Sawtooth Period Scalings

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Outline

Sawtooth

- Sawtooth oscillations: regular period reorganization of the core plasma surrounding the magnetic axis

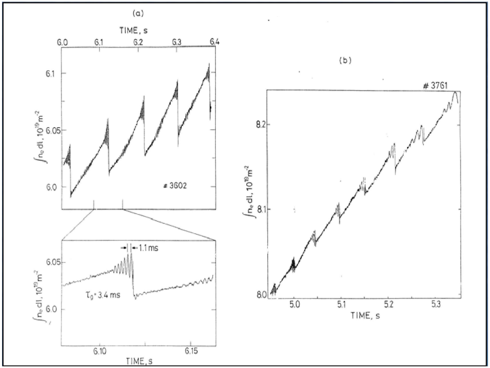
Three stages

- Ramp phase
- Precursor oscillation phase
- Crash

Deleterius

- Couple to the boundary of the confined plasma and trigger bursty modes that result into violent release of heat (Edge Localized Modes). Loss of confinement
- Trigger large "pressure driven islands" (neoclassical tearing modes), that cause plasma disruption. Loss of the whole plasma.

Early discharges in JET



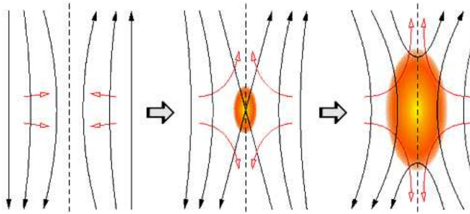
From Hastie (APSS, 1998)

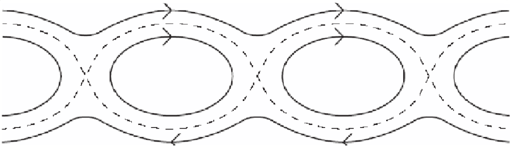
Magnetic Reconnection

- Kadomtsev (1976)

Arrangement of the structure of the magnetic field

- Breaking
- Merging
- Energy release
- Formation of magnetic structures (islands)





The breaking of the field lines happens at scales that depend on microscopic physics

far from the reconnection region the plasma is perfectly conducting (ideal Magnetohydrodynamics)

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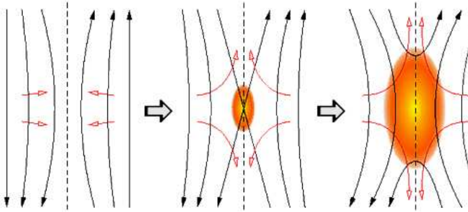

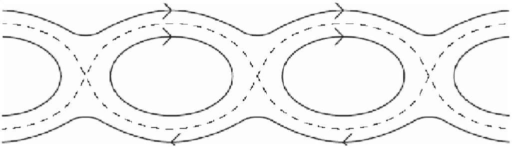

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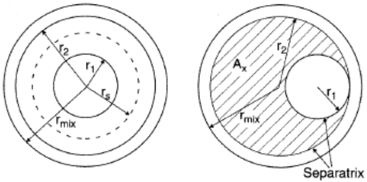
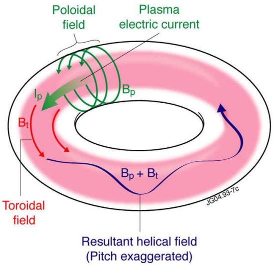
Toroidal Plasmas

- TOKAMAK (Magnetic Toroidal Chamber)

Reconnection?

- Section of the torus
- Plasma core displacement





Toroidal field $\mathbf{B} = \mathbf{B}_{\boldsymbol{p}} + \mathbf{B}_{\boldsymbol{t}}$

This system undergoes a number of hydromagnetic instabilities related to magnetic reconnection

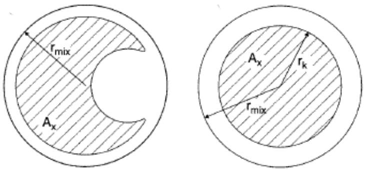
Magnetic field lines literally tear apart (tearing modes) and break

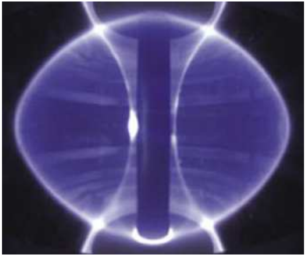
Toroidal Plasmas

- TOKAMAK (Magnetic Toroidal Chamber)

Reconnection!

- Section of the torus
- Croissant-shaped magnetic island





Spherical Tokamak MAST, Culham, UK

This system undergoes a number of hydromagnetic instabilities related to magnetic reconnection

Fundamental parameter for stabilty $q(r) = r/R(B_t/B_p)$, the "safety factor", field lines' pitch

Instabilities occur at $q(r_{n,m}) = n/m$ (rational surfaces)

The context

- In a recent work [Connor Hastie Zocco PPCF, **54**, 3 (2012) or arXiv:1110.2398] we studied the stability of those reconnecting/kink modes we believe are involved in the phenomenology of the Sawtooth in Tokamaks
- General theory of drift-tearing and internal kink modes with non-isothermal electrons (semicollisional) and gyrokinetic ions.
- Why? The accepted picture is that the Sawtooth in triggered when a stability threshold is crossed.
- However: (generally but no always) three phases $\Rightarrow$ ramp, instability (precursors, not always), crash.

- The process is periodic: we have to know what takes you to the pre-crash conditions after a crash.
- Here the pre-crash condition is believed to be the picture in Connor Hastie Zocco PPCF, **54**, 3 (2012), with all the boundaries
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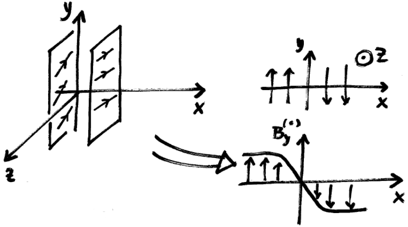
Very important

- Why semicollisional?
- Why the drift-tearing mode?

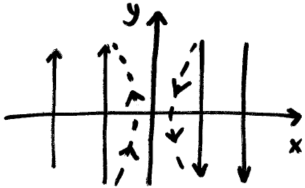
Equilibrium

Start with a sheared magnetic field



$$\mathbf{B} = \underline{\mathbf{e}}_z B_0 + \underline{\mathbf{e}}_z \times \nabla A_{\|,eq}$$

Add a small localized perturbation



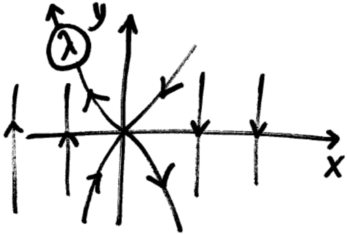
$$A_{\|}(x,y,t) = A_{\|,eq}(x) + \tilde{A}_{\|}(x)e^{iky-i\omega t}$$

This equilibrium is prone to formation of singularities.

Once the equilibrium is perturbed, the mode evolves to resolve the singularity by forming a magnetic island

Islands

If λ curvilinear variable moving along the field line



By the definition of magnetic field lines

$$\frac{dx(\lambda)}{d\lambda} = \delta B_x\left(x(\lambda), y(\lambda)\right) = \frac{\partial \delta A_\parallel}{\partial y(\lambda)}$$

$$\frac{dy(\lambda)}{d\lambda} = \delta B_y\left(x(\lambda), y(\lambda)\right) = -\frac{\partial \delta A_\parallel}{\partial x(\lambda)}$$

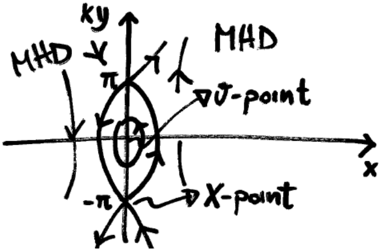
$A_\parallel$ is the Hamiltonian of the field lines

$\mathscr{X}$ and $\mathscr{O}$ Points

For an even localized perturbation in x, sinusoidal in y

$$\delta A_{\|} \approx A_{\|}(0)\left\{\frac{x^2}{2} - \cos(ky)\right\}$$

The phase portrait of the perturbed magnetic potential



The equilibria are $(x_1, ky_1) = (0,0)$ and $(x_2, ky_2) = (0,\pi)$

Around (x_1, ky_1) displaced field line Eq. $\ddot{x} + A_{\|}^2(0)k^2x = 0 \Rightarrow \mathscr{O}-\mathrm{point}$

Around (x_2, ky_2) displaced field line Eq. $\ddot{x} - A_{\|}^2(0)k^2x = 0 \Rightarrow \mathscr{X}-\mathrm{point}$

The current can flow along the perturbed magnetic field, the magnetic flux increases, the island grows BUT NONIDEAL PHYSiCS IS NEEDED

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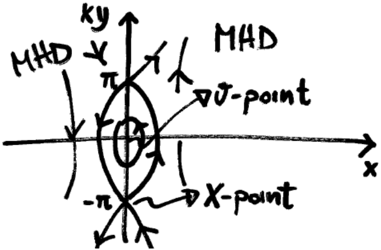

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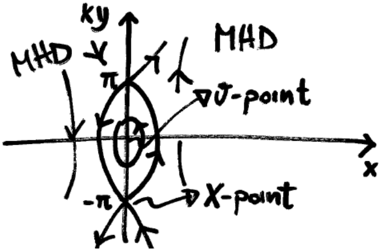

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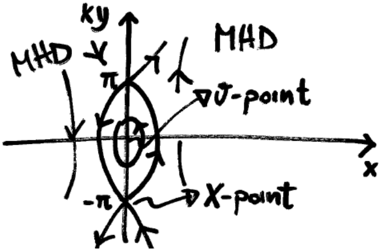

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Drift-tearing

Write *one* equation of motion of the electrons

$$\underset{\text{accel.}}{m_e(i\omega - \nu_{ei})\tilde{v}_\parallel} = \underset{\text{el. field + press. grad.}}{eE\left(1 - \frac{\omega_*}{\omega}\right)} + \underset{\text{visc. force}}{\frac{k_\parallel^2 T_{0e}}{\omega}\tilde{v}_\parallel}$$

There are regions where the current is limited by electron thermal conduction

$$m_e \nu_{ei} \tilde{v}_\parallel \sim \frac{k_\parallel^2 T_{0e}}{\omega}\tilde{v}_\parallel \Rightarrow \omega \nu_{ei} \sim k_\parallel^2 v_{the}^2$$

Once this is achieved, to maintain force balance

$$eE\left(1 - \frac{\omega_*}{\omega}\right) = 0 \Rightarrow \omega \approx \omega_*$$

the mode rotates in the electron direction.

The drift-tearing mode is a slowly growing rotating island

The island form because of small nonideal effects around the rational surface

No breaking of "frozen-in" law, no reconnection

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Why is this important?

Width of the mode $\delta_0 = \frac{\sqrt{\omega \nu_{ei}}}{k_y v_{the}} L_s$ ($L_s^{-1} \approx \partial_r q(r)$ magnetic shear length)

Introduce $\hat{\beta}_e = \beta_e \frac{L_s^2}{L_n^2}$ (L_n density gradient length, measure electr. diamagn.), given a resistive scale δ_η

- $\left(\frac{\delta_\eta}{\delta_0}\right)^2 \sim \frac{1}{\hat{\beta}_p} \Rightarrow$ for large density gradients and small magnetic shear the semicollisional theory is required
- $\frac{\delta_0}{\rho_i} \sim 0.1$ for typical JET parameters (ρ_i ion Larmor radius)

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Formulation

$k_\parallel v_{the} \ll \nu_{ei}$ neglect Landau damping

$\omega \sim k_\parallel^2 v_{the}^2/\nu_{ei}$ semicollisional effects

Braginskii's Eqs. for the electrons are valid

Can be derived in the collisional limit of the Kinetic Reduced Electron Heating Model [see Zocco-Schekochihin Phys. Plasmas, 18, 10, (2011)]

Here all background electron density and temperature gradients are kept.

- ***History...***
- Ion FLR stabilization [Antonsen Coppi (1982), BUT COLLISIONLESS]
- Diamagnetic stabilization, BUT COLD ION LIMIT [Drake et al. (1983)]
- Ion FLR stabilization [Cowley et al. (1985), semicoll. BUT SMALL Δ' (to be introduced)]
- Ion kinetic kink mode [Pegoraro et al. (1989), BUT no semicollisional physics]_
- We derive a unified theory for $\eta_e \sim \eta_i \sim \tau \sim \Delta' \rho_i \sim k_\perp \rho_i \sim 1$

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Double Asymptotic Matching

We can derive a general disperion relation for these modes

Within some subsidiary orderings we can study it analytically

- $\hat{\beta} \ll 1$
- $\hat{\beta} \gg 1$
- $\hat{\beta} \sim \eta_e \sim 1$, but $\omega/\omega_{*e} \to 1$ for the drift tearing mode
- $\hat{\beta} \sim 1$, but $\omega/\omega_{*e} \ll 1$ for the kink mode

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Transition to stability at finite $\hat{\beta}$

- We saw $\omega/\omega_{*e} \to 1$, for large Δ' but the electron region was solved imposing zero magnetic perturbation at the rational surface. [Drake et al. Phys. Fluids 26, 2509 (1983)]

- We can solve without imposing this constraint the complete fourth order differential electron equation in two separate electron sub-regions if $\eta_e \sim 2$.

- We match the two sub-regions, we match to the ion-region, and we get the shielding factor $\Lambda(\hat{\beta})$ missed before and calculate the critical $\hat{\beta}\eta_e$ for stabilisation

$$\hat{\beta} > \hat{\beta}_c = 0.34 \text{ (for } \eta_e \approx 2.53)$$

$$\hat{\beta} = \frac{\beta_0}{\varepsilon^2 [aq'(r_1)]^2} \approx 0.5/[aq'(r_1)]^2. \quad (1)$$

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- We saw $\omega/\omega_{*e} \to 1$, for large Δ' but the electron region was solved imposing zero magnetic perturbation at the rational surface. [Drake et al. Phys. Fluids 26, 2509 (1983)]

- We can solve without imposing this constraint the complete fourth order differential electron equation in two separate electron sub-regions if $\eta_e \sim 2$.

- We match the two sub-regions, we match to the ion-region, and we get the shielding factor $\Lambda(\hat{\beta})$ missed before and calculate the critical $\hat{\beta}\eta_e$ for stabilisation

$$\hat{\beta} > \hat{\beta}_c = 0.34 \text{ (for } \eta_e \approx 2.53)$$

$$\hat{\beta} = \frac{\beta_0}{\varepsilon^2 \left[aq'(r_1)\right]^2} \approx 0.5/\left[aq'(r_1)\right]^2. \tag{1}$$

Finite $\hat{\beta}$ theory of the internal kink mode

- If for these modes $\hat{\omega} \ll 1$, we can seek solutions based on an expansion in $\hat{\omega}$, rather that in $\hat{\beta}$ at finite $\hat{\omega}$.
- The small $\hat{\omega}$ expansion of the ion response fails for $k \sim \hat{\omega}^{-1} \gg 1$, we have to solve in this intermediate region before matching to the electron region
- The electron region is straightforward to solve iteratively in $\hat{\omega}$ (in the same way as in $\hat{\beta}$)
- After the matching we get a general dispersion relation. We derive an analytic expression for the boundary of stability $\hat{\gamma}(\lambda_H, \hat{\beta}) = 0$ ($\lambda_H^{-1} \propto -\Delta'$)

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Finite $\hat{\beta}$ theory of the internal kink mode

- The boundary of marginal stability is calculated analytically for the first time

$$\frac{\pi}{\delta_0 \Delta'} = H(\mu_1)\frac{\sqrt{1+\tau}}{8}\sqrt{\frac{d(\eta_e)}{d_1}} \times \left\{ \frac{\hat{\beta}}{1+\tau}\frac{\pi}{\cos(\pi\mu_1)}\sqrt{\frac{\sin(\pi\mu_1/2)}{\sin(3\pi\mu_1/2)}} - 1 - \frac{\hat{\beta}}{1+\tau}\left[\frac{3}{2} - k_0 - \left(1-\frac{\eta_i}{2}\right) I_2 + \ln\left(\frac{\delta_0}{\rho_i} H(\mu_1)\frac{\sqrt{1+\tau}}{8}\sqrt{\frac{d(\eta_e)}{d_1}}\right)\right]\right\} \quad (2)$$

with

$$H(\mu_1) = \left\{\frac{1/2+\mu_1}{1/2-\mu_1}\frac{\Gamma^2(-\mu_1)}{\Gamma^2(\mu_1)}\frac{\cos(\pi\mu_1)}{\cos(\pi\mu_1/2)+\sqrt{\sin(\pi\mu_1/2)\sin(3\pi\mu_1/2)}}\right\}^{\frac{1}{2\mu_1}}, \quad (3)$$

$2\mu_1 = \sqrt{1+4\hat{\beta}/(1+\tau)}$, $k_0 = \psi(1)+\psi(3)-\psi(3/2-\mu_1)-\psi(3/2+\mu_1)$, where ψ is the digamma function, $I_2(\eta_i,\tau) = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{dk}{k^2}\left[\frac{F_0}{G_0} - \frac{\sqrt{\pi}}{1-\eta_i/2}\frac{k^2}{(1+k)}\right]$, $G_0(k) = -(1-\hat{\omega}^{-1}) + F_0(k)$,
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The mode is unstable and rotates in the ion direction

$$\hat{\omega} = -\sqrt{d(\eta_e)\frac{1+\tau}{2\pi d_1}}\left(1-\frac{\eta_i}{2}\right)\frac{1}{\ln(\rho_i\hat{\beta}^2/\delta_0)}\frac{\delta_0}{\rho_i\hat{\beta}^2}e^{-i\frac{\pi}{4}}. \quad (4)$$

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$$\left. - \frac{\hat{\beta}}{1+\tau}\left[\frac{3}{2} - k_0 - \left(1-\frac{\eta_i}{2}\right) I_2 \; + \ln\left(\frac{\delta_0}{\rho_i}H(\mu_1)\frac{\sqrt{1+\tau}}{8}\sqrt{\frac{d(\eta_e)}{d_1}}\right)\right]\right\} \tag{2}$$

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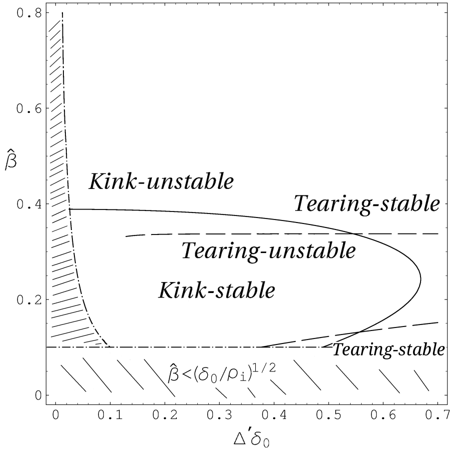
Finite $\hat{\beta}$ theory

Boundaries of marginal stability



$$\hat{\gamma}(\delta_0\Delta', \hat{\beta}) = 0$$

Finite $\hat{\beta}$ theory

Local critical shear [Electron Cyclotron Current Drive control showed the importance]

$$\hat{s}_c \approx \frac{1}{\Delta' r_1}\frac{R}{a}\frac{r_1^2}{a^2}\sqrt{\frac{\Omega_e}{0.5\nu_e}}2\pi^2\frac{\beta_e}{\sqrt{\frac{1+\tau}{4.26}(4.08-1.71\eta_e)}}. \qquad (5)$$

- Derived from the explicit expression of the boundary of marginal stability!!!
- Not given by the diamagnetic stabilisation condition $\hat{\gamma} \ll \omega_{*i}$ of Porcelli-Boucher-Rosenbluth [Plasma Phys. and Control. Fusion 38, 2163 (1996).]
- We can also derive from first priciples some heuristic constants introduced by Porcelli et al.
- Notice $\hat{s}_c \propto \delta W^{1/3}$, in the standard notation of MHD stability ($\delta W = \hat{s}^2/\Delta' r_1$).

Evolution of the q profile

- We saw that the stability of reconnecting and ki[Ge'lfand Shilov ('60)]nk modes can be described in the plane $(\hat{\beta}, \Delta')$.
- We could aim at a criterion for the onset similar to that Porcelli-Boucher-Rosenbluth (that is, the crossing of the boundary of marginal stability)
- It is the evolution of q after the crash that tells us when the boundary is crossed
- For this we need to derive a q equation coupled to transport equations
- We content ourselves with the exact boundary of stability to be implemented in transport codes, and proceed with a simple model for the neoclassical resistivity
- Idea (Gimblett and Hastie): the evolution of the safety factor on-axis can drive MHD modes to trigger the Sawtooth (1994).

Evolution of the q profile

- As first suggested by Park and Monticello [Nucl. Fusion 30, 2413 (1990)], we consider the importance of the trapped particles

Neoclassical resistivity is given approximately by (Hirshmann et al)

$$\eta(r) = \eta_{Sp}(r)/(1 - \sqrt{r/R_0})^2, \tag{6}$$

where η_{Sp} is the Spitzer resistivity. With the electron temperature profile given by $T_e(r) = T_0(1 - r^2/a^2)^{4/3}$, the Spitzer resistivity has the form

$$\eta_{Sp}(r) = \frac{\eta_0}{(1 - r^2/a^2)^2}. \tag{7}$$

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Evolution of the q profile

We construct the relevant diffusion equation for the q profile in the cylindrical Tokamak limit retaining one toroidal effect, namely the neoclassical correction to resistivity. Thus,

$$\begin{aligned} \frac{\partial B_\theta}{\partial t} &= -c\,(\nabla \times \mathbf{E})_\theta \\ &= c\,\frac{\partial}{\partial r}\,(\eta J_z) \\ &= \frac{\partial}{\partial r}\left[\frac{\eta c^2}{4\pi r}\,\frac{\partial}{\partial r}\,(rB_\theta)\right], \end{aligned} \tag{8}$$

and using the definition of the safety factor $q(r) = \frac{r}{R_0}\frac{B_z}{B_\theta}$.

$$\frac{\partial}{\partial \tau}\left(\frac{1}{q}\right) = 4\frac{\partial}{\partial x}\left[\hat{\eta}(x)\frac{\partial}{\partial x}\frac{x}{q}\right]. \tag{9}$$

$\tau = t/\tau_\eta$, $x = r^2/a^2$, with $\tau_\eta = 4\pi a^2/(\eta c^2)$,

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where $\varepsilon = a/R_0$. Clearly, the quartic power in the trapped electron correction to Spitzer resistivity generates (unphysical) singular behaviour, for $x \to 0$.

This is removed by including the transition from a neoclassical resistivity to Spitzer when

$$\nu_e > \frac{v_{the}}{R_0 q_0}\left(\frac{r}{R_0}\right)^{3/2}. \qquad (11)$$

Incorporating this correction, the expression for the resistivity becomes

$$\hat{\eta}(x) = \frac{1}{(1-x)^2(1-\frac{\sqrt{\varepsilon}x}{x^{3/4}+\nu_*})^2}, \qquad (12)$$

where $\nu_* = \nu_e/(\varepsilon^{3/2}\omega_{te})$, with $\omega_{te} = v_{the}/(qR_0)$ the transit frequency of thermal electrons.

- In JET or ITER, the dimensionless parameter ν_* is small, so that resistive evolution in the vicinity of the magnetic axis, though not singular there, is likely to be rapid

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By expanding Eq. (9) locally around $x = 0$, and employing Eq. (15), one obtains the solution

$$q_0(t) = q_0(0)\exp\left(-t/\tau_*\right), \tag{16}$$

with

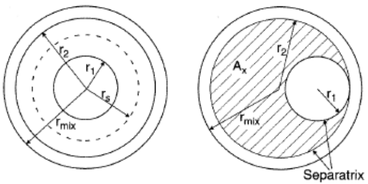
$$\tau_* = \tau_\eta \frac{\nu_*}{8\sqrt{\varepsilon}} \propto \frac{R_0^3 N_e}{T_e^{1/2}}. \tag{17}$$

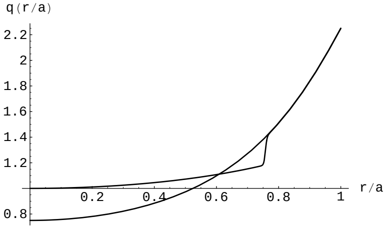
Hence, at early times, the safety factor undergoes an exponential decay on the timescale τ_*.

Evolution of the q profile

Kadomtsev Reconnection

- Section of the torus
- Plasma core displacement





Pre and post crash Kadomtsev states

$$\frac{2}{q_{pc}} = \frac{1-\tanh\left(\frac{r-r_{mix}}{\delta}\right)}{q_{fin}} + \frac{1+\tanh\left(\frac{r-r_{mix}}{\delta}\right)}{q_{in}}$$

where $q_{in} = q_0/(1-x^2+1/3x^2)$, $q_{fin} = 1/(1-0.27x)$, $q_0 = 0.75$, $r_{mix}/a = 9-\sqrt{144q_0-63}/4 \approx 0.573a$, and $\delta = 5\times 10^{-3}a$.

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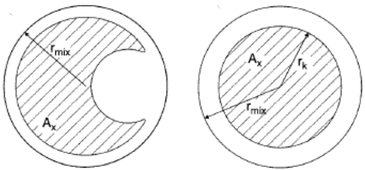
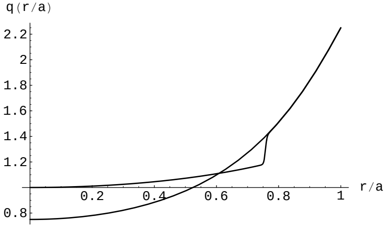




Pre and post crash Kadomtsev states

We could use the Gimblett-Hastie state [Plasma Phys. and Control. Fusion 36, 1439 (1994)] or

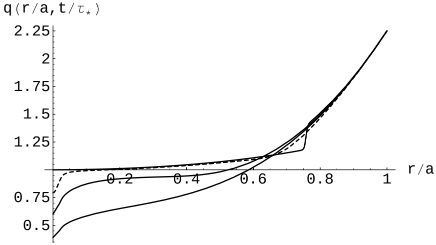
The incomplete reconnection state [C. G. Gimblett and R. J. Hastie, PPN/94/30 (Nov 1994)]

[F. Porcelli, D. Boucher, and M. N. Rosenbluth, Plasma Phys. and Control. Fusion 38, 2163 (1996)].

For our purposes, it is not important how you get the post-crash q profile!

Post-crash evolution of the q profile

Post-crash evolution



Two important facts

Rapid diffusive broadening of the initial current sheet at $r = r_{mix}$

Rapid downward evolution of $q(0, t)$

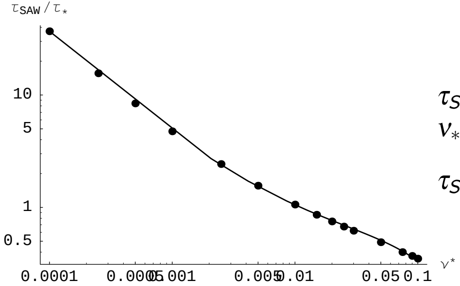
Axial criterion and Sawtooth Period

One might wonder what can limit such evolution of q on axis.

- From the theory of ideal MHD, for $q_0 \leq 1/2$, an $m = 1$, $n = 2$ mode becomes unstable in a cylindrical plasma, [True also in a torus Bussac et al PRL (1975)]
- Phenomenologically, having $q_0 \approx 0.75$, if it is not a sufficient condition, surely is necessary for the sawtooth trigger
- Hence, it is tempting to look for a correlation between the crossing of $q \leq q_0$, and the Sawtooth period.
- Solve for the time at which $q(0, \tau_{SAW}) - 0.75 = 0$.

Axial Crieterion and Sawtooth Period

Sawtooth period



Scalings

$\tau_{SAW} \sim \tau_* \nu_*^{-1/3} \propto R_0^{8/3} N_e^{2/3} T_e^{1/6}$ sec.$001 \lesssim \nu_* \lesssim .1$

$\tau_{SAW} \sim \tau_\eta \propto T_e^{3/2} a^2$ sec.

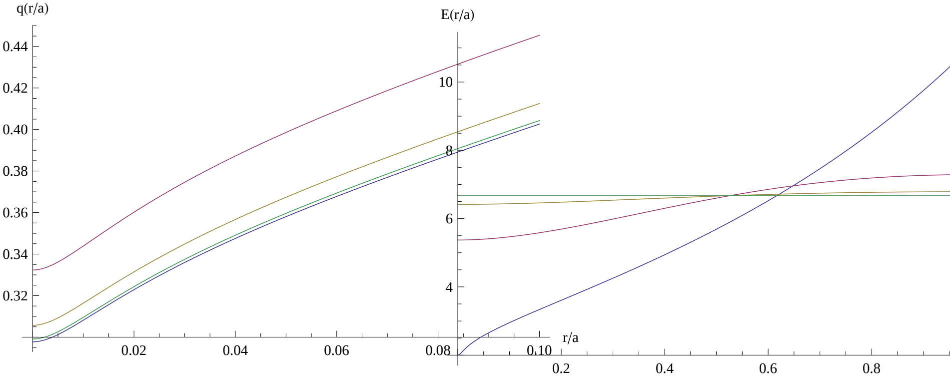
JET	ITER
$a = 1\,\mathrm{m}$	$a = 3\,\mathrm{m}$
$T_e = 4\,\mathrm{keV}$	$T_e = 25\,\mathrm{keV}$
$\tau_\eta \sim 400\,\mathrm{sec}$	$\tau_\eta \sim 24 \times 10^3\,\mathrm{sec}$
$\tau_* \sim 0.86\,\mathrm{sec}$	$\tau_* \sim 3\,\mathrm{sec}$
$\nu_* \sim 0.01\,\mathrm{sec}$	$\nu_* \sim 6 \times 10^{-4}\,\mathrm{sec}$
$\delta_* \sim 4.6\,\mathrm{cm}$	$\delta_* \sim 1.4\,\mathrm{cm}$

Conclusion

- We discussed the role of neoclassical resistivity and local magnetic shear in the prediction of the sawtooth period in Tokamaks.
- We calculated the new critical shear for stabilisation of the dissipative kink mode with grokinetic ions and semicollisional electrons, improving previous results.
- We then considered the influence of neoclassical resistivity on the evolution of the safety factor on-axis, $q(0,t)$. This evolves on a new time scale much shorter than the resistive diffusion time, and is characterised by the formation of a structure of size $\delta_* \sim \nu_*^{2/3} a$, with$a$ the minor radius.
- We explored the possibility of having the Sawtooth triggered by the the ideal MHD instability $m = 1$, $n = 2$, which can be driven when $q(0,t) \approx 0.75$.
- When $.001 \lesssim \nu_* \lesssim .01$, we find a "sawtooth period" scaling as $\tau_{SAW} \sim R_0^{8/3} N_e^{2/3} T_e^{1/6}$ sec. For smaller ν_*, the width δ_* becomes negligible compared to the position of the resonant surface, and cannot change the global resistive dynamics.
- For ITER, we estimate values of the Sawtooth period much shorter than what one would expect from a simple resistive diffusion model of the q profile: $\tau_{SAW} \lesssim 100\,\text{sec}$.

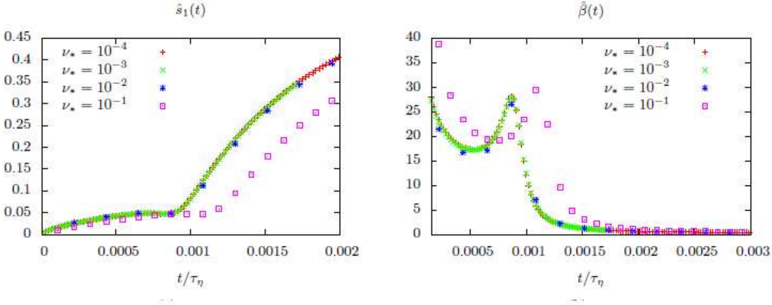
Post-crash evolution of the q profile

Long time equilibrium q



Long time equilibrium Electric field

Post-crash evolution of the q profile



Evolution of the q profile