FAST PARTICLE DRIVEN INSTABILITIES IN TOKAMAKS

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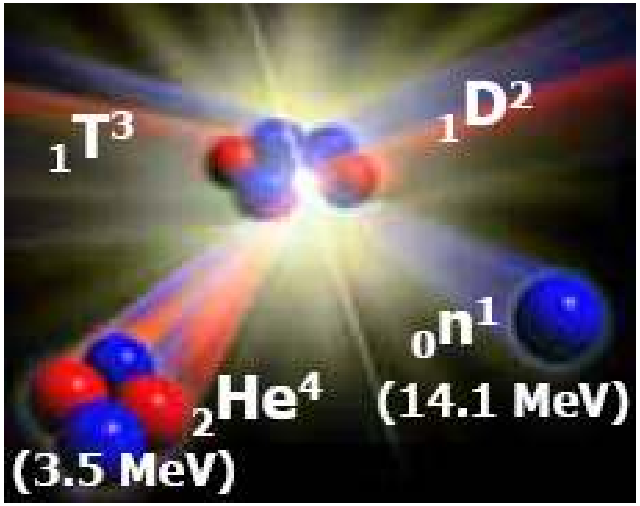
- SUMMARY

INTRODUCTION

FUSION OF HYDROGEN ISOTOPES



- Nuclear fusion reaction D+T = He + n +17.6 MeV of hydrogen isotopes deuterium (D) and tritium (T) is the "easiest" to access.
- Fusion alpha-particles (20% of fusion energy) heat the plasma and balance heat loss; neutrons (80% of energy) breed new tritium and generate steam.
- Deuterium is naturally abundant (0.015% of all water), Tritium must be obtained from lithium, $^6Li + n = T + ^4He$. Raw materials are water & lithium.

ENVIRONMENTAL ADVANTAGES OF FUSION

- To generate 1GW for 1 year (equivalent to a large industrial city):

  COAL: 2.5 Mtonnes – produces 6 Mtonnes CO_2;
  FISSION: 150 tonnes U – produces several tonnes of fission waste;
  FUSION: 1 tonne Li + 5 Mlitres water.

- Fusion gives no “greenhouse” gasses.

- Fusion reactor structure will become activated but will decay to a safe level in < 100 years. Tritium is radioactive: half life is 13 years.

- No plutonium or long-lived (thousands of years) active waste from fuel cycle.

SELF-SUSTAINING FUSION REACTION

- Three key parameters for fusion to occur in plasma:

  KeV-range ion temperature to overcome Coulomb force between D and T;

  Good energy confinement time τ_E = Plasma energy/ Heat loss;

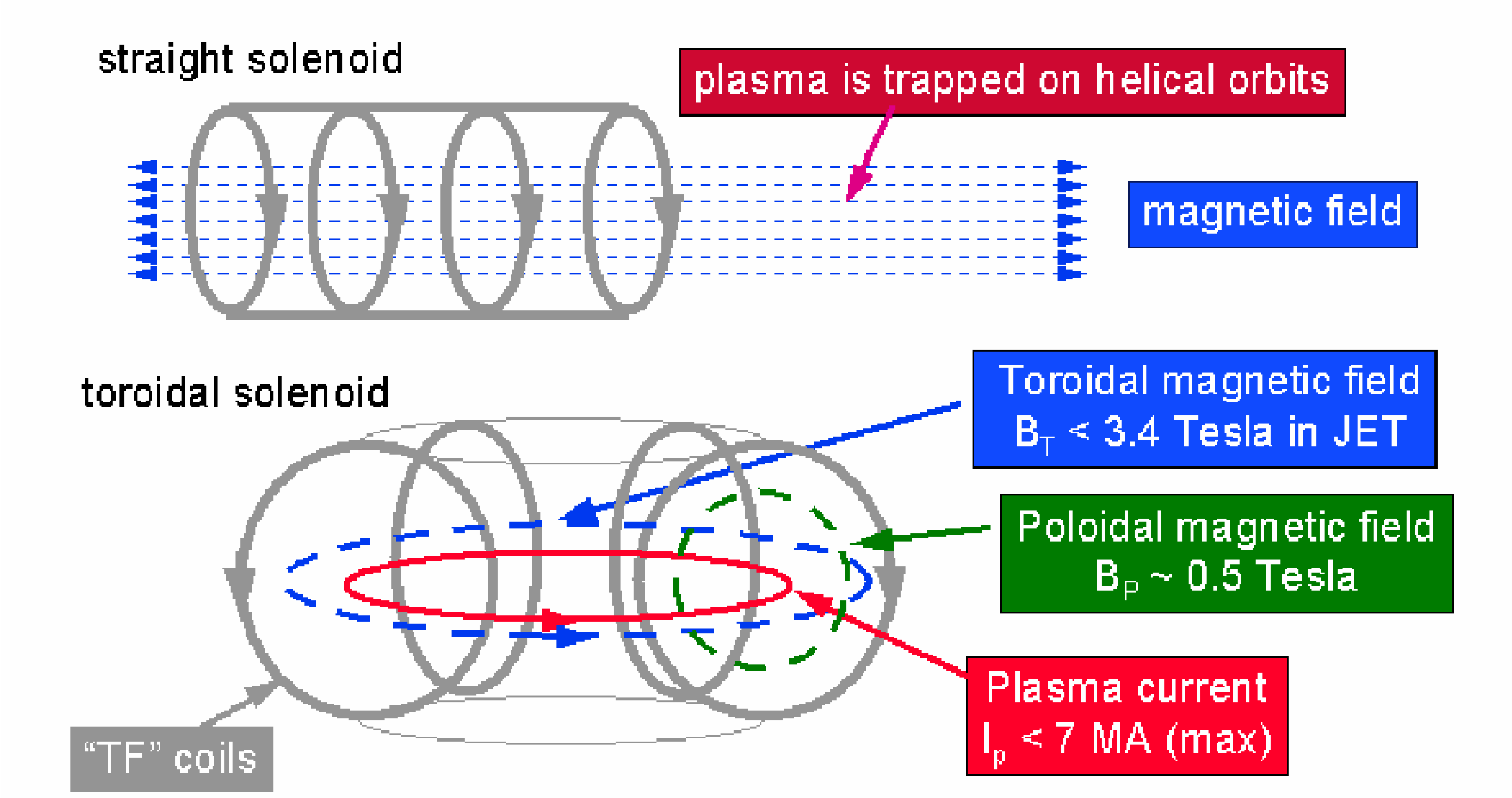
  Fuel density n_D and n_T must be high enough;

- The "ignition" condition for self-sustained fusion reaction

$$n\, T\, \tau_E > 5 \times 10^{21}\ m^{-3}\ keV\ s\ (\approx 10\ atm\ s)$$

- Methods of confinement:
  Gravity (Sun and stars) – works well but dimensions are too large;
  Inertial (Hydrogen bomb, lasers or beams) – works well, needs pressure 10^{12} atm for very short times 10^{-11} s;
  Magnetic – few atms x few seconds, plasma is confined by magnetic field B.

THE IDEA OF MAGNETIC CONFINEMENT OF PLASMA



- **In the presence of strong magnetic field, charged particles of plasma are trapped on helical orbits attached to magnetic field lines**

WAYS OF ACHIEVING IGNITION IN MAGNETIC FUSION

The ignition criterion can be re-written in the form

$$\beta\, \tau_E\, B^2 > 4\ T^2\ s, \quad \text{where } \beta = P_{plasma}/P_{magnetic} = 4\mu_0(nT)/B^2$$

- Increasing τ_E : larger size fusion reactors since energy balance for steady-state is determined by $P_\alpha = 0.2\ P_{FUSION}$:

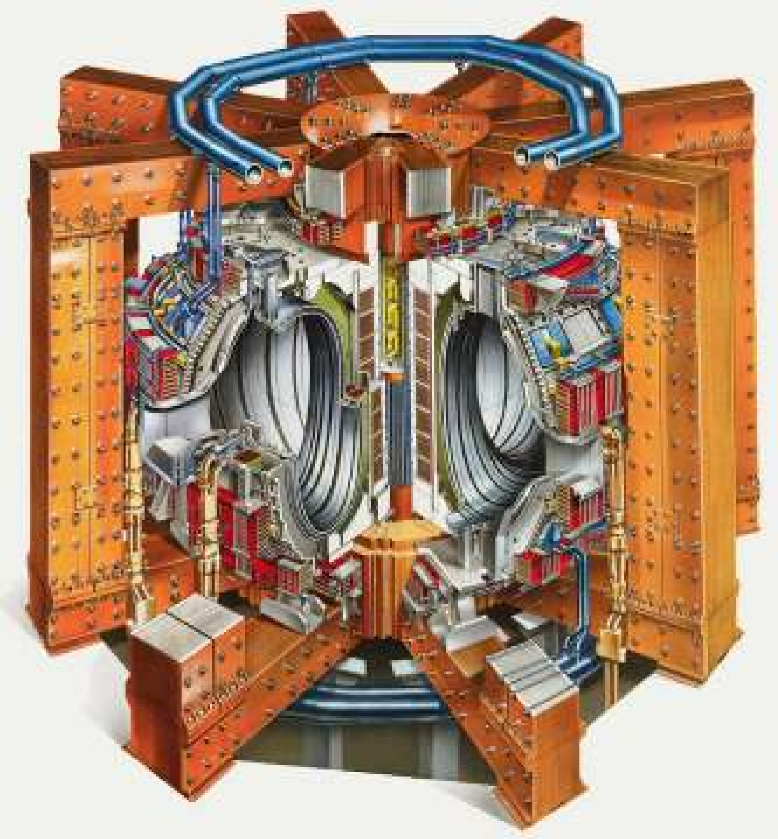
  $$dW/dt = -W/\tau_E + P_\alpha = 0 \rightarrow P_\alpha = W/\tau_E = n\,T\,(V/\tau_E)$$

  "Critical" plasma volume for 5 T is about 1000 m^3
  Present day JET (EU) ≈ 100 m^3 (sub-critical), next step ITER ≈ 800 m^3

- Increasing B : technologically challenging to obtain B > 5 T.
  Present-day Alcator C-MOD (US), next step: IGNITOR (Italy), FIRE (US)

- Increasing β : this is limited by MHD instabilities at a level of few %. However spherical tokamaks with a/R ≈ 1 achieve volume averaged $<\beta> \approx 40\%$
  Present day MAST (UK), NSTX (US), next step project, e.g. STPP (UK)

TOKAMAK JET (JOINT EUROPEAN TORUS)



Volume = 100 m^3; B$_{max}$ = 4 T; I$_{max}$ = 7 MA; P$_{FUS}$ = 16 MW

IGNITED AND BURNING FUSION PLASMAS

PLASMA SELF HEATING

- New element in burning plasmas: plasma is self-heated by fusion alphas → plasma is exothermic and highly nonlinear medium

- The leading-order effects may be identified in accordance with $Q=P_{FUS}/P_{IN}$,

$Q \approx 1$ – at the threshold

$Q \approx 5$ – alpha-effects on heating profile and Alfvén instabilities

$Q \approx 10$ – nonlinear coupling between alphas, MHD stability, bootstrap current, turbulent transport, interaction plasma-boundary

$Q \geq 20$ – burn control and transient ignition phenomena

- Modelling of such plasmas is challenging even if plasma transport is classical

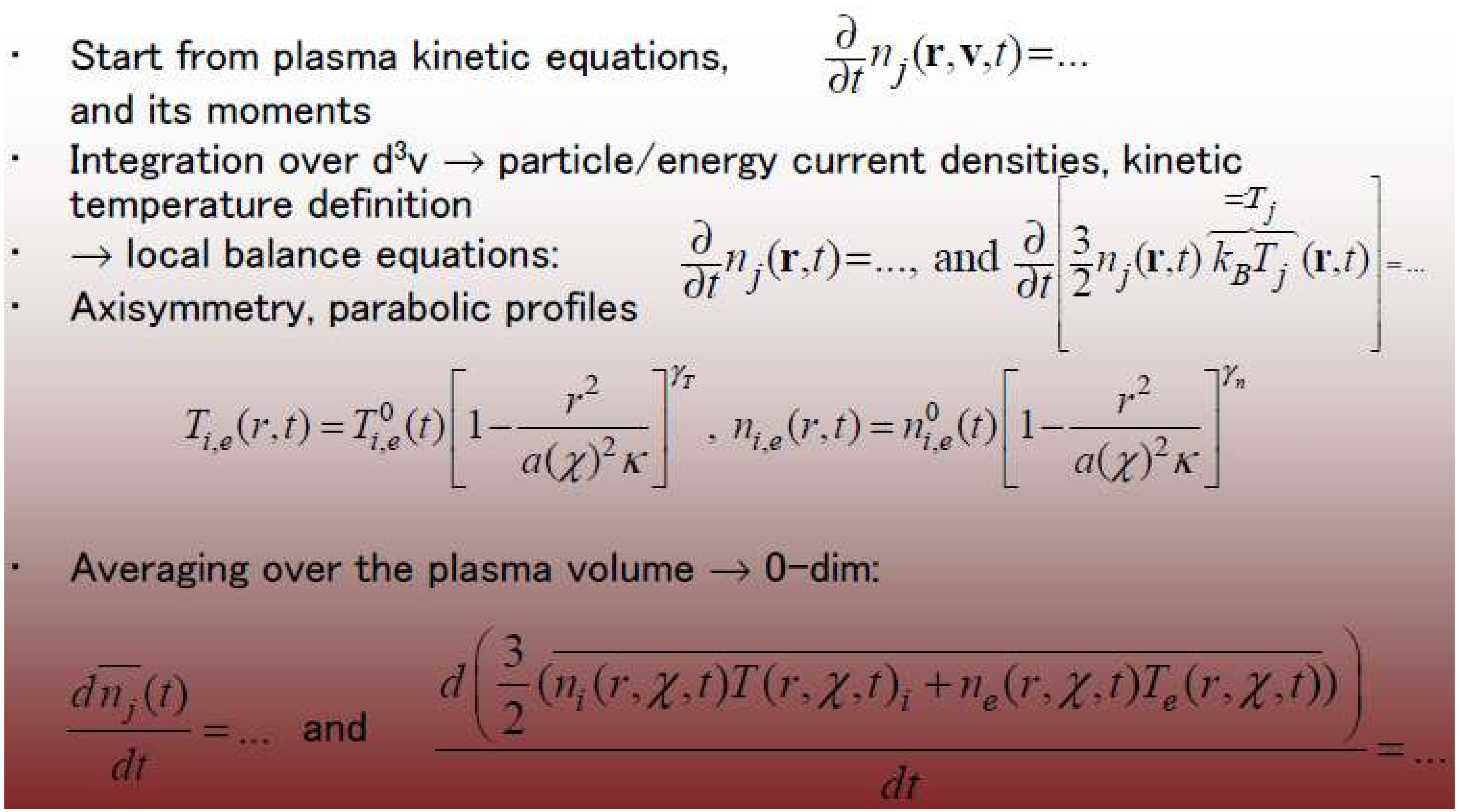
IGNITED ITER-94 WITH CLASSICAL TRANSPORT

(K.Schoepf et al. ICENES-7 Conference, 2007)

PARAMETERS OF ITER EDA-94

R=8.1 m …major plasma radius	κ=1.6-2.0 … plasma elongation
a =3.0 m … minor plasma radius	δ=0.3 … triangularity
B_0=5.7 T … toroidal magnetic field on axis	N=16 (24) … number of toroidal field coils
I_p=18-24 MA … total plasma current	R_f= 0.9 … first wall reflectivity
$q(a)$ = 3 … safety factor at the edge	Z_{eff}=1.6
C_T = 3÷4 … Troyon factor	$\beta<\beta_T$,', SBL: $\tau_E(\beta/\beta_T)$
A_i=2.5 amu … average fuel ion mass	$n<n_G$

EXAMPLE OF IGNITED ITER WITH CLASSICAL TRANSPORT (cont'd)



- Start from plasma kinetic equations, and its moments $\frac{\partial}{\partial t} n_j(\mathbf{r},\mathbf{v},t)=\ldots$
- Integration over d^3v → particle/energy current densities, kinetic temperature definition
- → local balance equations: $\frac{\partial}{\partial t} n_j(\mathbf{r},t)=\ldots$, and $\frac{\partial}{\partial t}\left[\frac{3}{2} n_j(\mathbf{r},t)\overbrace{k_B T_j}^{=T_j}(\mathbf{r},t)\right]=\ldots$
- Axisymmetry, parabolic profiles

$$T_{i,e}(r,t)=T^0_{i,e}(t)\left[1-\frac{r^2}{a(\chi)^2\kappa}\right]^{\gamma_T},\quad n_{i,e}(r,t)=n^0_{i,e}(t)\left[1-\frac{r^2}{a(\chi)^2\kappa}\right]^{\gamma_n}$$

- Averaging over the plasma volume → 0-dim:

$$\frac{d\overline{n_j}(t)}{dt}=\ldots \quad \text{and} \quad \frac{d\left(\frac{3}{2}\overline{(n_i(r,\chi,t)T(r,\chi,t)_i+n_e(r,\chi,t)T_e(r,\chi,t))}\right)}{dt}=\ldots$$

THE SET OF TRANSPORT EQUATIONS

$$\frac{d\overline{n_i}(t)}{dt} = \overline{s}(t) - \frac{\overline{n_i}(t)}{\tau_p} - \frac{1}{2}\overline{n_i^2(r,\chi,t) < \sigma v >_{dt} (T(r,\chi,t))}$$

$$\frac{d\left(\frac{3}{2}\overline{(n_i T_i + n_e T_e)}\right)}{dt} = \overline{P_{aux}(r,\chi,t)} + \overline{P_{Ohm}(r,\chi,t)} + \eta_\alpha \overline{P_\alpha(r,\chi,t)}$$

$$- \frac{\frac{3}{2}\overline{(n_i T_i + n_e T_e)}}{\tau_E} - \overline{P_{brems}(r,\chi,t)} - \overline{P_{cycl}(r,\chi,t)} - \overline{P_{line}(r,\chi,t)}$$

GLOBAL ENERGY CONFINEMENT TIME

- based on theoretical calculations of heat conductivities as well as on semiempirical scaling laws

- extended formulation to iclude all operation regimes (Ohmic heating, L-mode, L-H transition, H-mode)

- modified by a soft-ß limit (SBL) factor:

$$SBL = 1 + \frac{35(\beta/\beta_T)^2}{\pi\left[1 + 25(1 - \beta/\beta_T)^2 (3\beta_T/C_T)^2\right]} \;, \quad \beta_T[\%] = C_T \frac{I_p[\mathrm{MA}]}{a[\mathrm{m}]\; B[\mathrm{Tesla}]}$$

3 COUPLED NONLINEAR DIFFERENTIAL EQUATIONS

$$\frac{d\overline{n_i}}{dt} = f(\overline{n_i}, \overline{T_i}, \overline{T_e}, \overline{s})$$

$$\frac{d\overline{T_i}}{dt} = g(\overline{n_i}, \overline{T_i}, \overline{T_e}, \overline{P_{aux \to i}})$$

$$\frac{d\overline{T_e}}{dt} = h(\overline{n_i}, \overline{T_i}, \overline{T_e}, \overline{P_{aux \to e}})$$

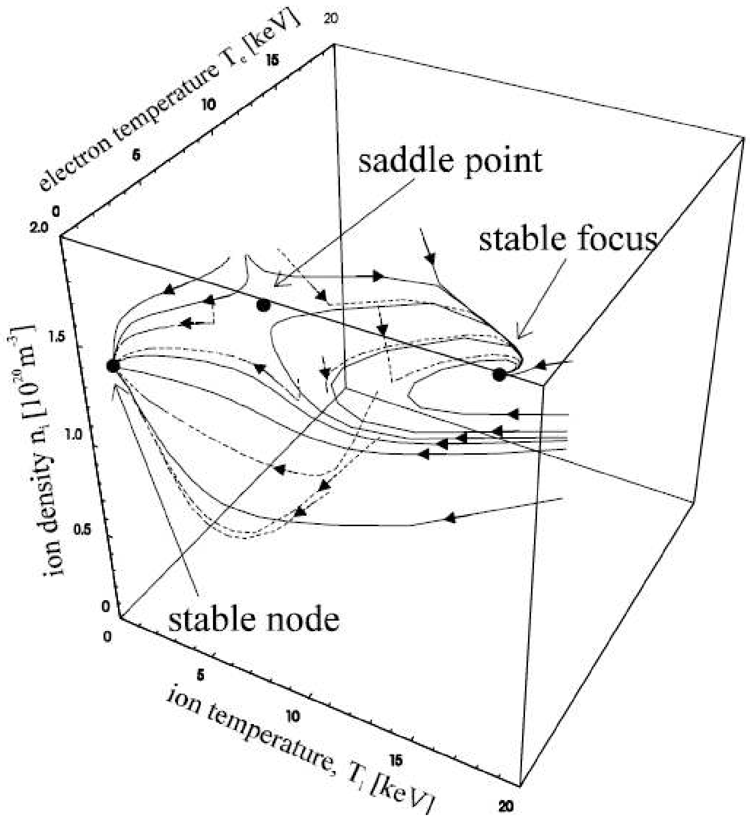
3 fixed points (equilibrium):

$\dot{T}_i = 0 \wedge \dot{T}_e = 0 \wedge \dot{n}_i = 0$
$\rightarrow \dot{E}_p = 0$
$\Rightarrow$ ignition, if $P_{aux} = 0$

-- the stable node at low temperature just refers to the equilibrium when Ohmic heating balances all loss powers → Ohmic ignition (practically no fusion power).

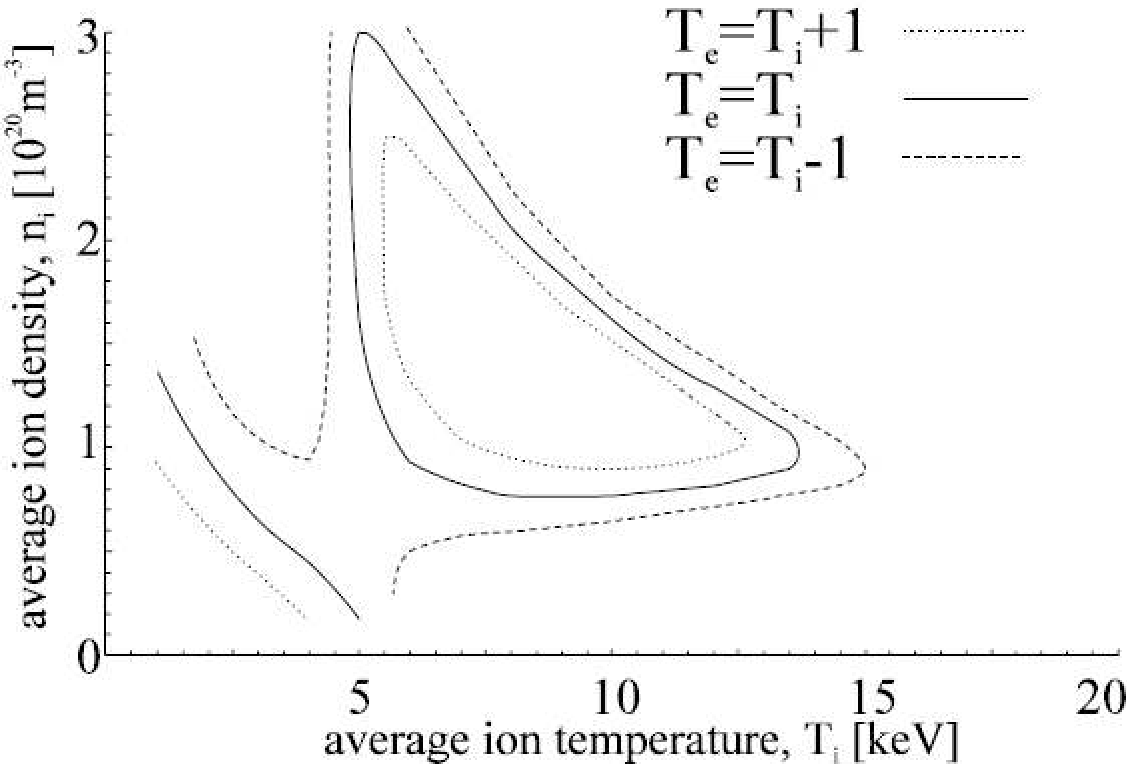
-- the attracting high-T equilibrium corresponds to fusion ignition as well as the unstable saddle point that repells the trajectories.

3D DYNAMICS OF THE IGNITED PLASMA



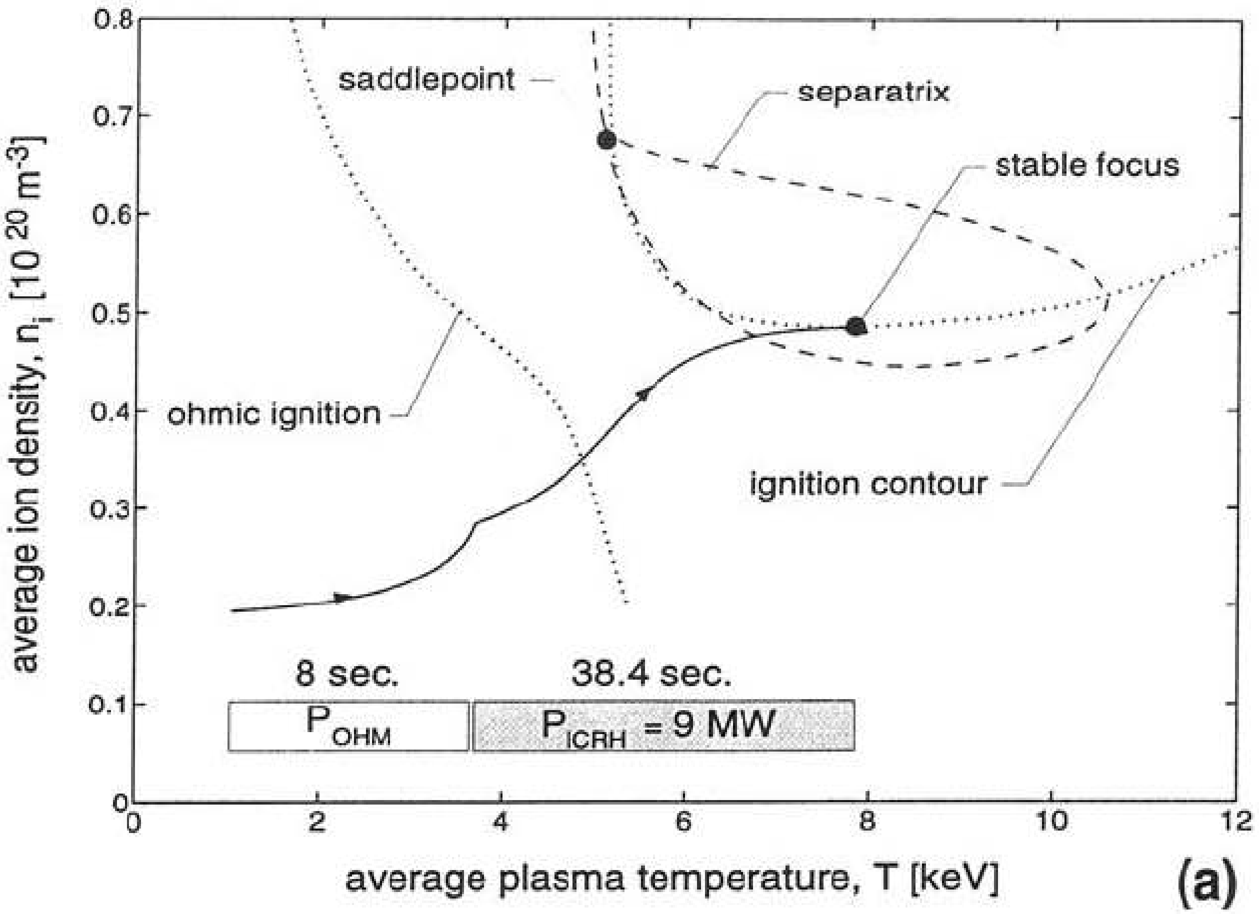
- The system evolves depending on initial conditions along the trajectories shown if no auxiliary heating is applied, the fuelling rate is constant, and no other impurities than He ash are present

2D PLOTS ASSUMING T_e/T_i = const

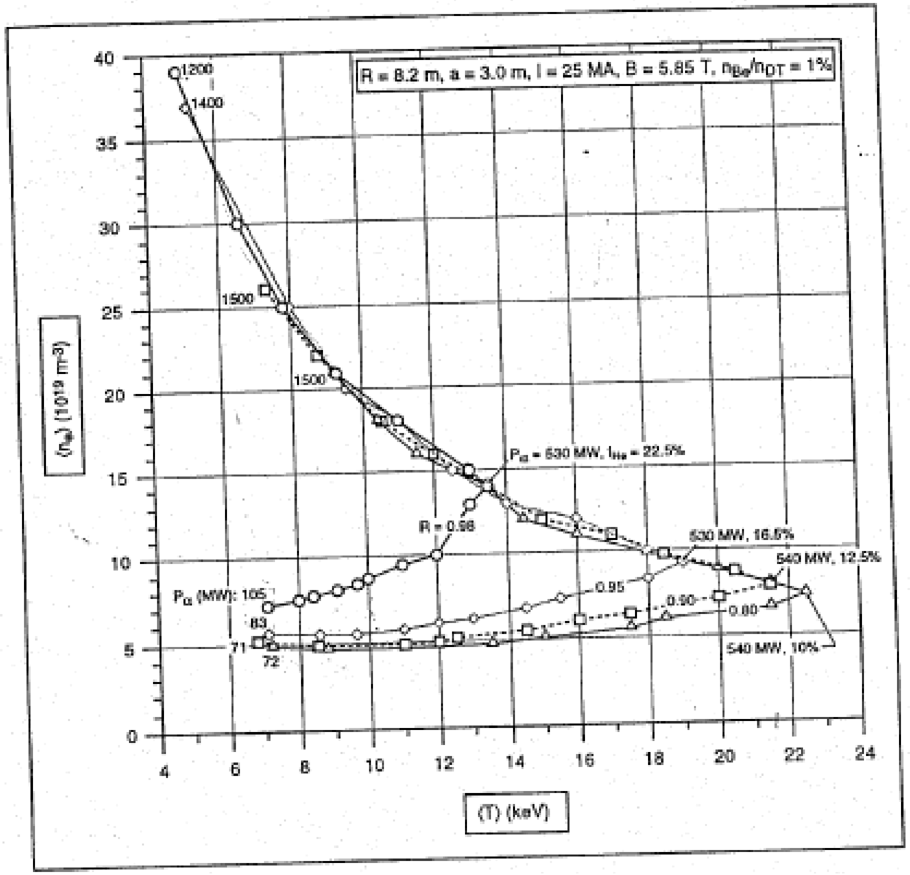


Ignition contours in the (n, T)-plane

PREFERABLE PATH TO IGNITION



“ITER-94” PATH TO IGNITION (from TAC-6 Report, 93-12-17 F)



ALFVÉN INSTABILITIES DRIVEN BY ENERGETIC IONS

FAST IONS IN FUSION BURNING PLASMA

- Alpha-particles (He^4 ions) are born in deuterium-tritium nuclear reactions with birth energy 3.52 MeV, i.e. these fusion-born ions are *super-Alfvénic*,

$$V_{Ti} \ll V_A < V_\alpha \ll V_{Te}$$

- During slowing-down of alpha-particles, they cross the resonance condition

$$V_A = V_{\parallel\alpha}$$

  and may excite Alfvén waves if the drive due to alphas exceeds wave damping due to thermal plasma:

$$\gamma_\alpha \cong Cq^2 (d\beta_\alpha / dr) \cdot F\left(V_\alpha / V_A\right) > \left|\gamma_e + \gamma_i\right|$$

- Free energy source: radial gradient of alpha-particle pressure. The instability results in radial re-distribution /losses of alpha-particles if the Alfvén wave amplitude is high.

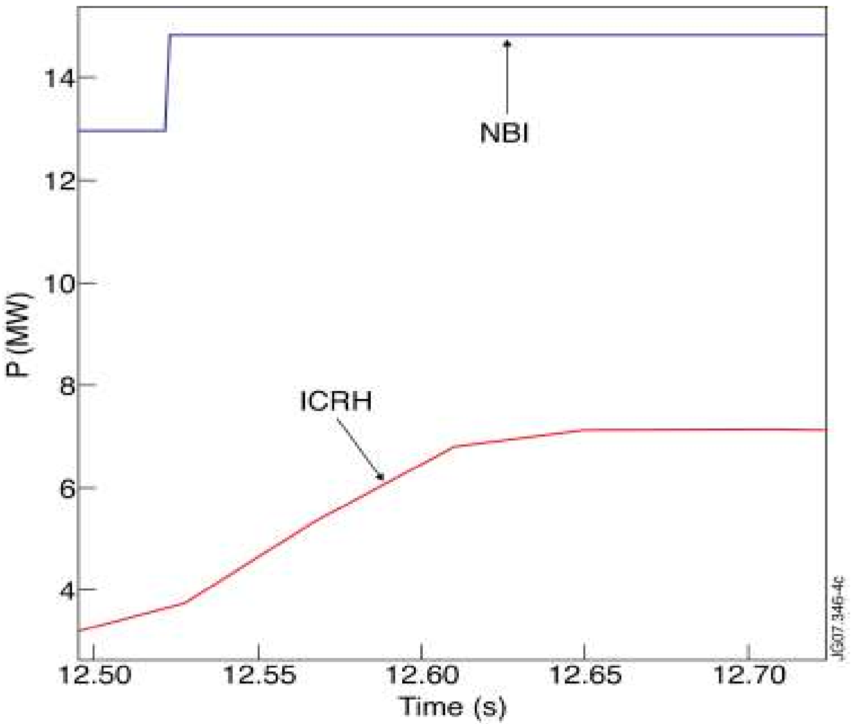
ALFVÉN INSTABILITIES IN PRESENT DAY MACHINES

- **On present day tokamaks, fast particles produced by Ion-Cyclotron Resonance Heating (ICRH) and Neutral Beam Injection (NBI) do excite numerous Alfvén instabilities**

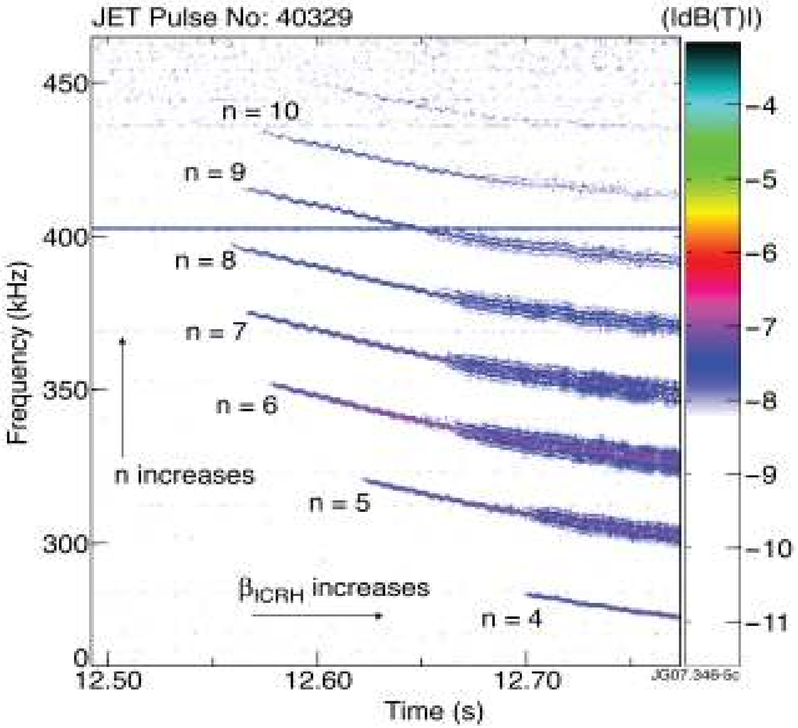
- **These instabilities usually have discrete spectrum in frequency since the instability criterion $\gamma_\alpha > \gamma_{Damping}$ is easier to satisfy for so-called "weakly-damped Alfvén Eigenmodes" (AEs) with small values of $\gamma_{Damping}$**

- **Among weakly-damped Alfvén Eigenmodes, Toroidal Alfvén Eigenmodes (TAEs) have extremely high quality factors, $Q=\omega/\gamma_{Damping}$ = 10^2 – 10^3 (i.e. very low damping caused by thermal plasmas) and are easiest to excite with fast particles**

TAEs DRIVEN BY ICRF-ACCELERATED IONS ON JET



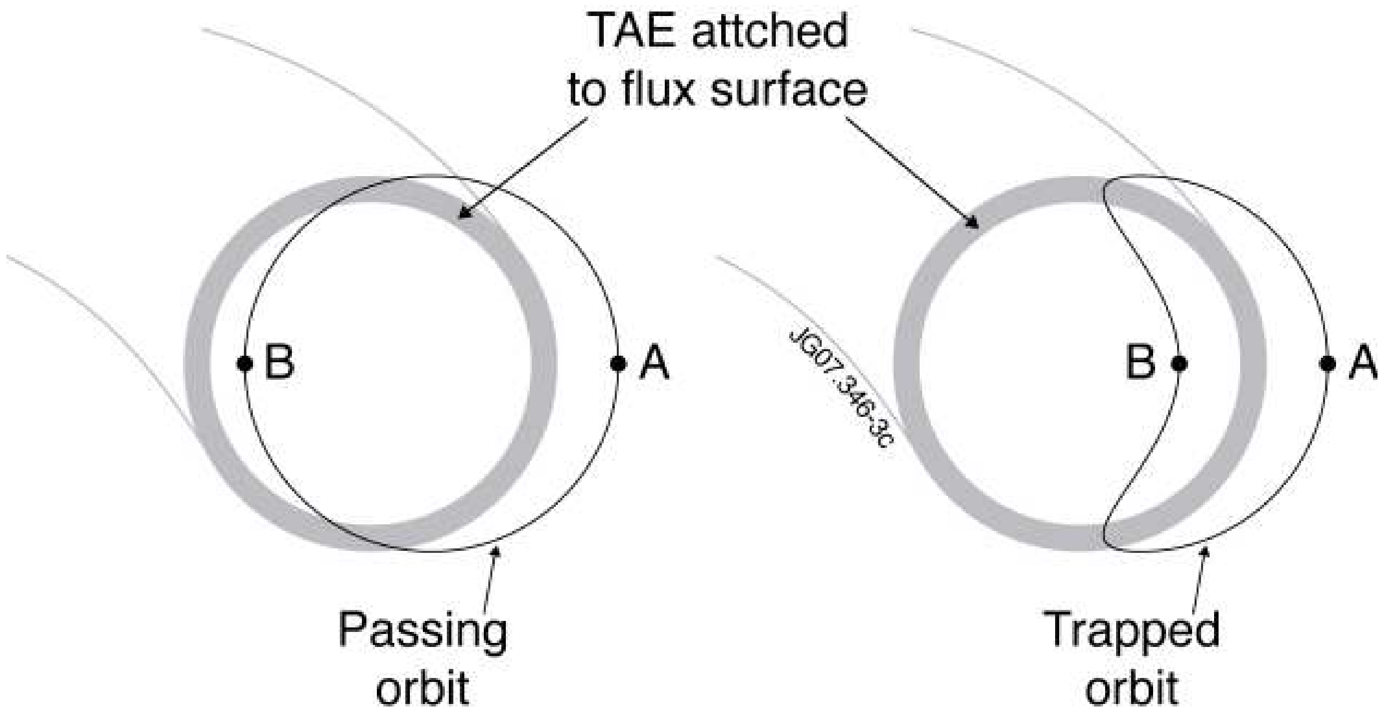
Power waveforms of ICRH driving TAE and NBI (NBI provides damping)



TAEs with toroidal mode numbers from n=4 to n=10 are seen separated by $f_{Rotation}$ ~ 25 kHz

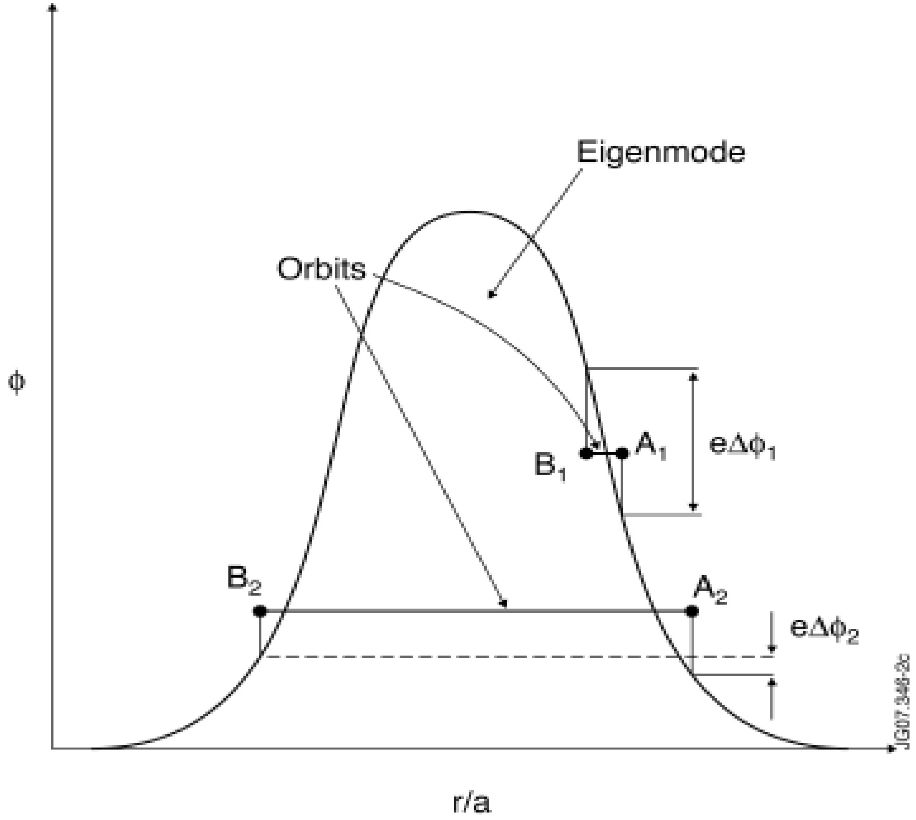
Why TAEs with different n's are excited at different P_{ICRH} ?

FAST PARTICLE DRIVE: QUALITATIVE PICTURE - 1



- **TAE modes are attached to magnetic flux surfaces, while the resonant ions, both passing and trapped, experience drift across the flux surfaces and TAE mode structure**

FAST PARTICLE DRIVE: QUALITATIVE PICTURE - 2



- When resonant ion moves radially across TAE from point A to point B, the mode and the ion exchange energy $e\Delta\phi$ as Figure shows.

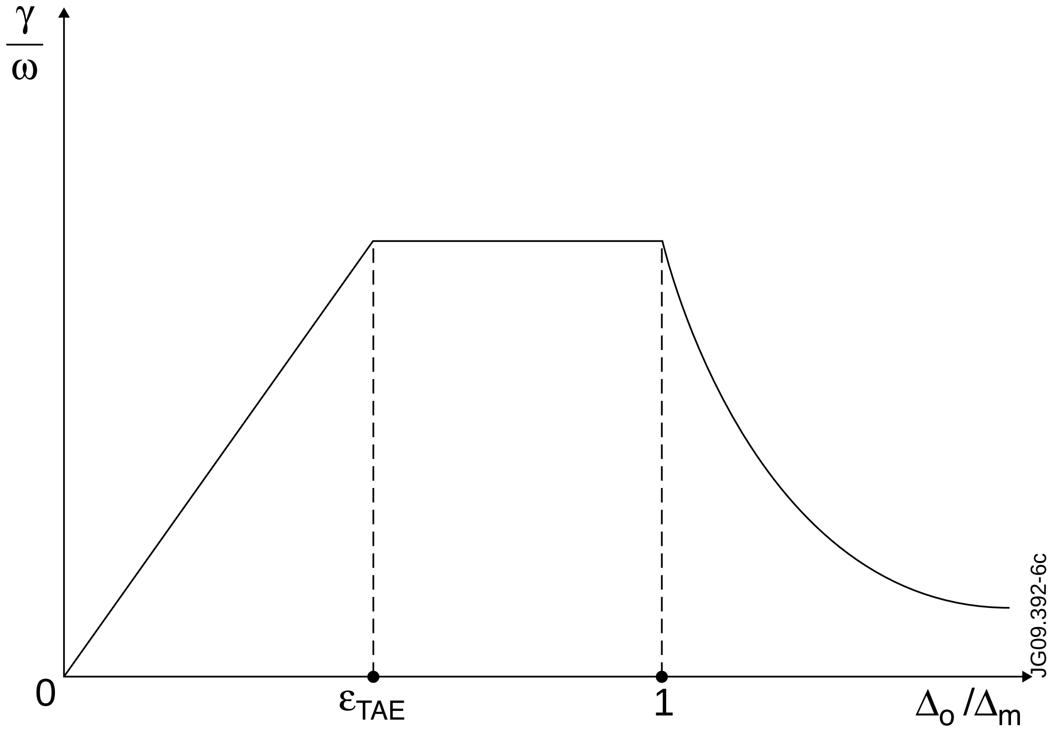
POWER TRANSFER FROM ENERGETIC PARTICLES TO WAVE

- **Within the guiding centre approximation, the general expression is**

$$P_\alpha = \int d\zeta d\vartheta dr \cdot Rr \int d^3 v \left(- e_\alpha \mathbf{V_d} \cdot \delta \mathbf{E}\right) \cdot f$$

where $\mathbf{V_d}$ is the unperturbed guiding center drift velocity, and f is the linear perturbation of the guiding centre distribution function.

FAST PARTICLE DRIVE: QUALITATIVE PICTURE - 3



- **The maximum of $e\Delta\phi$ occurs when the orbit size, Δ_{orbit}, is equal to the mode width, $\Delta_{TAE} \approx r_{TAE}/m$, where m is poloidal mode number. This is why energetic ion drive is expected to have a maximum at $m \approx nq \approx r_{TAE}/\Delta_{orbit}$.**

NUMERICAL TOOLS USED FOR ALFVÉN INSTABILITIES

Grad-Shafranov equilibrium solver HELENA [G.Huysmans et al., 1991]

For computing the modes:

Resistive compressible MHD spectral code CASTOR [W.Kerner et al., ~1992]

The CSCAS code for computing continuum in toroidal geometry [S.Poedts 1993]

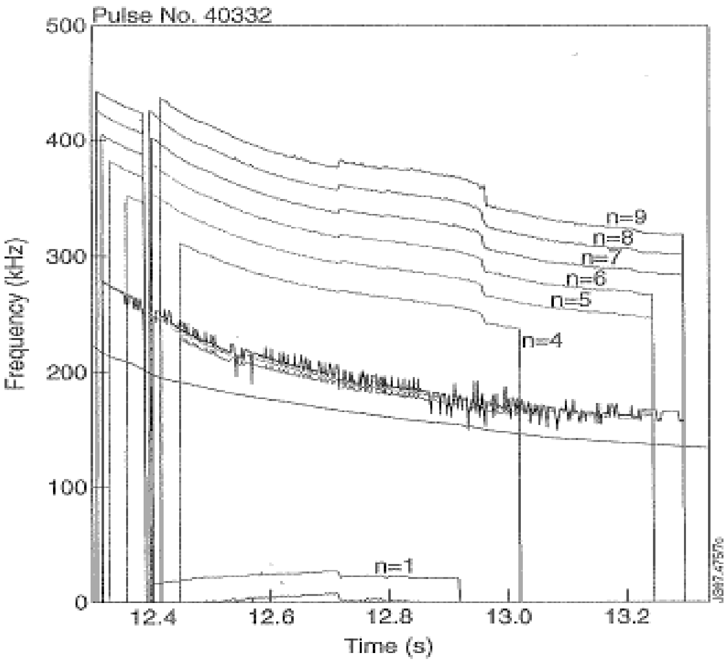
Ideal MHD spectral MISHKA-1 code [Mikhailovskii et al., 1998]

For computing the fast ion orbits and wave-particle energy exchange:

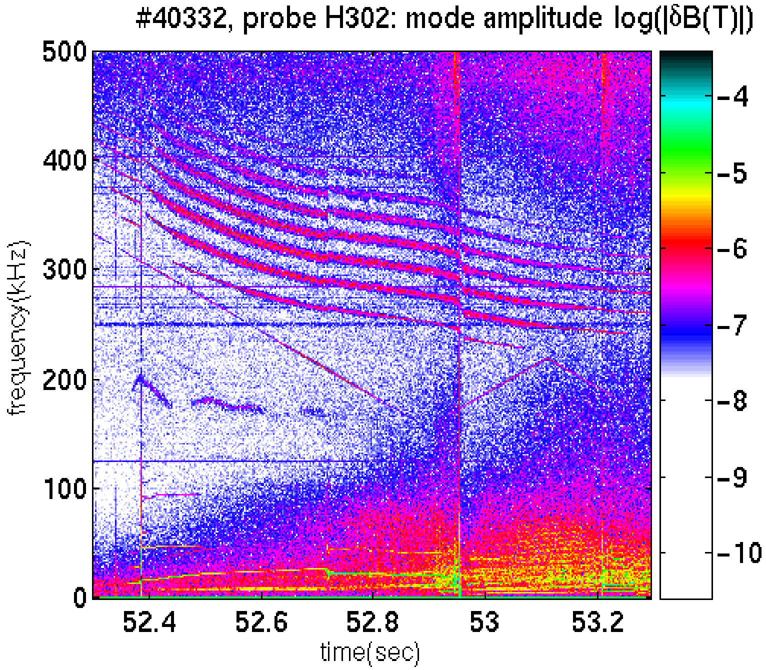
CASTOR-K for computing particle-to-wave power transfer giving linear γ_α , γ_e, γ_i [D.Borba, W.Kerner et al., ~1997]

HAGIS initial value particle-following code giving nonlinear re-distribution of energetic particles due to the modes [S.Pinches et al., ~1998]

COMPUTED VERSUS OBSERVED TAEs



Eigenfrequencies of TAEs with n=4…9 computed for equilibrium in JET discharge #40332. Added Doppler shift matches the experiment



Discrete spectrum of TAE observed in JET discharge #40332. Plasma starts at t=40 sec. Frequency changes due to plasma density increase, $f \sim B/\sqrt{n_i M_i}$.

ALFVÉN INSTABILITIES ON MAST

- Tight aspect ratio ($R_0\,/a \sim 1.2 \div 1.8$) limits the value of magnetic field at level $B_T \sim 0.15 \div 0.6$ in present-day STs ⇒ Alfvén velocity in ST is very low

$$V_A = B_T\,/\,(4\pi n_i m_i)^{1/2} \cong 10^6\ \mathrm{ms}^{-1}\ \text{(MAST)}$$

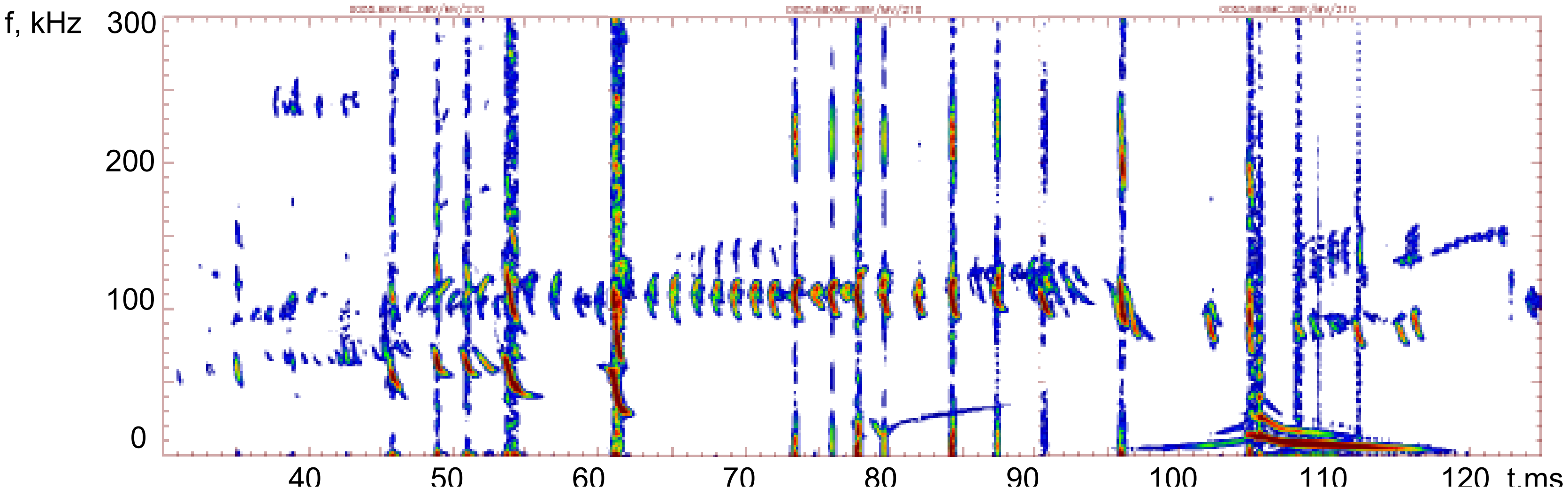
(compare, e.g. to Joint European Torus (JET), where $V_A \cong 7\times10^6\ \mathrm{ms}^{-1}$)

- Even a relatively low-energy NBI, e.g. 30 keV hydrogen NBI is super-Alfvénic

$$V_{NBI} \cong 2.4\times10^6\ \mathrm{ms}^{-1} > V_A\ ,$$

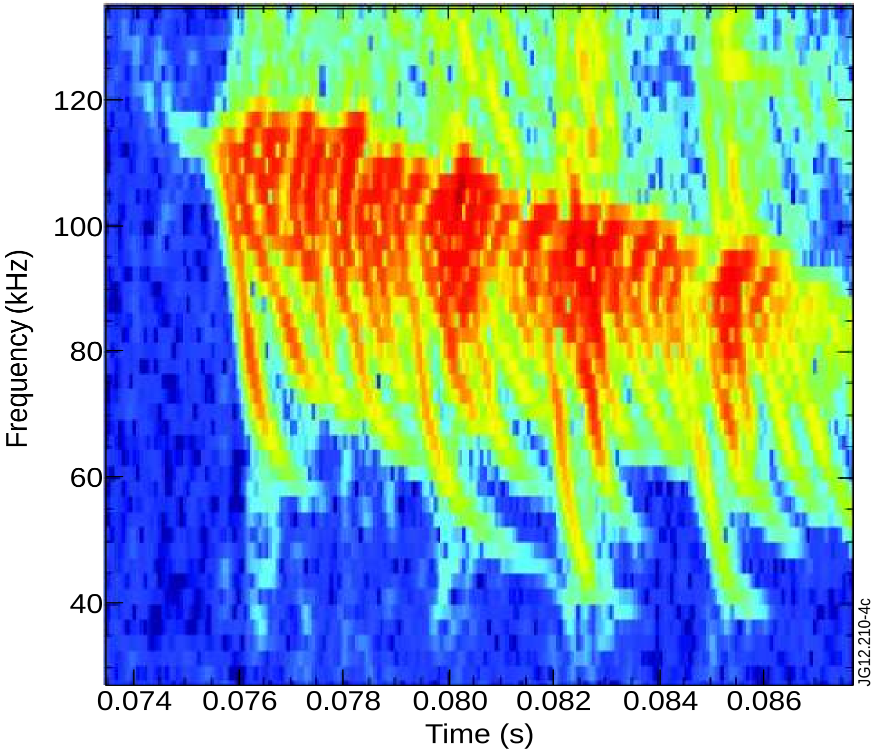
The super-Alfvénic NBI can excite Alfvén waves via the fundamental resonance $V_{||NBI} = V_A$ as in the case of fusion alphas.

NBI-DRIVEN AEs ARE DIFFERENT THAN ICRH-DRIVEN

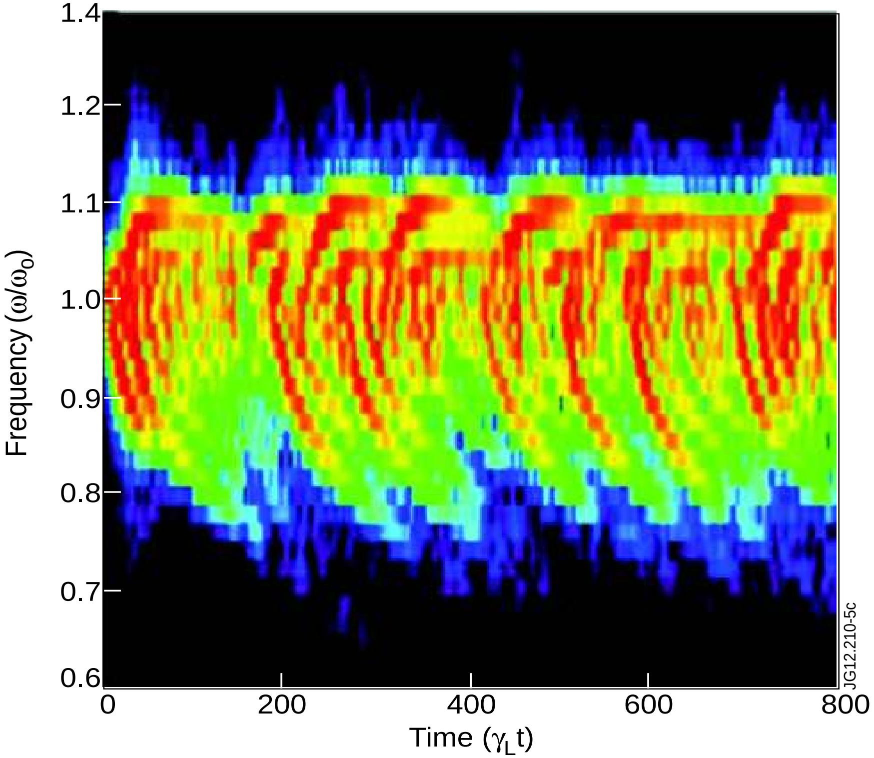


- **AEs are seen in bursts not as steady-state modes**

BURSTING TAEs ON MAST VERSUS HAGIS MODELLING



Magnetic spectrogram showing bursting frequency-sweeping Alfvén modes driven by NBI in MAST discharge #27177.



Nonlinear HAGIS simulation of TAE with unlocked phase driven by slowing-down beam distribution function.

TRANSPORT DUE TO TAE MODES

QUALITATIVE ESTIMATES – 1

- The *unperturbed* orbit of a particle is determined by three invariants:

$$\mu \equiv \frac{Mv_\perp^2}{2}; \qquad E \equiv \frac{Mv^2}{2}; \quad P_\varphi \equiv -\frac{e}{c}\psi(r) + RMv_\varphi$$

- In the presence of a *single TAE* mode with perturbed quantities $\propto \exp i(n\varphi - \omega t)$, the wave-particle interaction is invariant with respect to transformation

$$t \to t + \tau; \quad \varphi \to \varphi + \frac{\omega}{n}\tau$$

- In the presence of the TAE, neither E nor P_φ is conserved for particle orbit, but *their following combination is still invariant*:

$$E - \frac{\omega}{n} P_\varphi = const$$

- Change in the particle energy is related to change in particle radius produced by TAE

$$\Delta E = \frac{\omega}{n}\Delta P_\varphi \cong \frac{\omega e}{nc}\psi'\Delta r$$

- The relative change in particle energy is much smaller than in particle radius:

$$\frac{\Delta v}{v} = \frac{\omega}{\omega_{*\alpha}} \cdot \frac{\Delta r}{L_\alpha}; \quad \text{where } \omega_{*\alpha} \equiv \frac{nq\rho_\alpha v}{2rL_\alpha} >> \omega$$

QUALITATIVE ESTIMATES – 2

- **The interaction between TAE and fast particles causes *radial transport of the particles at nearly constant energy***

- **This type of interaction is extremely unpleasant as it may deposit a population of fusion born alphas too close to the first wall**

- **Losses of fusion born alphas must be minimised down to few percent (<5% on ITER) for avoiding the first wall damage**

- **The radial redistribution also gives a non self-consistent alpha-heating profiles etc. and may affect the burn**

TWO MAIN TYPES OF THE TAE-INDUCED TRANSPORT

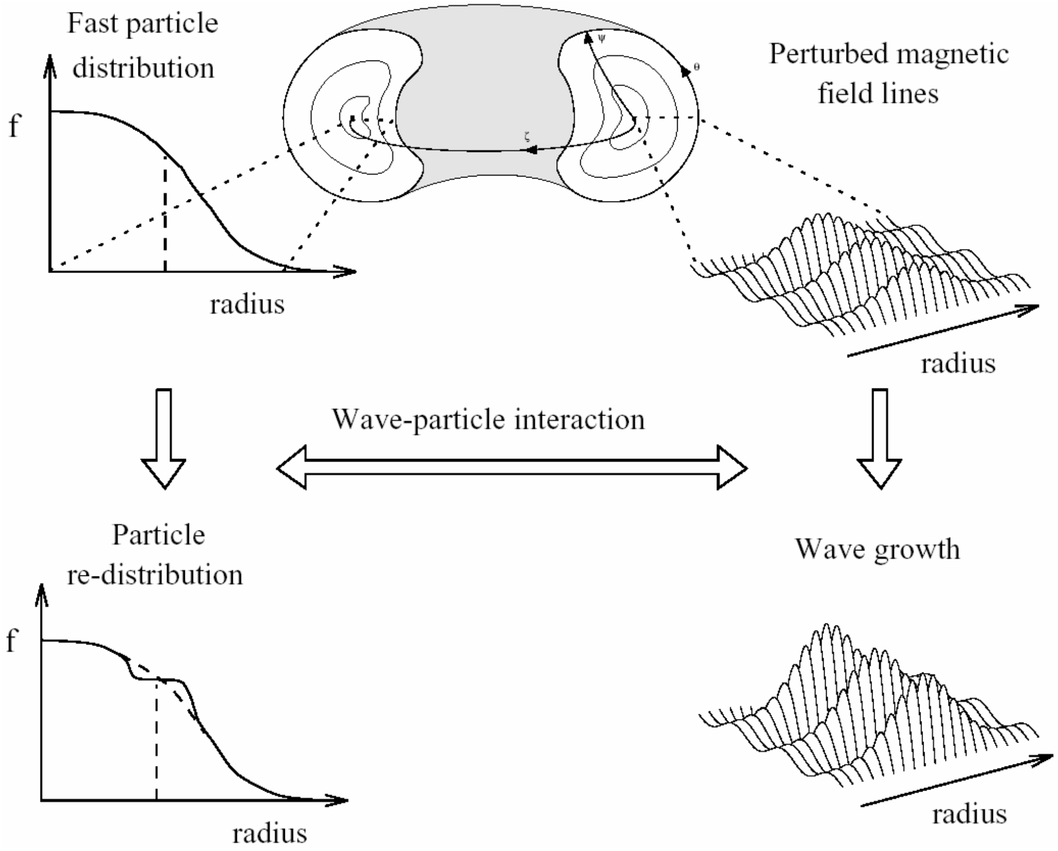
- Fast ion orbits comparable to the machine radius, $\rho_\alpha / a \cong 10^{-1} \div 1$. A single-mode *'convective' transport* is observed in present-day machines (DIII-D, TFTR, JET, JT-60U). TAE-induced enhancement of prompt losses is important, losses $\propto \delta B_{TAE}$

- For ITER with parameter $\rho_\alpha / a \cong 10^{-2}$ the dominant channel of alpha-particle transport is predicted to differ from present-day machines.

- On ITER, higher-*n* (*n* > 10) TAEs will be most unstable. The radial width of a poloidal harmonic will be more narrow, $\Delta_{\mathrm{mod}\,e} \propto r_{AE} / nq$, but the number of unstable modes may be significantly larger than in present-day tokamaks

- Resonance overlap will lead to a global stochastic diffusion of energetic ions over a broad region with unstable Aes, with transport $\propto \delta B_{TAE}^2$

MODELLING TAE-ORBIT INTERACTION (HAGIS CODE)



S.D.Pinches et al., Computer Physics Communications 111 (1998) 133

HAMILTONIAN APPROACH FOR δf

Trajectory of each individual macro-particle follows the Hamiltonian approach [White & Chance, Phys. Fluids 27 (10) 1984] leading to equations of the type:

$$\frac{\partial \psi_p}{\partial \vartheta} = \frac{1}{D}\left[I \frac{\partial \tilde{A}_\zeta}{\partial \vartheta} - g \frac{\partial \tilde{A}_\vartheta}{\partial \vartheta} \right]; \quad \frac{\partial \psi_p}{\partial \zeta} = \frac{1}{D}\left[I \frac{\partial \tilde{A}_\zeta}{\partial \vartheta} - g \frac{\partial \tilde{A}_\vartheta}{\partial \zeta} \right]; \quad \frac{\partial \psi_p}{\partial P_\vartheta} = \frac{g}{D}; \quad \frac{\partial \psi_p}{\partial P_\zeta} = -\frac{I}{D}$$

For the shear Alfvén modes, the assumption $\tilde{\mathbf{A}} = \tilde{\alpha}(\mathbf{x}, t) \cdot \mathbf{B}_0$ is used;

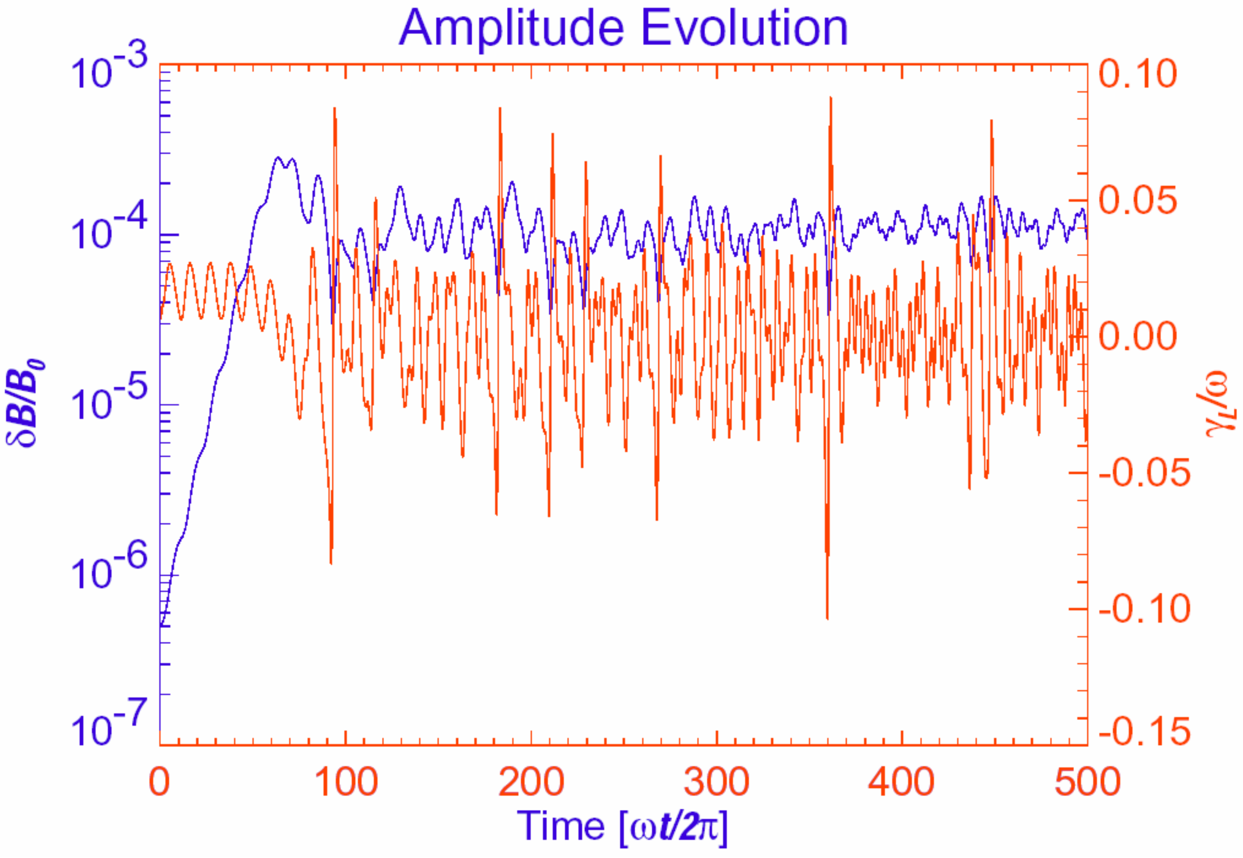
Nonlinear code: for the eigenmode structure provided by CASTOR or MISHKA, the mode amplitude and phase are evolving through (schematically):

$$\frac{dA}{dt} = A_0 + \sum_{particles} (\ldots) - \gamma_{damp} A; \qquad \frac{d\varphi}{dt} = \varphi_0 + \sum_{particles} (\ldots),$$

for unchanged mode structure

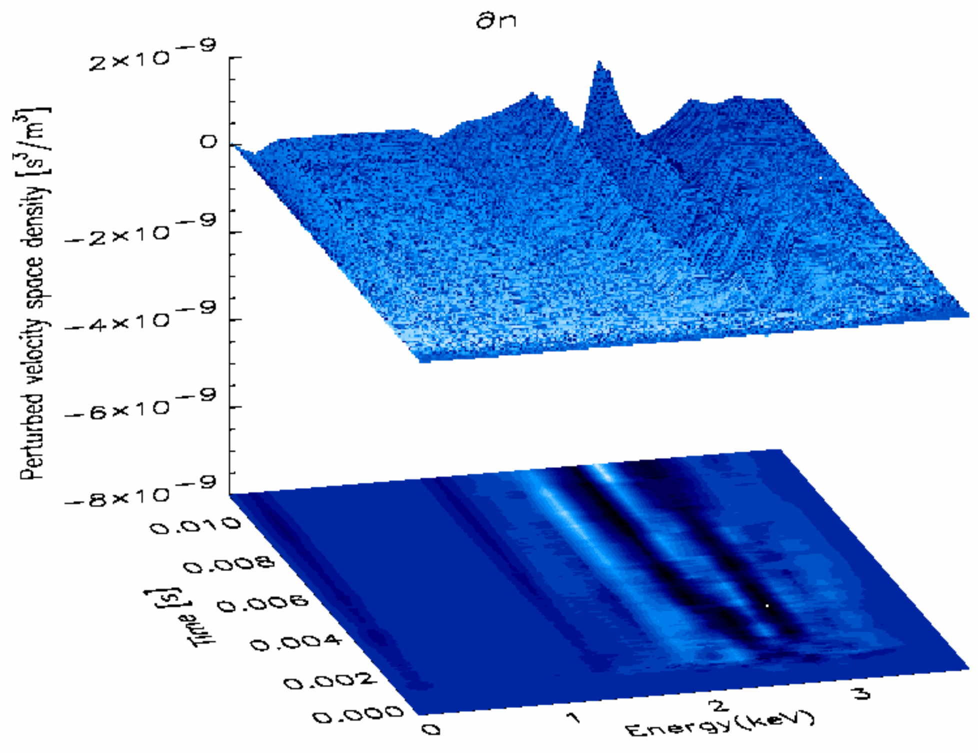
δf low-noise technique is used for deviation from f_0 computed by launching $>10^5$ macro-particles

MODE EVOLUTION FROM HAGIS

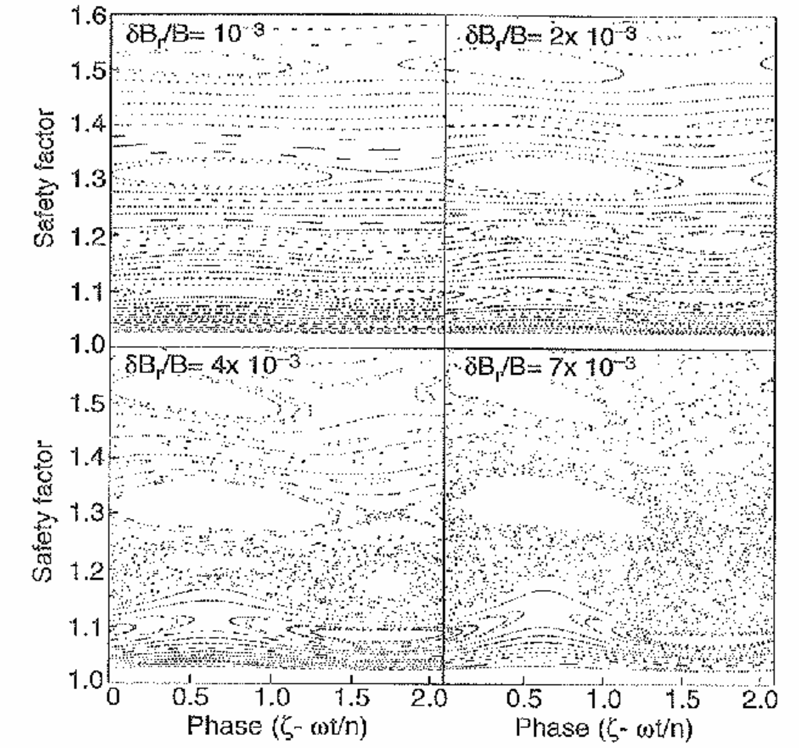


γ_d/ω = 2%, $\beta\langle f\rangle = 3\times10^{-4}$

FAST ION REDISTRIBUTION



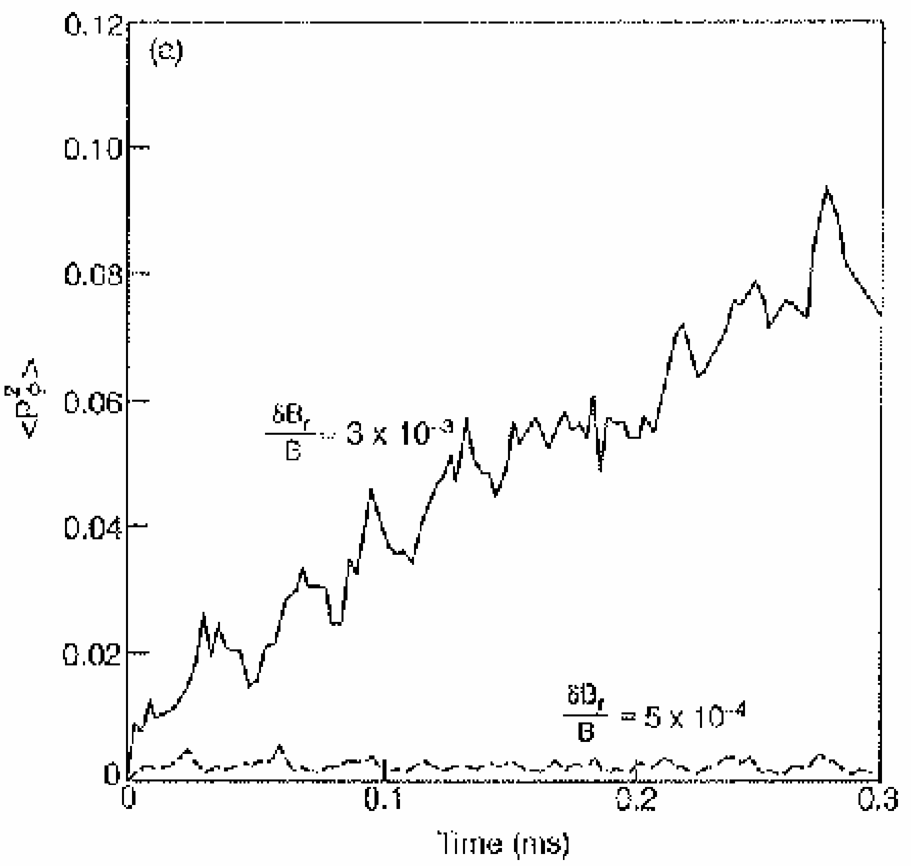
DRIFT ORBIT STOCHASTICITY (HAGIS MODELLING FOR JET)



- The analytically derived stochasticity threshold (**Berk et al Phys. Fluids B5, 1506, 1993**) is close to that obtained numerically:

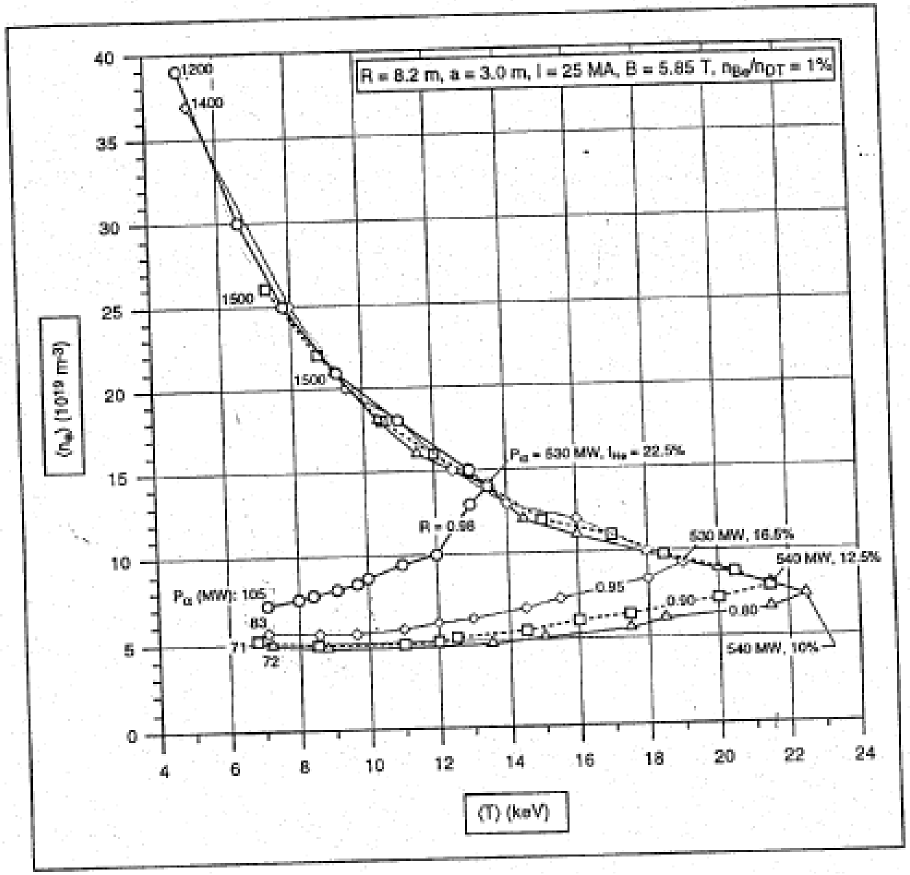
$$\delta B_r / B_0 > r_{TAE} \cdot \left(64 m R_0 q S\right)^{-1} \cong 1.5 \times 10^{-3} / m$$

STOCHASTIC TRANSPORT OF ALPHAS ON JET (HAGIS-95)



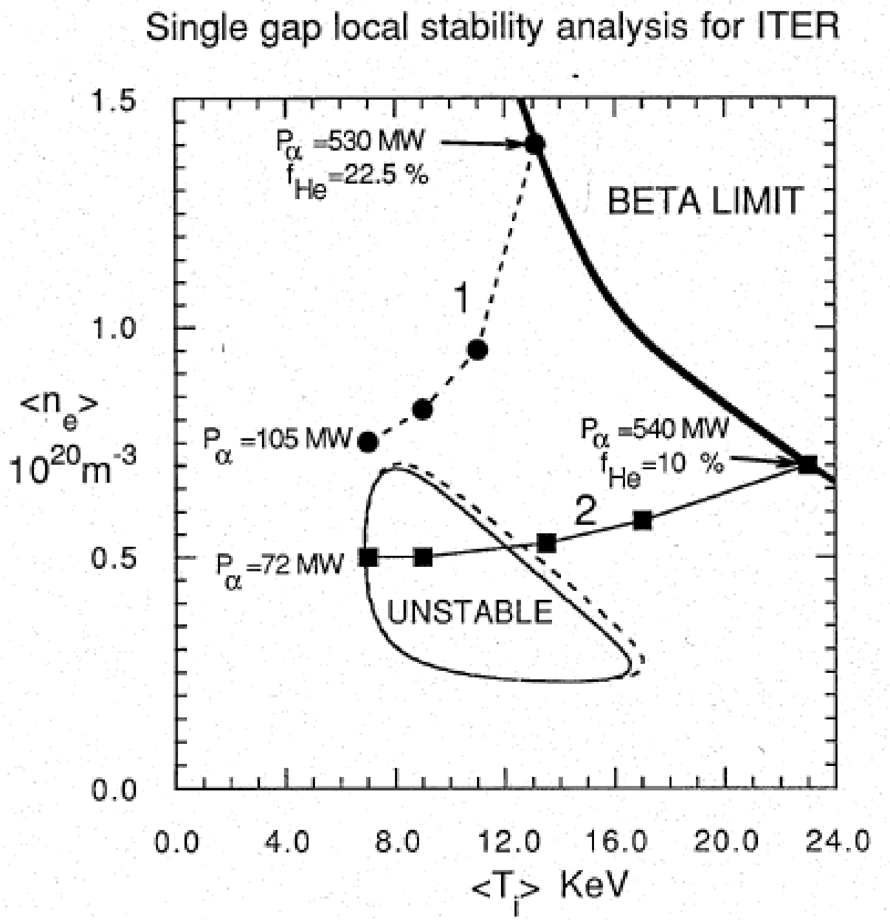
WHAT ANALYSIS COULD AND SHOULD BE DONE FOR ITER

“ITER-94” PATH TO IGNITION (from TAC-6 Report, 93-12-17 F)



S.E. Sharapov, IST Seminar, Lisbon, 17 May 2013

OUR ASSESSMENT OF TAE IN "IGNITED ITER" (≈1994)



“OLD ITER” ASSESSMENT (≈1994)

- ITER was aiming at ignition → alphas only were considered as the source of instabilities
- H-mode scenario was the only case of interest (positive shear, q(0)<1, etc.)
- TAE and Kinetic TAE (no radiative damping, somewhat higher frequency than TAE) were only considered
- Alpha drive, thermal ion Landau damping, electron collisional damping, radiative damping were taken into account
- The “ignited operational point” and the “path to ignition” were both looked at

IMPORTANT EFFECTS IN “OLD ITER” ASSESSMENT (≈1994)

- Although the “ignited operational point” was found to be stable with respect to TAE, the “path to ignition” was found to be unstable to TAE with n>10

- Thermal ion Landau damping

$$\gamma_i / \omega \propto \exp\left(-\frac{m_i}{2T_i} \times \left(\frac{V_A}{3} \right)^2 \right)$$

was the main stabilising effect with D contribution >> T contribution

- The role of impurities (for D:T=50:50): exponentially sensitive,

$$V_A = \frac{B_0}{\sqrt{4\pi m_p \left(2.5 n_{DT} + 4 n_{He} + 9 n_{Be}\right)}}$$

NOW IT IS NECESSARY TO INVESTIGATE “NEW” BURNING PLASMA ITER

- ITER is not ignited → alphas should be considered together with 1 MeV (super-Alfvénic) NBI and ICRH-accelerated tail, Q=10 means $P_{alpha} / P_{NBI} = 2$

- H-mode scenario + hybrid scenario + ITB scenario

- Not only TAE should be considered, but also Alfvén Cascades in advanced tokamak scenario with reversed magnetic shear and fishbones in hybrid

- The “operational points” and the “paths to these points” in all scenarios should be looked at

“NEW” ITER

A comprehensive analysis of new ITER was done so far by N.N.Gorelenkov, H.L.Berk, R.Budny, and A.Polevoi (e.g. BP Workshop, Oak Ridge, 8 Dec. 2005). This included:

- Alphas and NBI,
- Three scenarios (Normal Shear, Hybrid, Reversed Shear),
- TAE only

CONCLUSIONS DRAWN

- AEs driven by alphas alone are marginally stable in all scenarios;
- 1 MeV NBI is driving TAEs as much as alphas → lowering NBI energy to 500 keV may be needed for TAE stability;
- Hybrid scenario is the most unstable with growth rate up to 6%→non-perturbative theory may be required

MISSING BITS FOR TAE ANALYSIS

- Paths to the operational points may be more unstable
- Uncertainty associated with D:T mixture, He and Be concentrations should be assessed
- Effect of ICRH-accelerated ions should be assessed – these may be even stabilising TAE

SUMMARY

- Burning/ ignited magnetic fusion plasmas constitute a very challenging physics field, which require strong effort for minimising uncertainties for the forthcoming ITER experiment
- Super-Alfvénic alpha-particles may excite Alfvén instabilities, which give undesirable re-distribution and losses of alphas
- Present-day experiment and modelling of Alfvén instabilities are quite mature, but ITER needs a dedicated effort in building a reliable instrument working fast enough for forecasting TAE in every ITER scenario
- The CASTOR-K code developed and available at IST has the potential of assessing linear TAE stability fast and this makes CASTOR-K a valuable asset in ITER modelling
- Nonlinear evolution of TAE and alpha-particle re-distribution and losses will be the next essential step in ITER modelling, after linear TAE stability is assessed