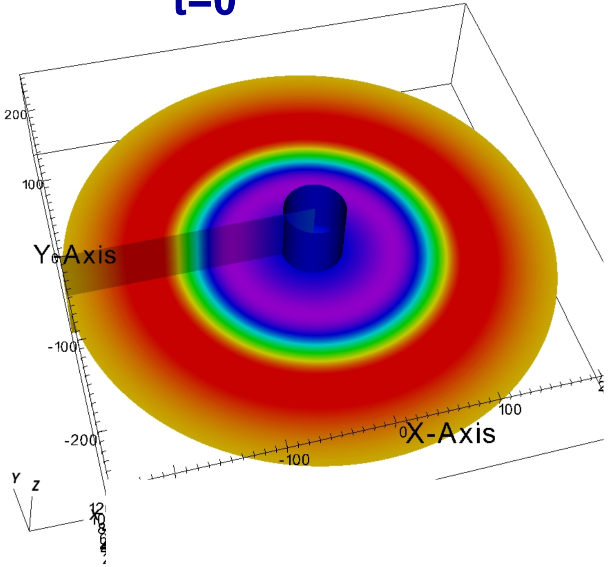
Numerical Results – Higher Order Moments



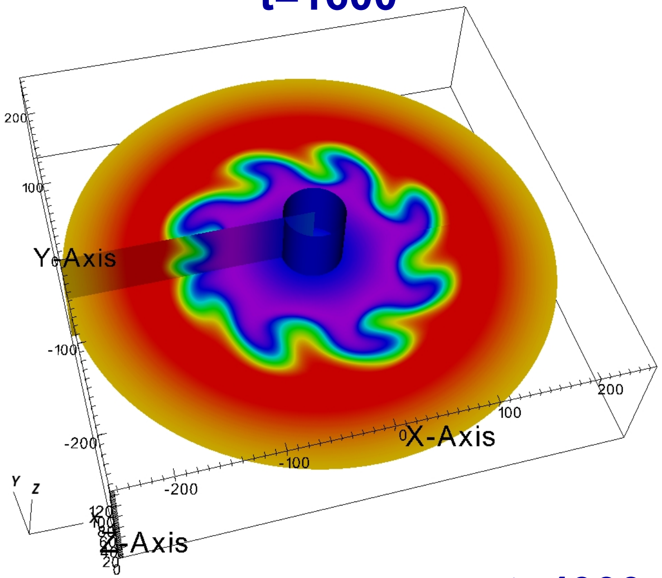
- The time evolution of the heat flux: $Q(t) = \frac{1}{2}\int f\, v_{\parallel}^2 v_{GC_r} \frac{d\theta}{2\pi}\frac{dz}{L_z} dv_{\parallel}$

Numerical Results - Distribution Function

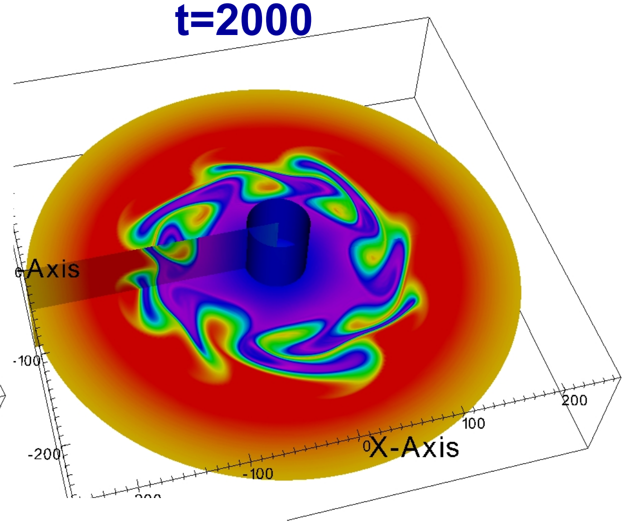
t=0



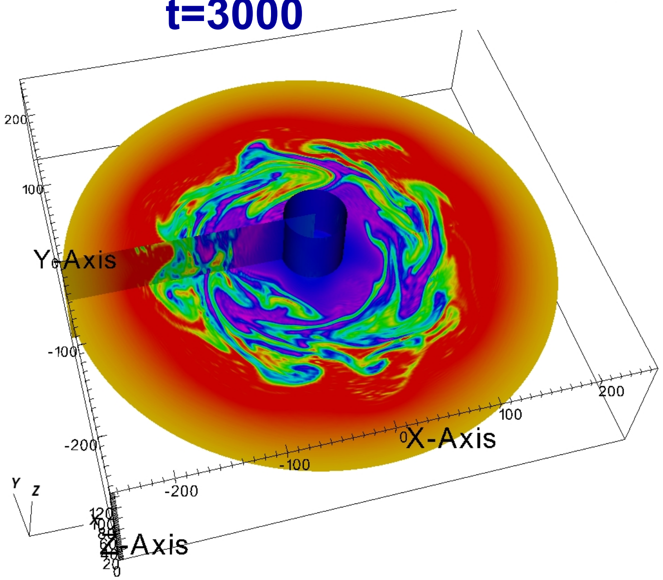
t=1600



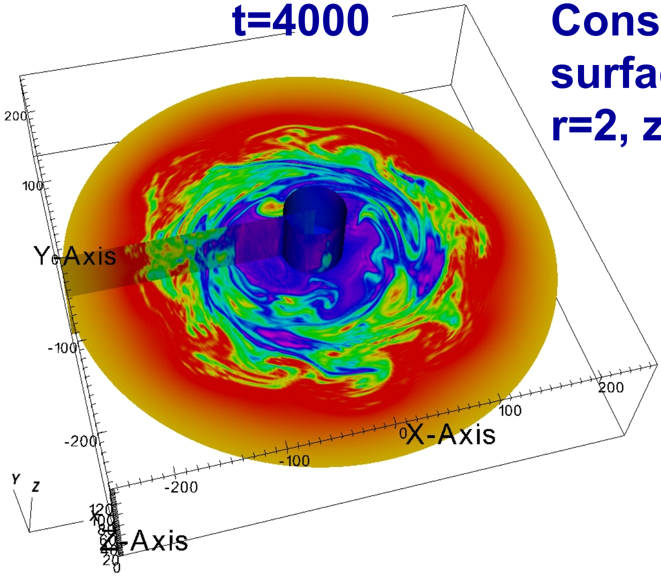
t=2000



t=3000



t=4000



Constant coordinate surfaces visualized
r=2, z=0, θ=π

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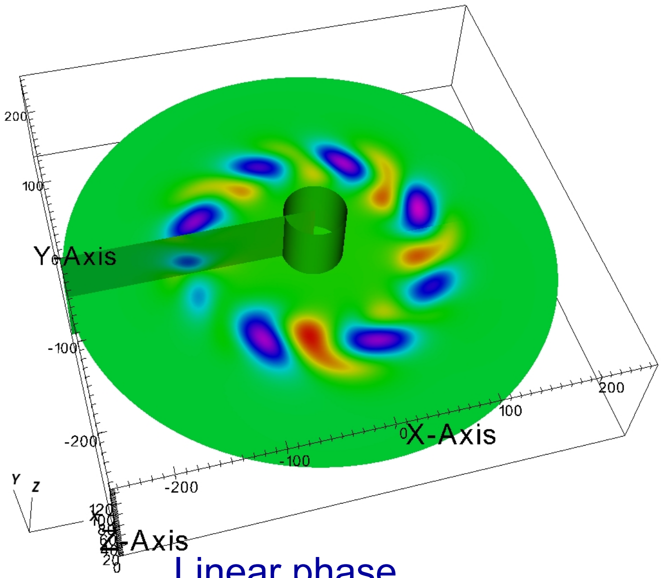
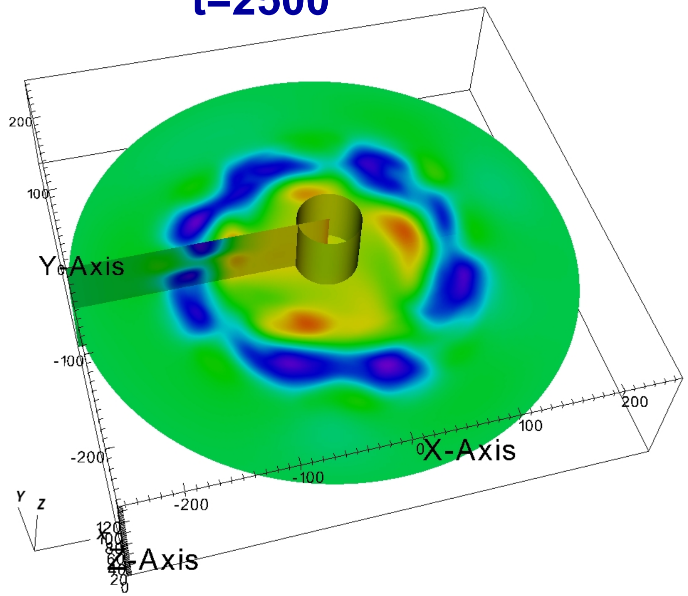
Numerical Results - Potential

t=1600

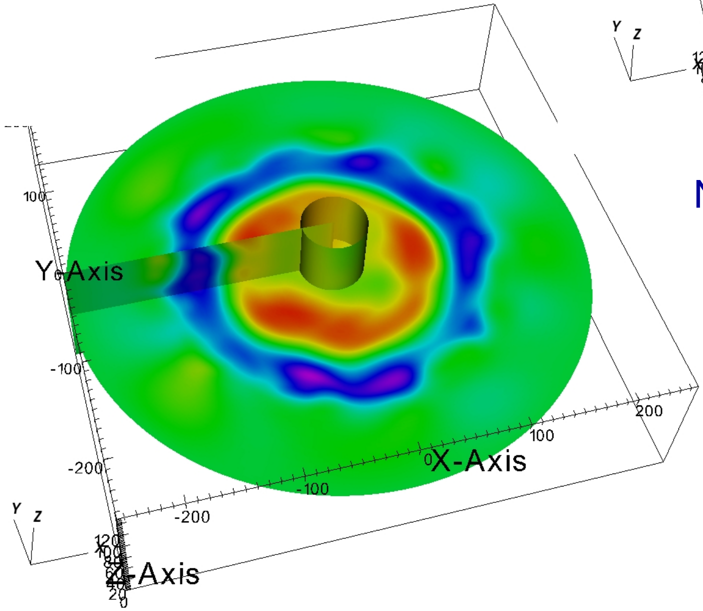


Linear phase

t=2500



Non-linear phase



t=4000

Energy conservation

- Variation of the kinetic energy:

$$\delta\varepsilon_{kin} = \int m_i \frac{v_{\parallel}^2}{2}(f - f_{eq})\,dV dv_{\parallel}$$

- Variation of the potential energy:

$$\delta\varepsilon_{pot} = \frac{q_i}{2}\int (n_i - n_0)\phi\, dV$$

- Conservation of the total energy

$$\delta\varepsilon_{tot} = \delta\varepsilon_{kin} + \delta\varepsilon_{pot} = \text{constant}$$

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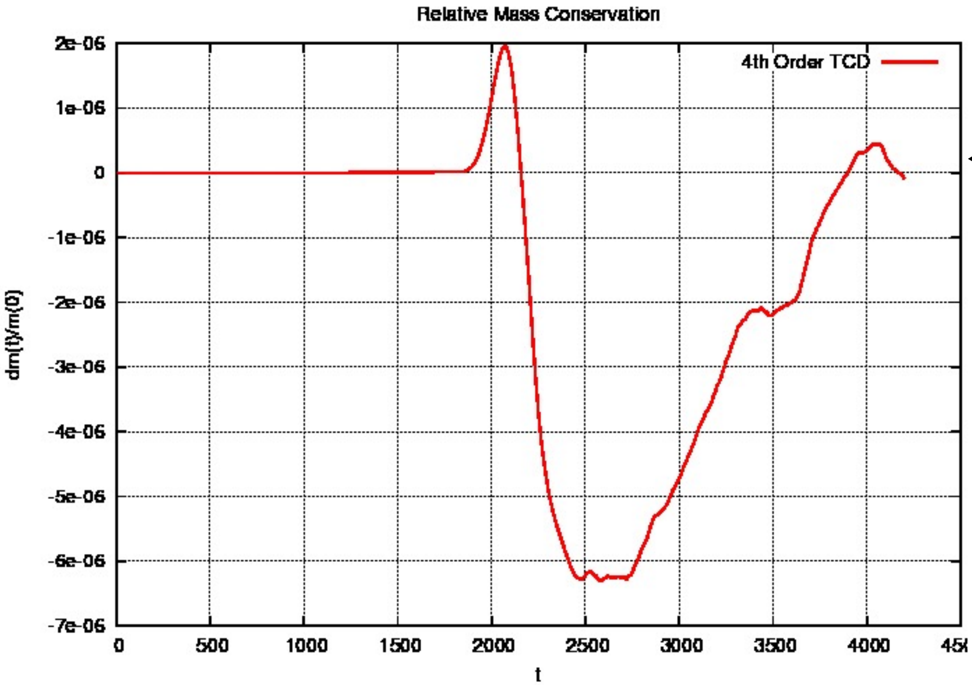
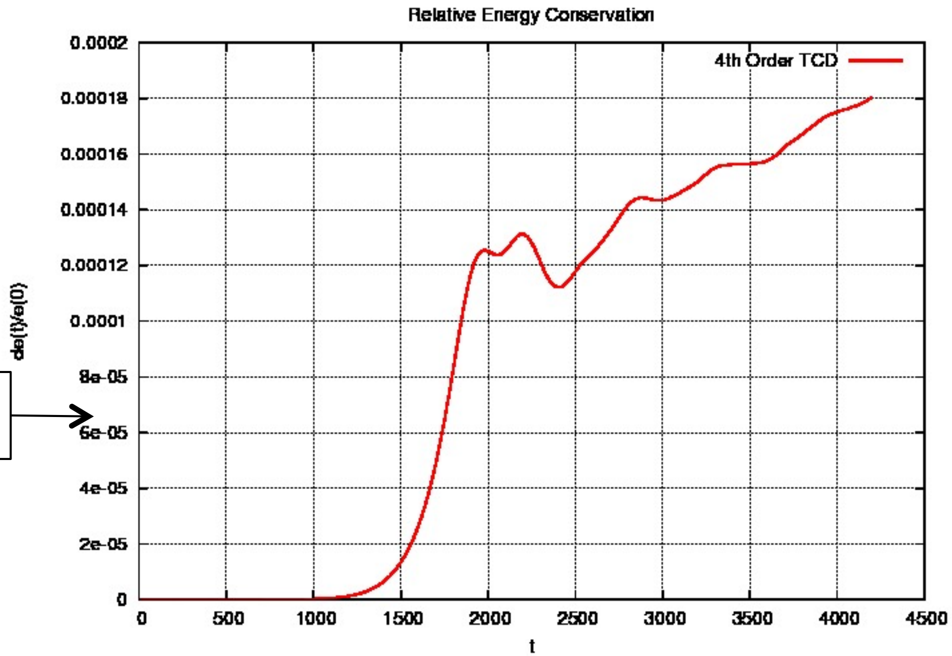
Numerical Results - Conservation

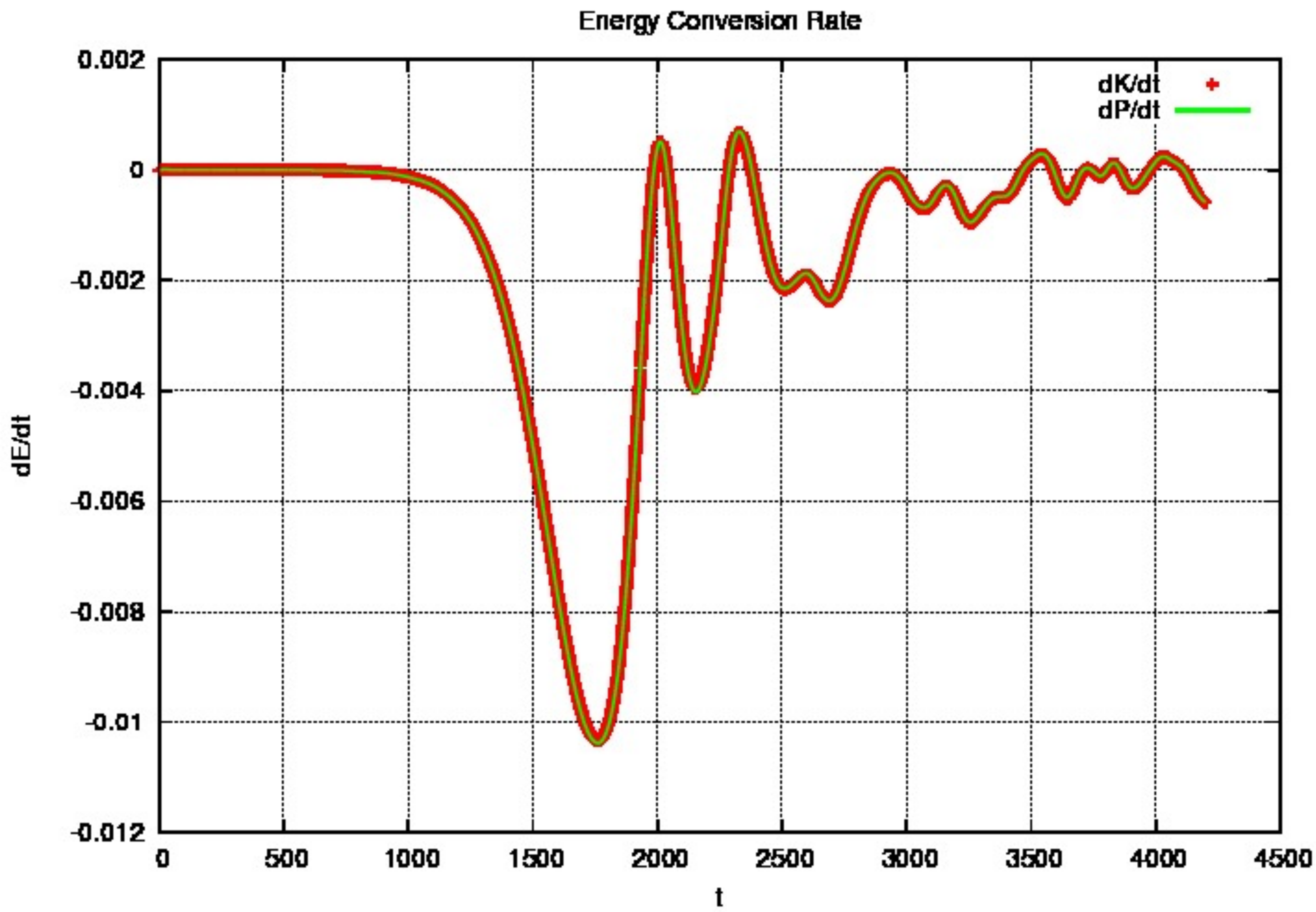


Mass conservation < 10^{-3} %

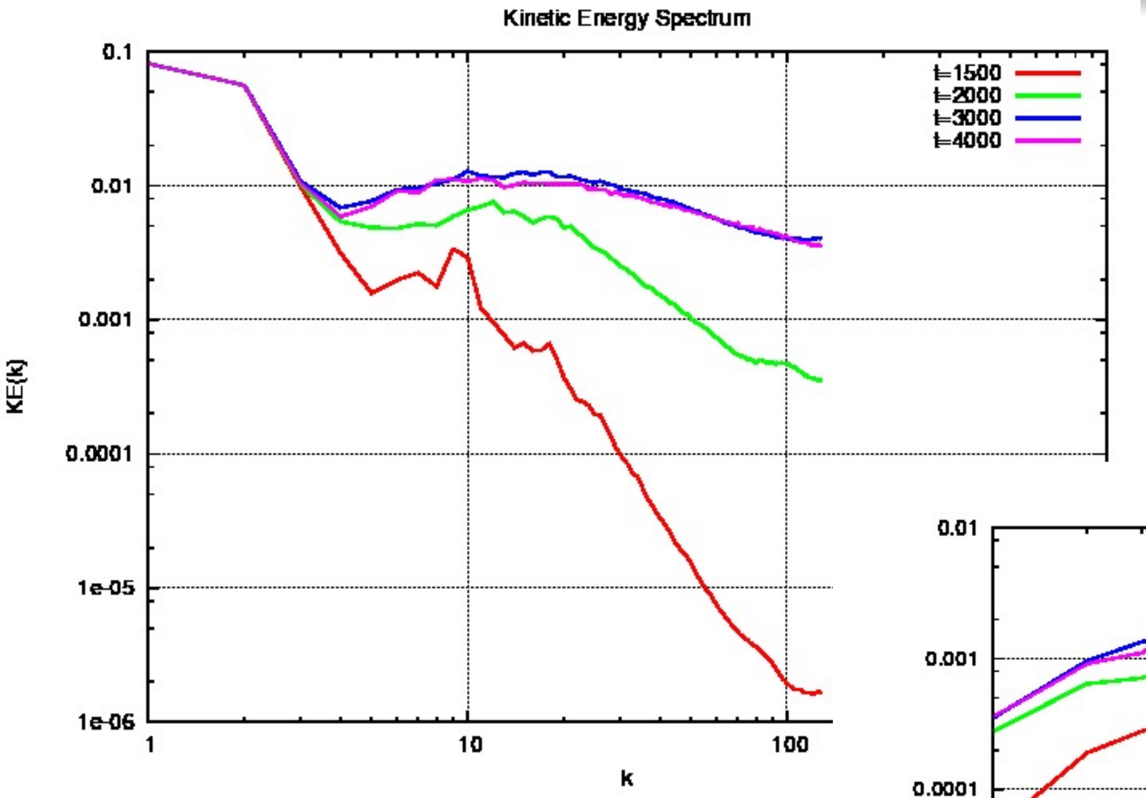


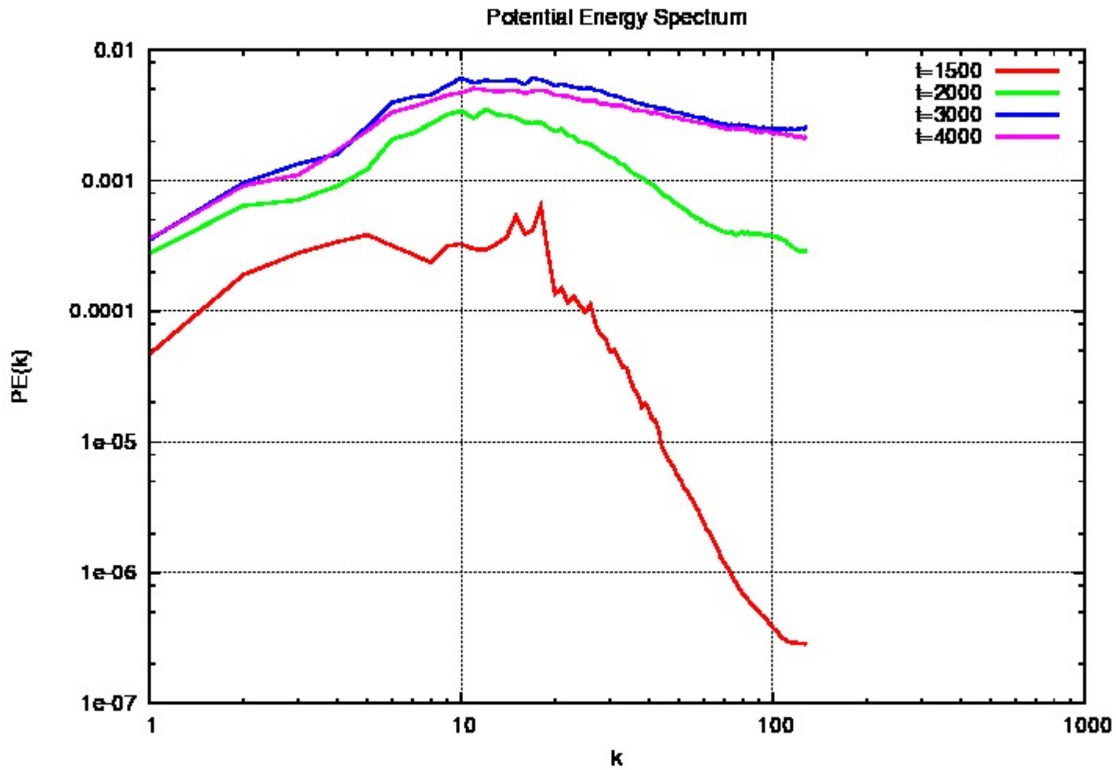
Energy conservation within 0.02%

Numerical Results – Energy Conversion



Numerical Results - Energy Spectrum





Vlasov-Poisson System – Ill-posedness

- From the numerical results, and in particular the energy spectrum, we observe generation of small scales
  - *What is the physical cut-off?*

- <u>Claim</u>: The drift-kinetic Vlasov-Poisson system of equations is scale-free
  - *$x \rightarrow \alpha x'$, $t \rightarrow \alpha t'$, $B \rightarrow B'/\alpha$, $\Phi=\Phi'/\alpha^2$, $E \rightarrow E'/\alpha$, $f=f'$, $n=n'$, $v_{\|}=v'_{\|}$*
  - *With this transformation the equations are unchanged*

- In the nonlinear regime, there is no physical cut-off for the small scales. The system of equations is ill-posed and requires either a physical cut-off mechanism or a numerical regularization
  - *Without regularization the generation of small scales proceeds ad-infinitum*
  - *Convergence with mesh refinement will not be achieved*
  - *In fact, after a certain critical time the code ought to blow up as energy keeps piling at the small-scales*
  - *Frequently the phrase "velocity space filamentation" is used (see, for example, Klimas, JCP 1987 and references therein for a discussion of this)*
  - *Tatsuno et al. (PRL 2008) introduce a non-dimensional number D (a la Reynolds number) to characterize the scale separation in gyrokinetic turbulence*

$$\frac{\delta v_{\perp c}}{v_{\rm th}} \sim \frac{1}{k_{\perp c}\rho} \sim D^{-3/5}, \qquad D = \frac{1}{\nu \tau_\rho},$$

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Vlasov-Poisson System – Regularization

- A Laplacian "viscosity" term in the Vlasov equation
  - *Motivated by shock-hydrodynamics*
  - *Viscosity coefficient depends on the local gradients so that in relatively smooth regions this term is inactive*
  - *Results presented so far used this regularization*

- A hyperviscosity term in the Vlasov equation
  - *Motivated by hydrodynamic turbulence simulation literature*
  - *4th order hyperviscosity term which provides a numerical cut-off at high wave numbers.*
    - *Employed, for example, A. Bañón Navarro et al. , Free Energy Cascade in Gyrokinetic Turbulence, PRL, 2011. Also by Howes et al. PRL 2008*

- Implicit numerical dissipation provided by upwinding methods

- Collision physics model
  - *Example: Howes et al. (PRL 2011) work on gyrokinetic simulations of solar wind used collision models developed by Abel (PoP 2008) and Barnes (PoP 2009)*
  - *Most of the recent papers acknowledge that we need collisions to provide the physical cut-off*

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Model Collision Operator

- Generalization of the operator defined in Rathman and Denavit

$$\frac{\partial f}{\partial t} + \vec{v}_{GC}.\vec{\nabla}_{\perp} f + v_{\|}\frac{\partial f}{\partial z} + \dot{v}_{\|}\frac{\partial f}{\partial v_{\|}} = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

where the collision operator:

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{\partial}{\partial v_{\|}}\left[\gamma v_{\|} f + D\frac{\partial}{\partial v_{\|}}(\gamma f)\right]$$

$$= \frac{\partial}{\partial v_{\|}}\left(\gamma v_{\|} f\right) + D\frac{\partial^2}{\partial v_{\|}^2}\left(\gamma f\right)$$

- $\gamma = \dfrac{C}{\left(2\langle v_{\|}^{2}\rangle + v_{\|}^{2}\right)^{\frac{3}{2}}}$ where $C = \left(3\langle v_{\|}^{2}\rangle\right)^{\frac{3}{2}}\gamma_0$

- $D = \langle \gamma v_{\|}^{2}\rangle / \langle \gamma \rangle$ where $\langle \cdot \rangle = \int \cdot\, f\, dV dv_{\|}$

- 4th order central difference for discretization

$$f' = \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$$

$$f'' = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2}$$

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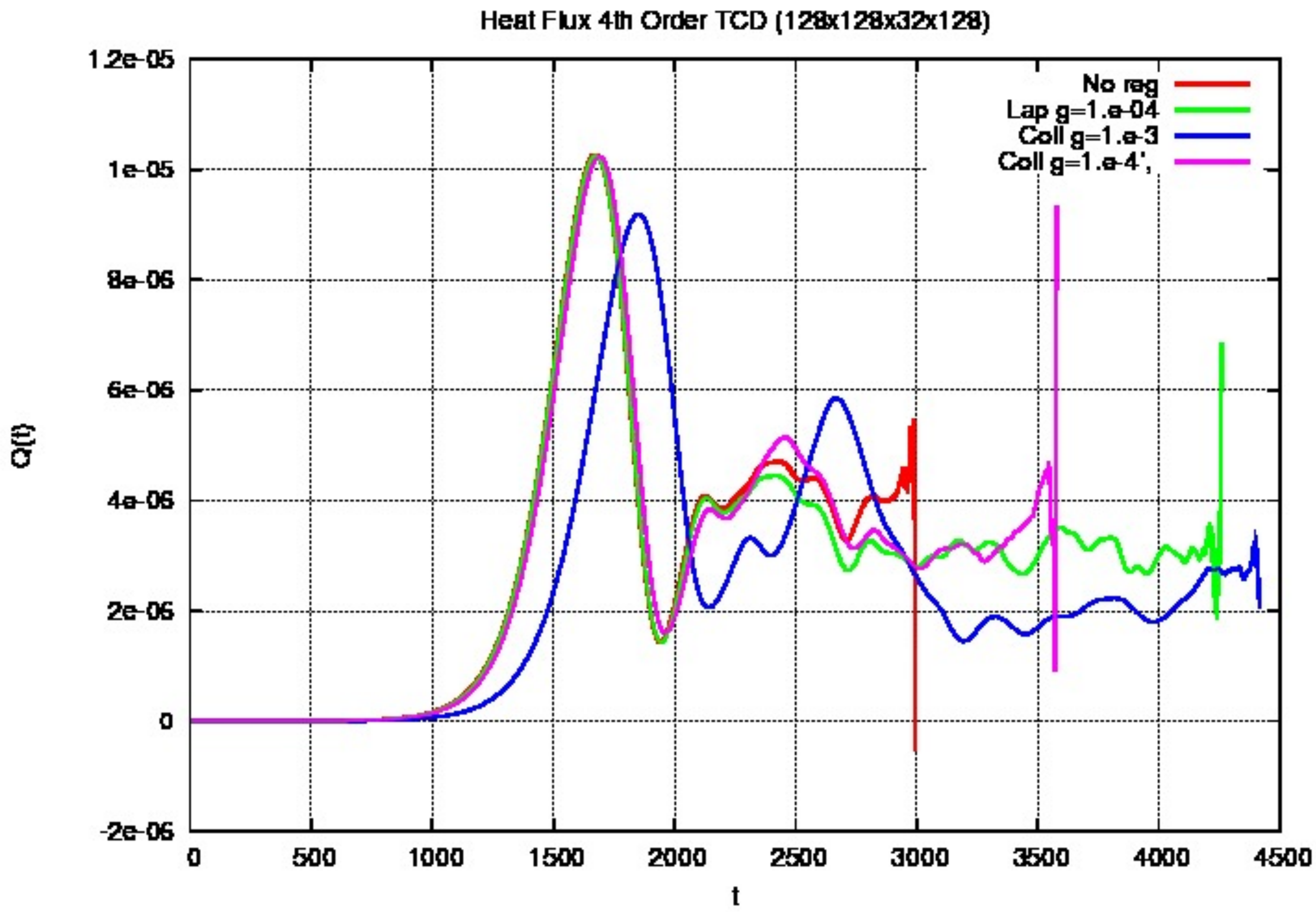
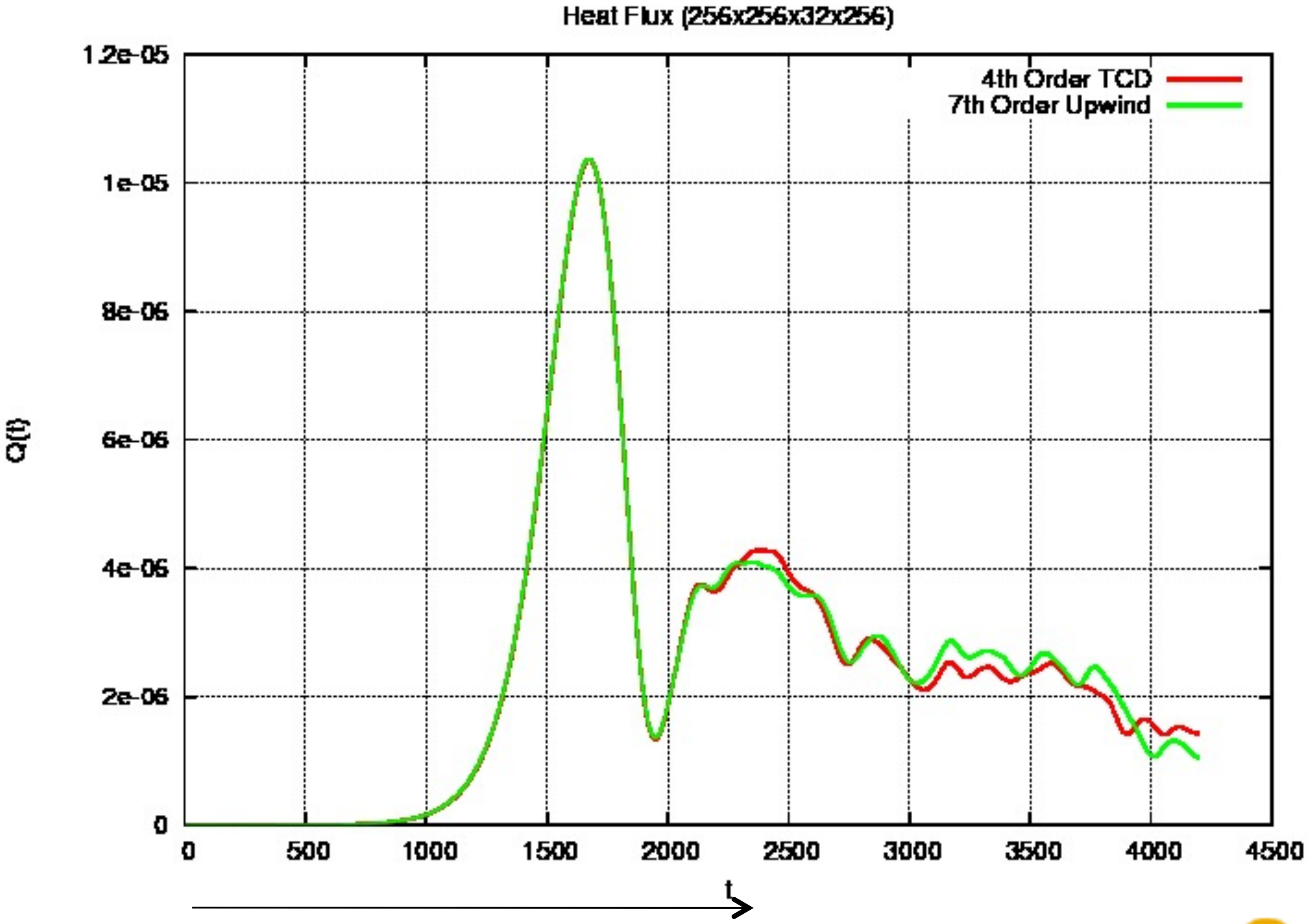
Regularization of Central FD Scheme



Comparing Central vs. Upwind



Less than 1% difference until t≈2300

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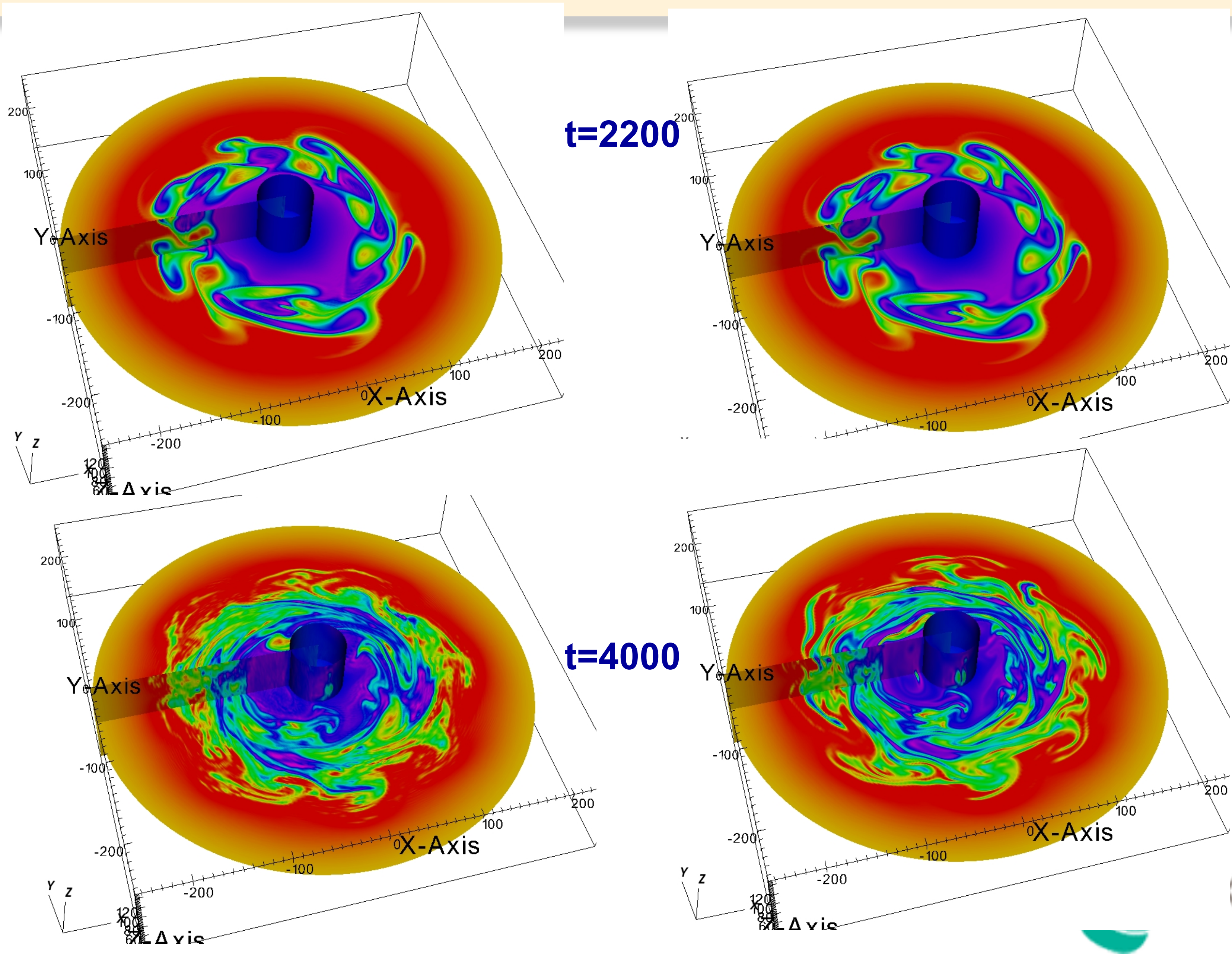
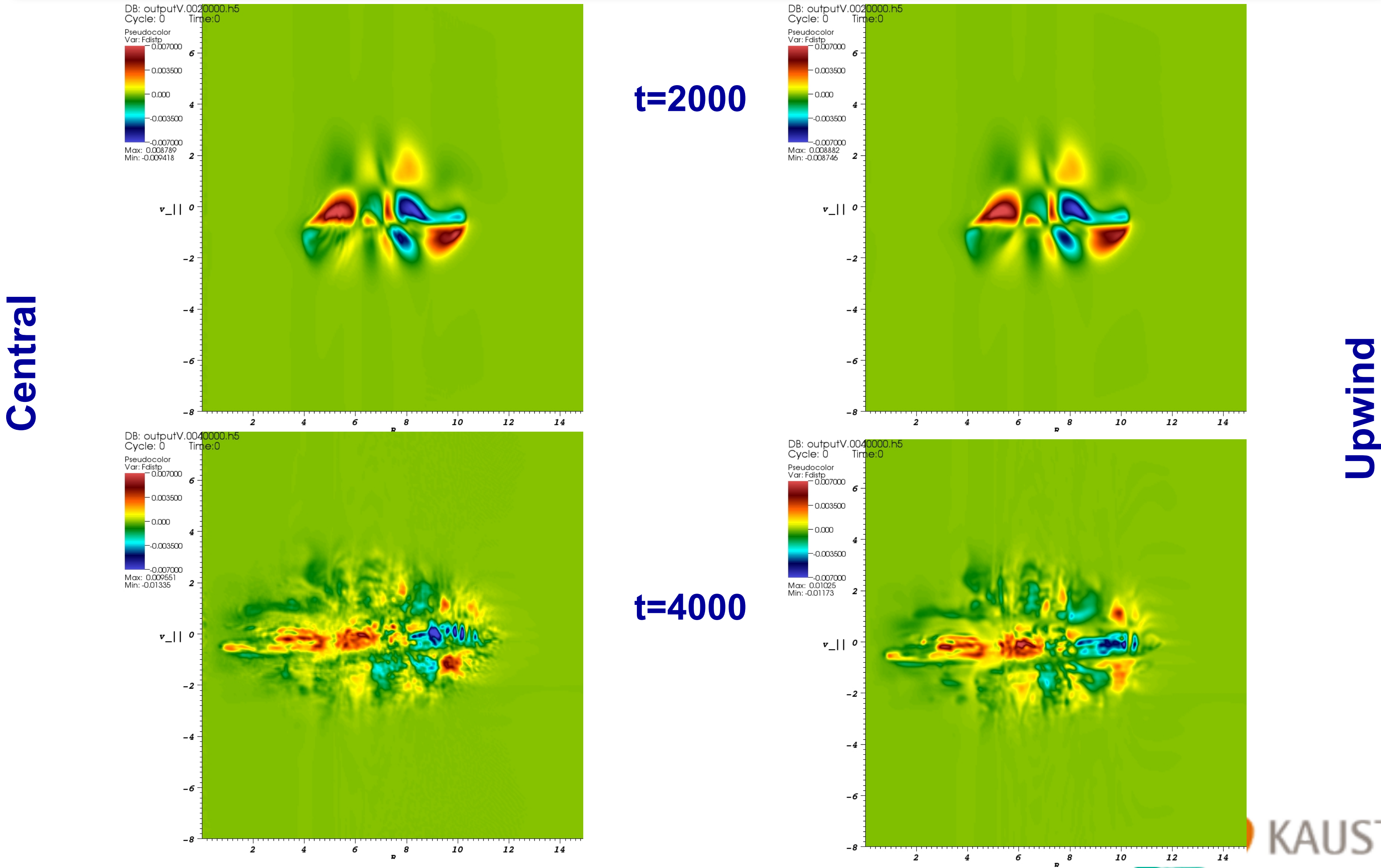
Comparing Central vs. Upwind



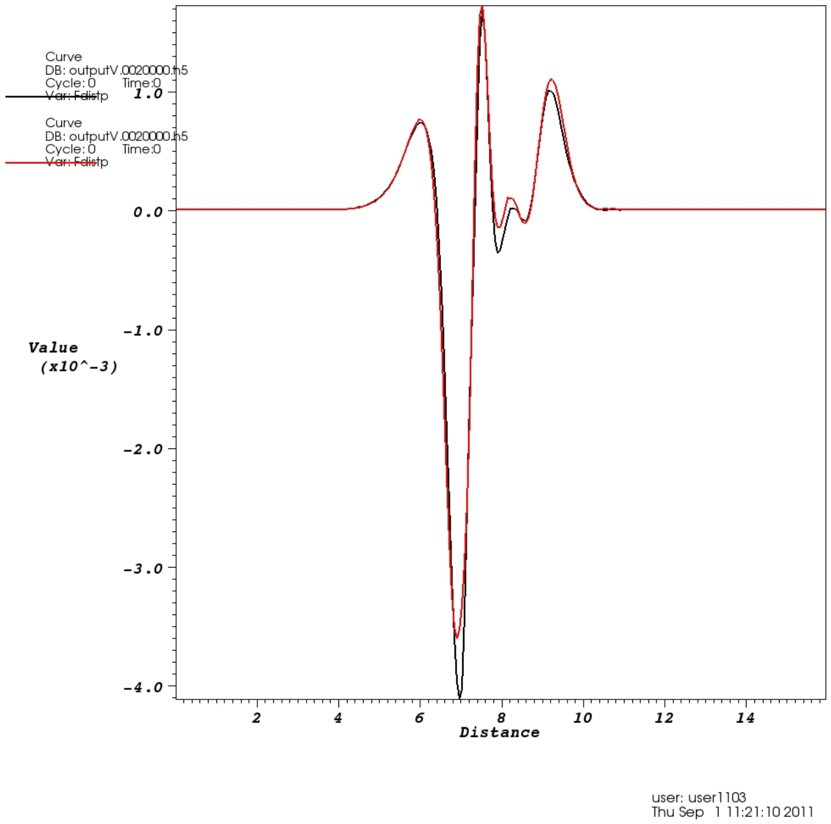
Central

Upwind

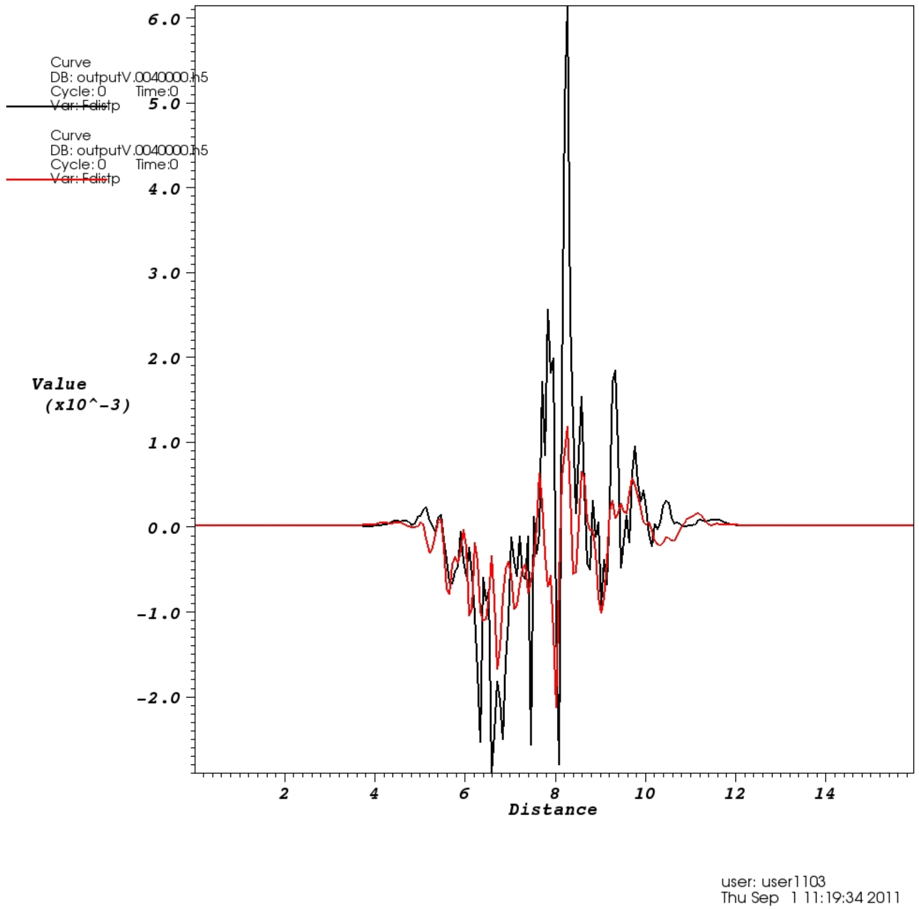
Distribution function $f(r,3\pi/2,0,v_{||})$
Central vs. Upwind



Distribution function f(7.5,3π/2,0,$v_{||}$) Central vs. Upwind



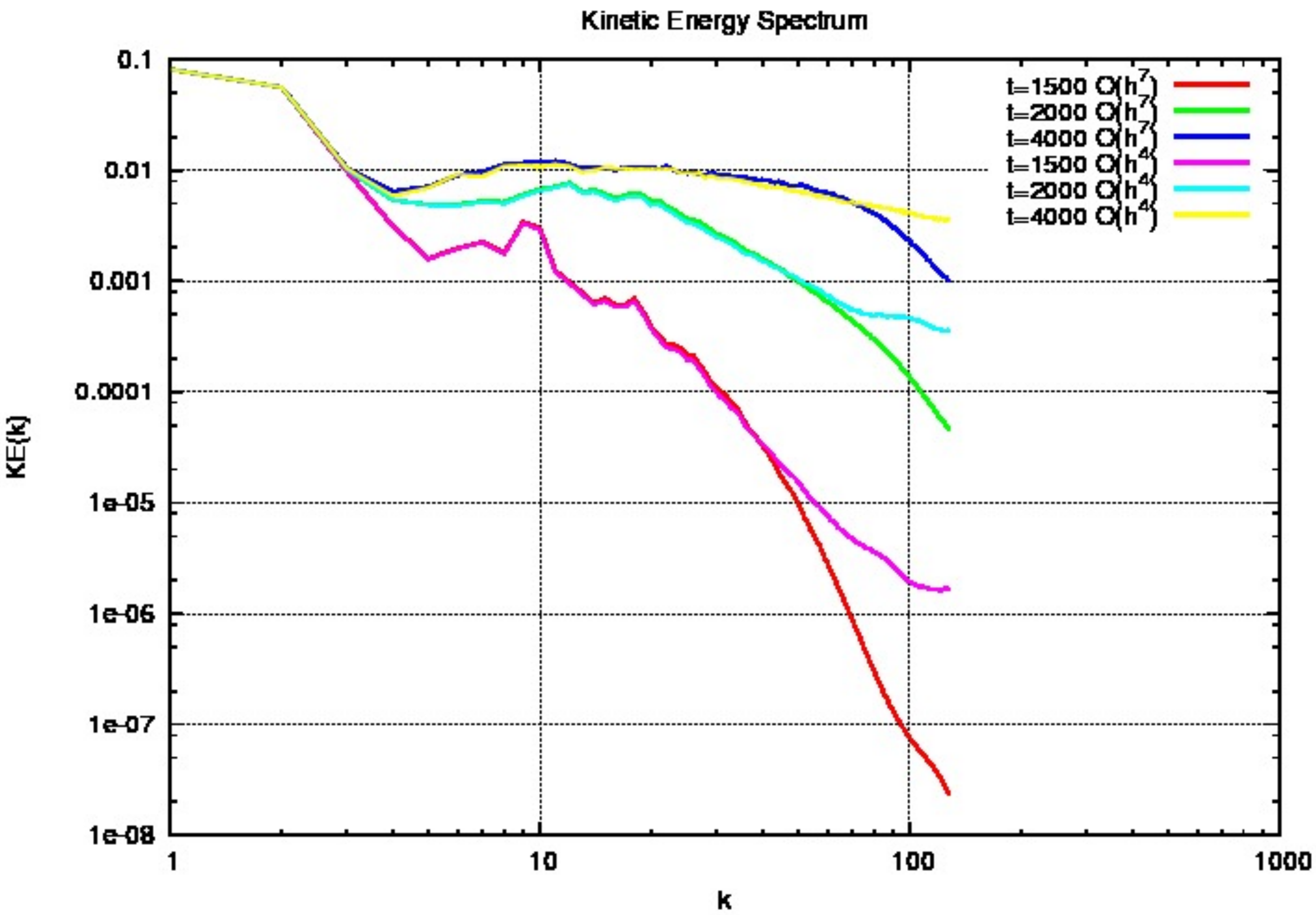
t=2000



t=4000

Good agreement until early non-linear stage. Upwind method gives smoother distribution function at late times compared with central FD.

Kinetic Energy Spectrum: Central vs. Upwind



Good agreement up to k=60-70.
Upwind method results in steeper spectrum for high k

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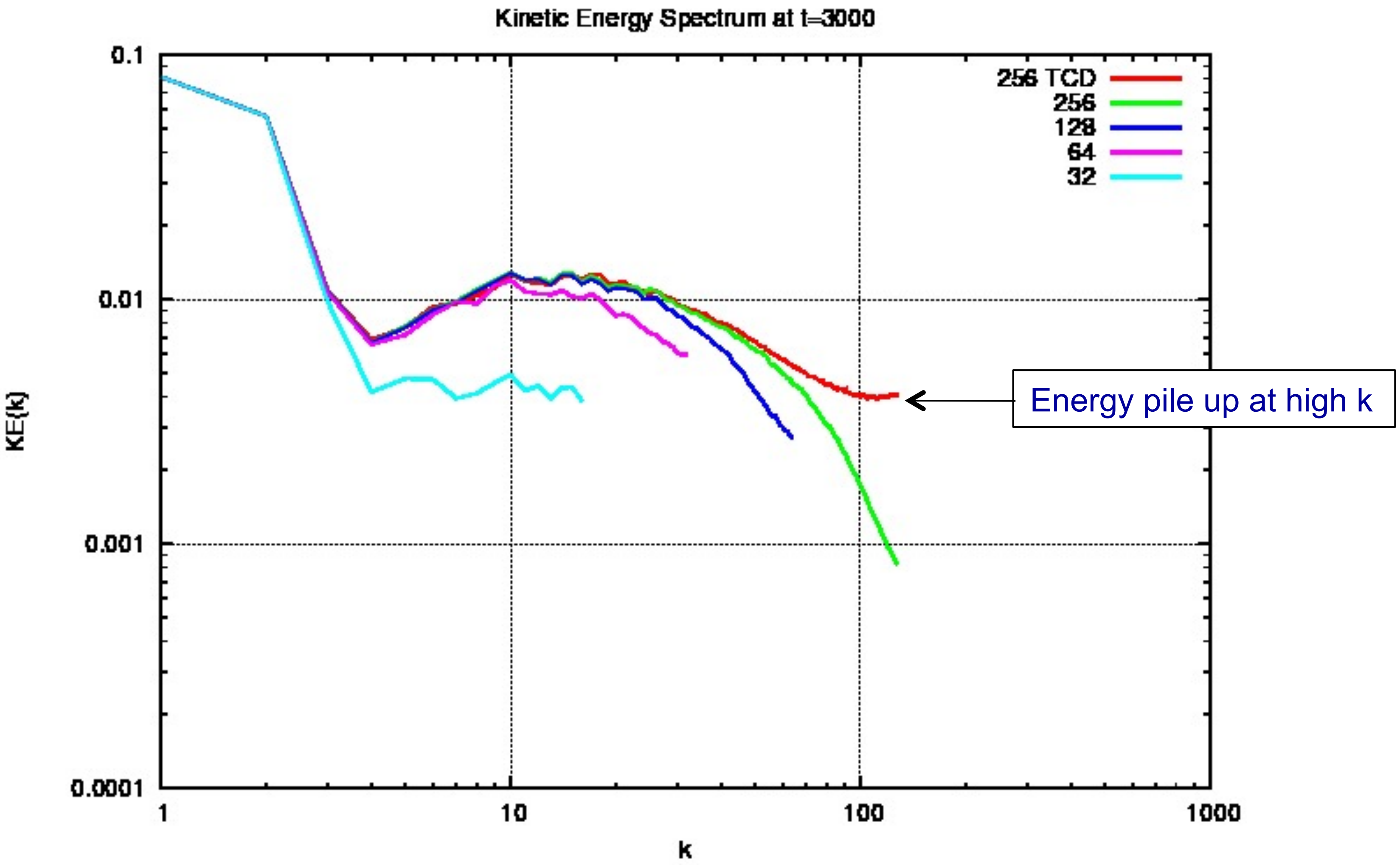
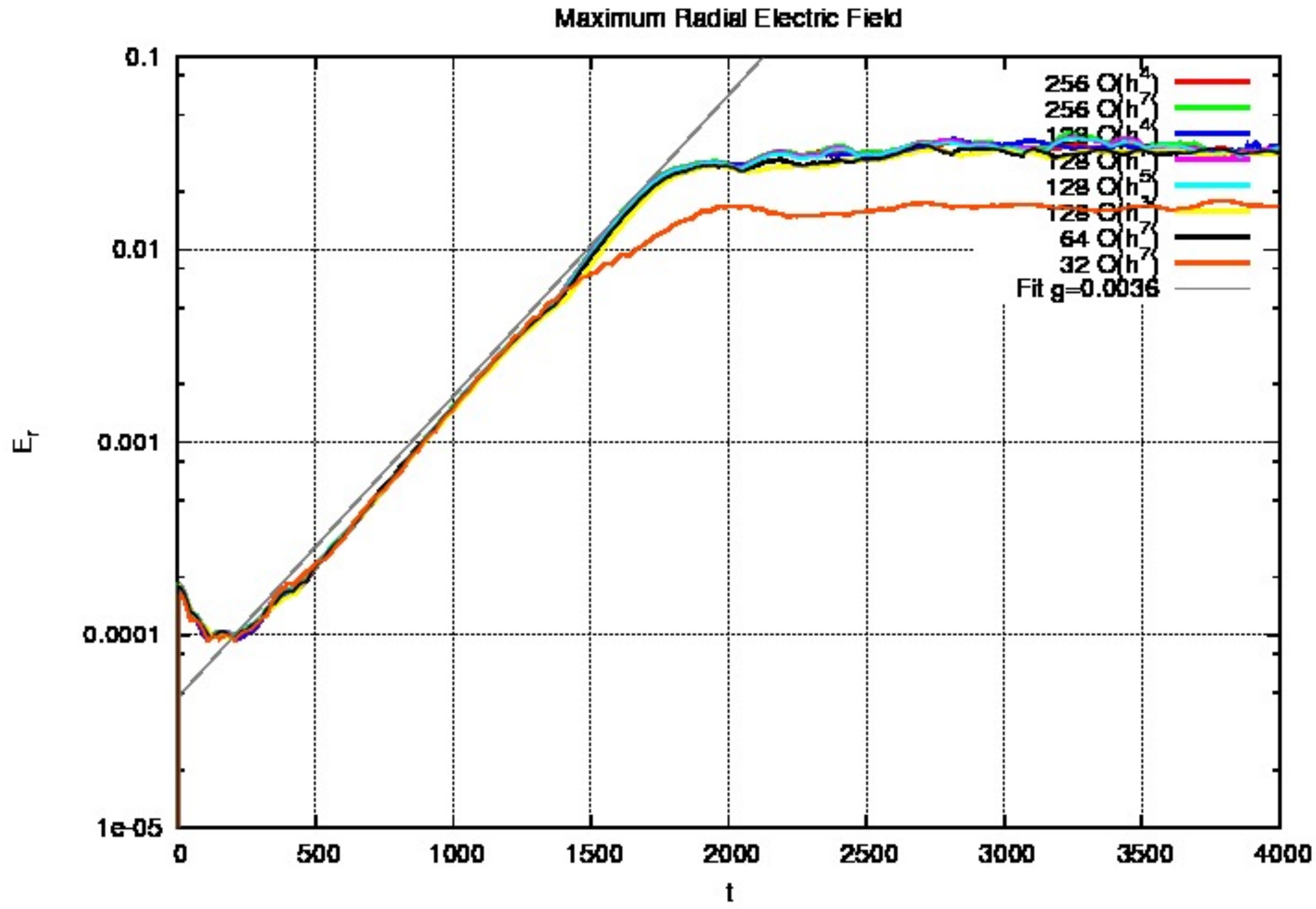
Kinetic Energy Spectrum: Mesh Resolution



Radial Electric Field: Different Methods and Mesh Resolutions



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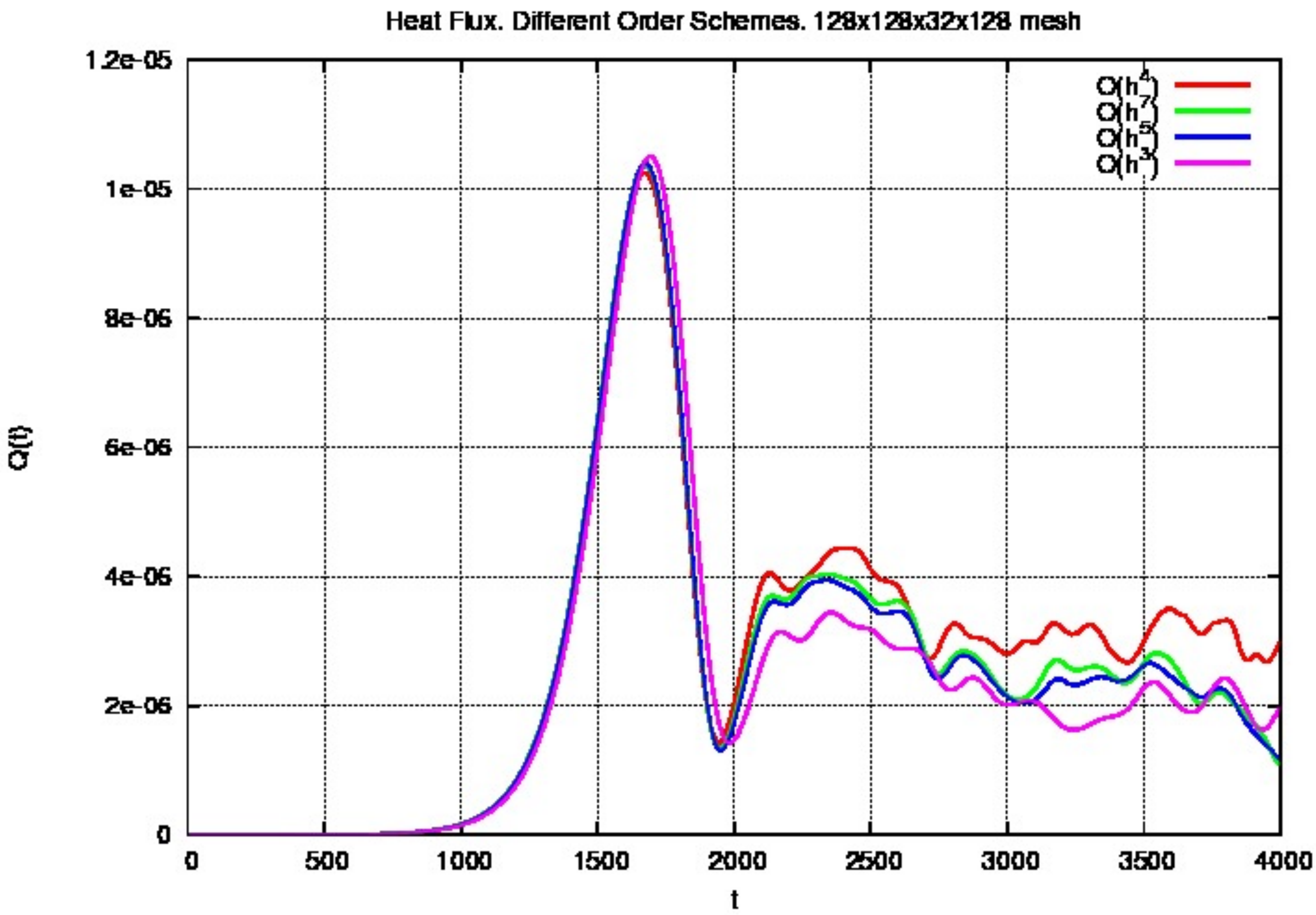
Heat Flux: Comparison of Order



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Hydrodynamics Turbulence - Primer

- Central idea
  - *For high Reynolds numbers, there exists a range of scales (aka the inertial range) in which effects of viscosity, boundaries, and large scale structures. Dimensional analysis leads to the well-known universal power-law spectrum (Kolmogorov 1941)*

$$E(k) = c_K \, \varepsilon^{2/3} \, k^{-5/3}.$$

- DNS: Direct Numerical Simulation
  - *All scales, i.e. turbulent fluctuations resolved up to the Kolmogorov scale where dissipation takes place*
  - *Computationally expensive for large Reynolds numbers*
    - *No of grid points scales as* $Re^{9/4}$
- LES: Large Eddy Simulation
  - *Only those turbulent fluctuations resolved as determined by the mesh resolution. Scales smaller than the mesh, i.e. sub-grid-scales must be modeled*

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Filtering the Navier Stokes Equations

- Consider a filter $G_\Delta(\mathbf{x})$
  - *For example a Gaussian filter* $G_\Delta^{\text{gaus}}(\mathbf{x}) = [6/(\pi\Delta^2)]^{3/2} \exp(-6x^2/\Delta^2)$,
- Convolve the velocity field with the filter (~ below indicates the convolved velocity field)
  - *Δ is the filter width below which the scales are eliminated*

- The resulting equations, which are amenable to numerical discretization at spatial resolution Δ, are

$$\partial_t \tilde{\mathbf{u}} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\frac{1}{\rho} \nabla \tilde{p} + \nu \nabla^2 \tilde{\mathbf{u}} - \boxed{\nabla \cdot \boldsymbol{\tau}^\Delta}, \quad \nabla \cdot \tilde{\mathbf{u}} = 0$$

- The additional term above dubbed the "SGS (sub-grid-scale)" stress tensor is

$$\boxed{\tau_{ij}^\Delta = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j.}$$

  - *SGS stress tensor must be modeled in terms of the resolved (filtered) velocity field*

Leonard, Adv. Geophysics, 1974
Pope, "Turbulent Flows", Cambridge, 2000
Rogallo & Moin, Ann. Rev. Fluid Mech, 1984
Lesieur & Metais, Ann. Rev. Fluid Mech, 1996

"LES" for Kinetic Equations

- Present work explores applying similar ideas to kinetic equations (Vlasov, Fokker-Planck)
  - *Question: Can we filter the 6D kinetic equation and derive analogous SGS terms which must be modeled if all scales are not resolved?*
  - *Benefit: It is likely that even a modest reduction in size may result in vast savings in computations*
- Filter the kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + v \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(E + v \times B)\frac{\partial f_\alpha}{\partial v} = C_{\alpha,\beta}(f_\alpha)$$

$$f = \overline{f} + f' \qquad \overline{f} = \int G_\Delta(x - x', v - v') f \, dx' dv'$$

$$\frac{\partial \overline{f}}{\partial t} + v \cdot \nabla \overline{f} + \overline{a}\,\frac{\partial \overline{f}}{\partial v} + \overline{a'\frac{\partial f'}{\partial v}} = 0$$

Extra term resulting from correlations between sub-grid quantities; a' contains sub-grid magnetic and electric fields

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Examining the SGS terms

Define $f = \bar{f} + f'$

the filtered field has $\bar{\bar{f}} = \bar{f}$ and $\bar{f}' = 0$

Filtered Vlasov equation: $(v_\| = \bar{v}_\|)$

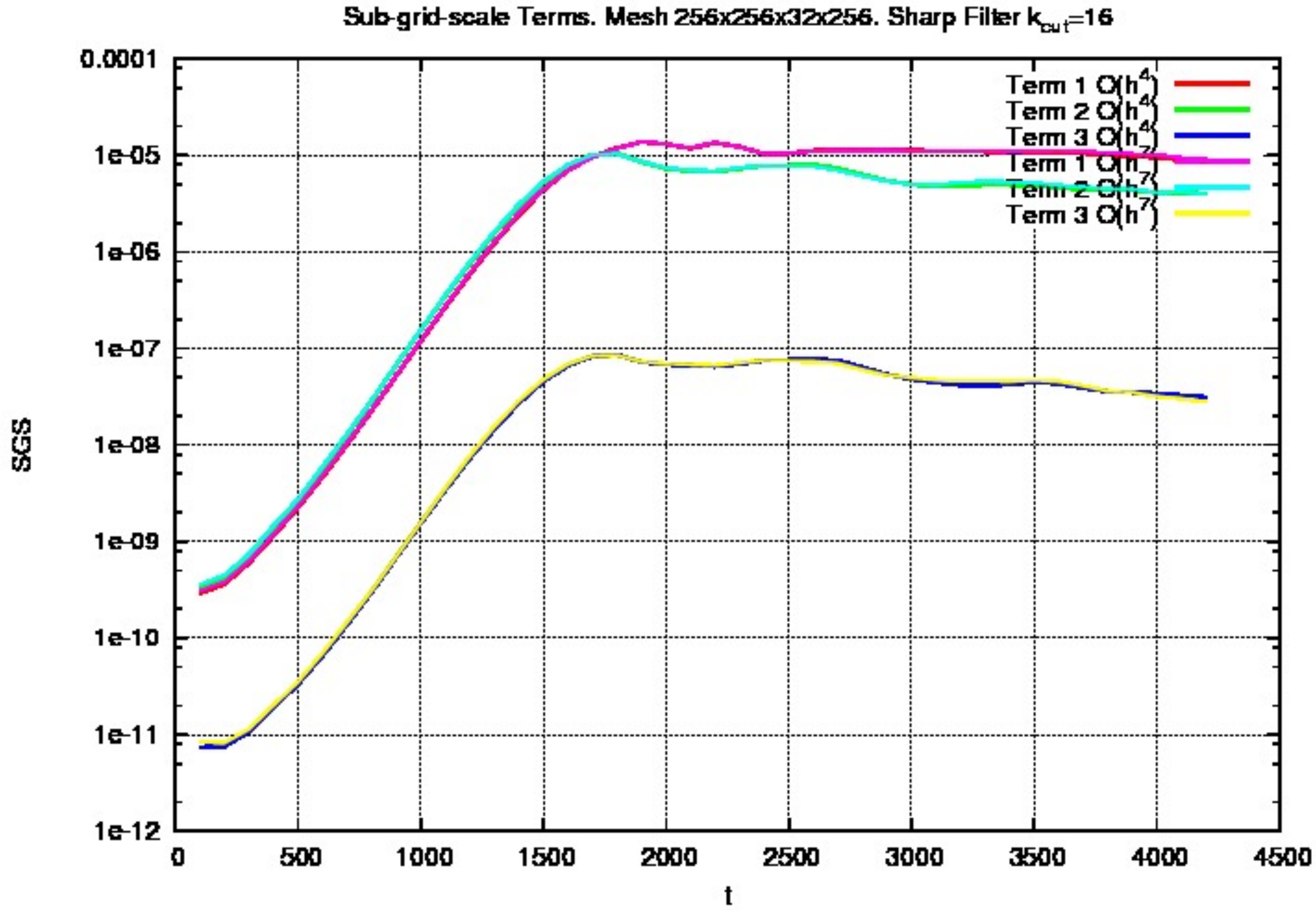
$$\frac{\partial \bar{f}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{v}_{GC_r}\bar{f}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\bar{v}_{GC_\theta}\bar{f}\right) + \frac{\partial}{\partial z}\left(v_\|\bar{f}\right) + \frac{\partial}{\partial v_\|}\left(\bar{\dot{v}}_\|\bar{f}\right) + SGS = 0$$

Calculate the last three SGS terms

$$SGS = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\overline{v_{GC_r} f} - \bar{v}_{GC_r}\bar{f}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\overline{v_{GC_\theta} f} - \bar{v}_{GC_\theta}\bar{f}\right] + \frac{\partial}{\partial v_\|}\left[\overline{\dot{v}_\| f} - \bar{\dot{v}}_\|\bar{f}\right]$$

Numerical Results: SGS Terms

- SGS terms evaluated by filtering the simulation results
  - *Filter is a "sharp cut-off" in Fourier space k_c=16*



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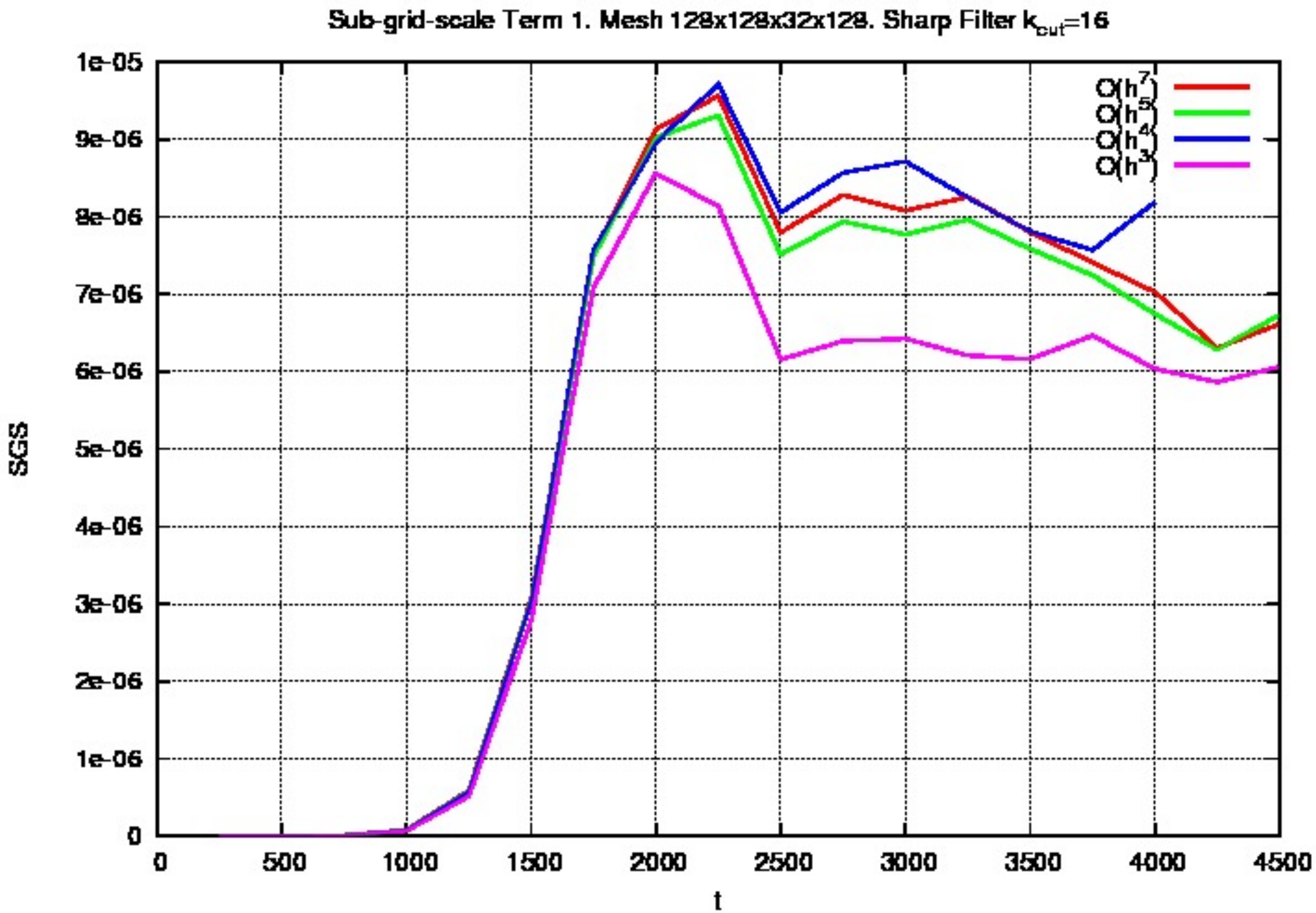
SGS Terms: Comparison of Order



After t>2500 fraction of SGS terms compared with $\overline{fv}$ is in excess of 20-30%

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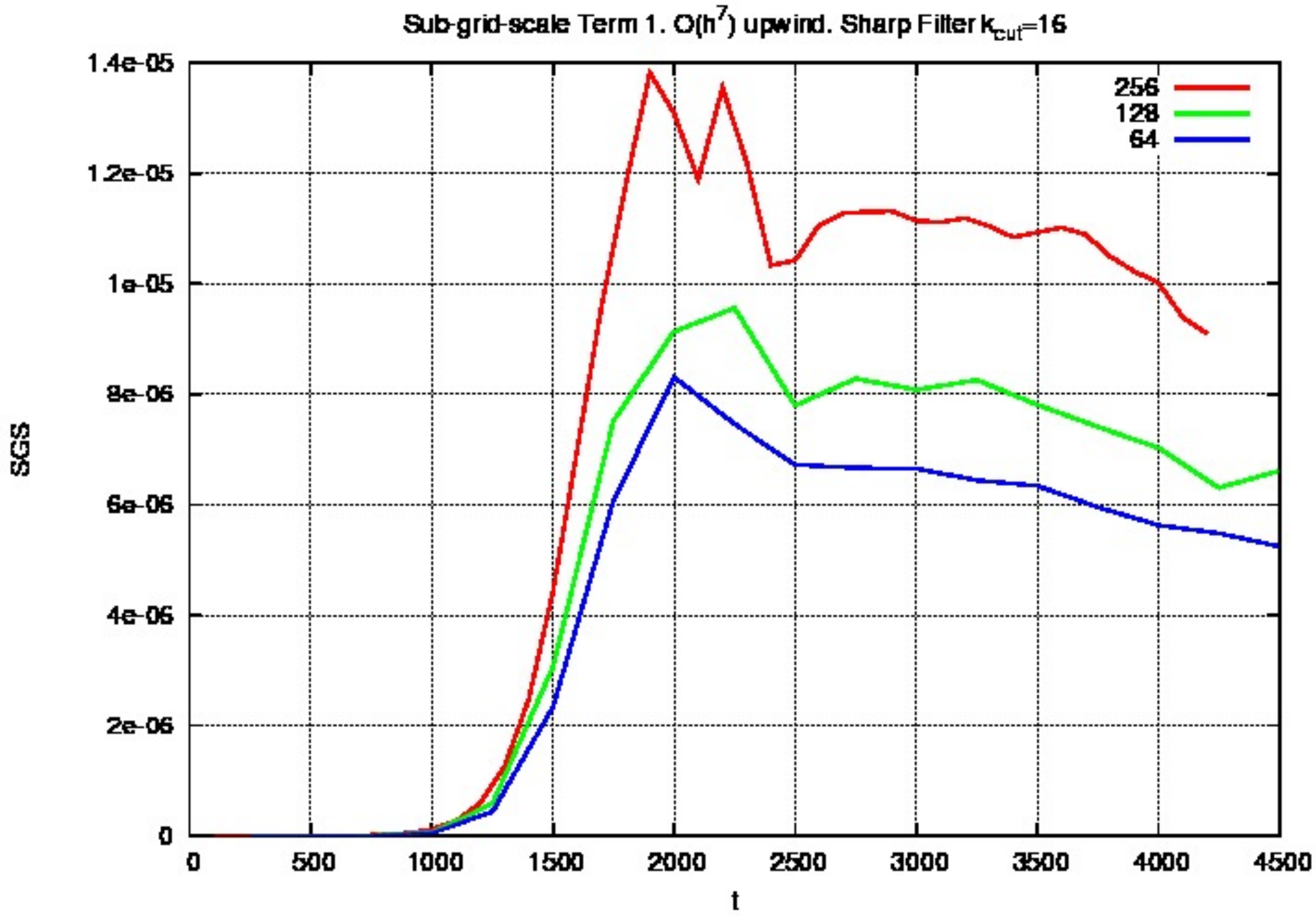
SGS Terms: Mesh Resolution



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Summary & Future Work

- Numerical results presented from a recently developed Eulerian Vlasov code
  - *High-order fluxes with 4th order Poisson solver*
  - *Regularization (either diffusion/hyperdiffusion/upwinding/model collisions) required for code stability*
  - *A true DNS will require cut-off provided by physical collisions which must be resolved*
  - *Presented preliminary estimates of SGS terms which are in excess of 20-25%.*
    - *Under-resolved simulations will miss this contribution unless modeled*
    - *Back-scatter effects from fine to coarser scales will render these missed SGS terms even more important*

- **Main challenge:** Developing physically accurate models for SGS terms in terms of resolved quantities especially because cascade to fine scales is not as straightforward as in hydro turbulence
- Future Work
  - *Extend to 5D gyrokinetic*
  - *More diagnostics and quantification of SGS terms*
  - *Develop SGS models*

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