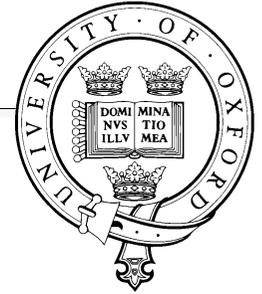




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IPFN, IST, Lisbon 12 September 2014



# *Toward a Theory of Plasma Dynamo*

*Magnetic Fields and Microinstabilities in a Weakly Collisional Plasma*

**Alexander Schekochihin (Oxford)**

Steve Cowley (UKAEA)

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Rincon, AAS & Cowley, arXiv:1407.4707 (2014)

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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# *Toward a Theory of Plasma Dynamo*

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**best astro theorist  
on the job market**

→ **Matt Kunz (Princeton)**

Scott Melville (Oxford) ← clever undergraduate

Federico Mogavero (ENS Paris) ← clever undergraduate

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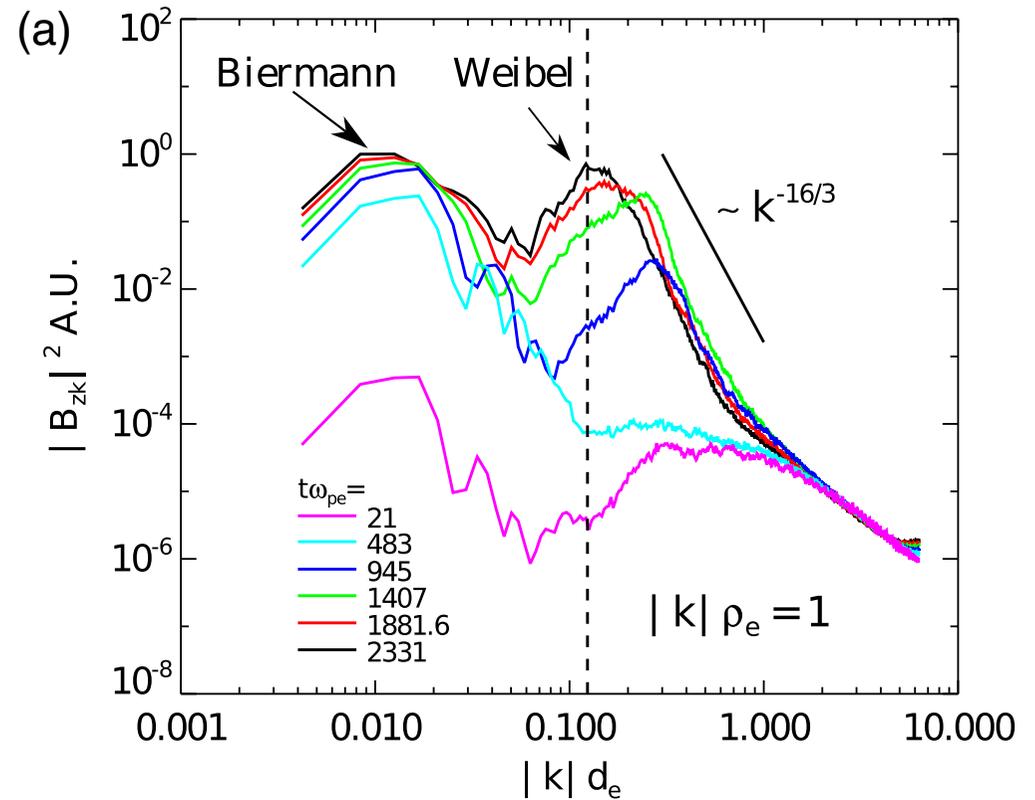
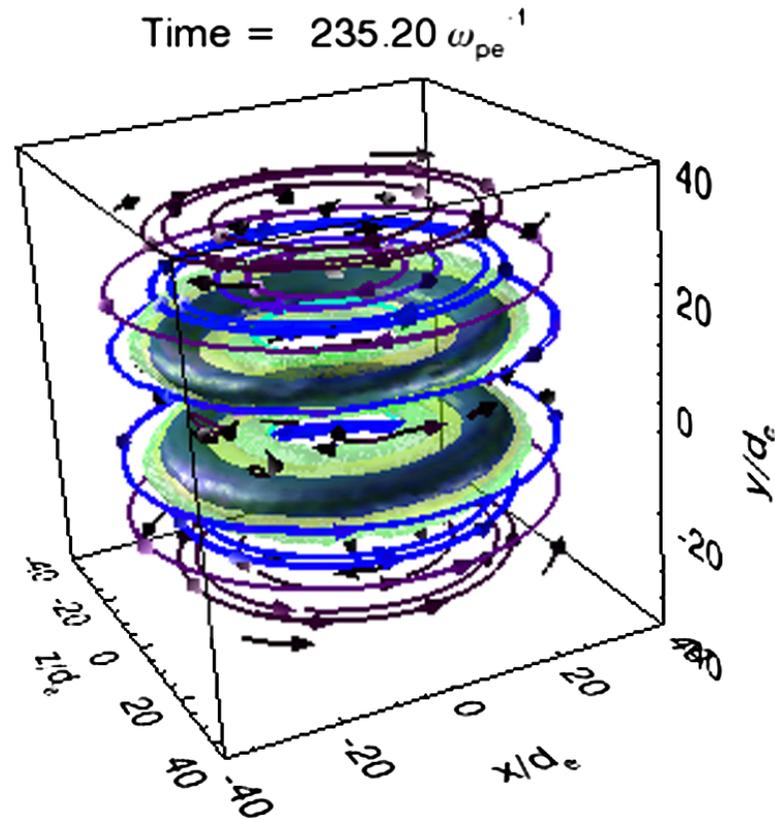
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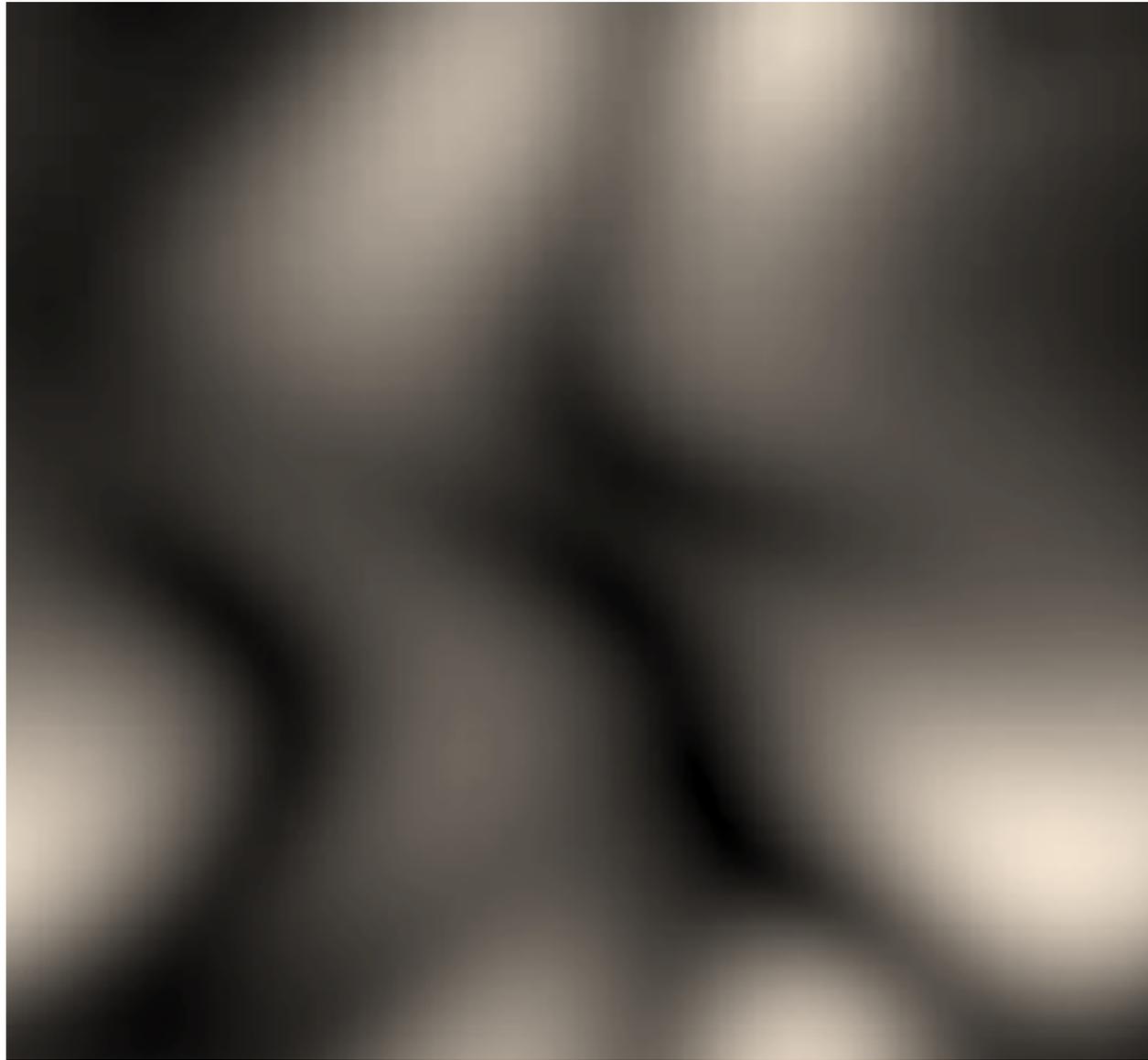
# What I Will Not Talk About



[Schoeffler, Loureiro, Fonseca & Silva 2014, PRL 112, 175001]

# Standard Turbulent MHD Dynamo

---



# Standard Turbulent MHD Dynamo



This was the solution of

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{d\mathbf{B}}{dt} \equiv \frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u}$$

$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla \mathbf{u})B \equiv \gamma B$$

$$\ln B \sim \int^t dt' (\mathbf{b}\mathbf{b} : \nabla \mathbf{u})(t')$$

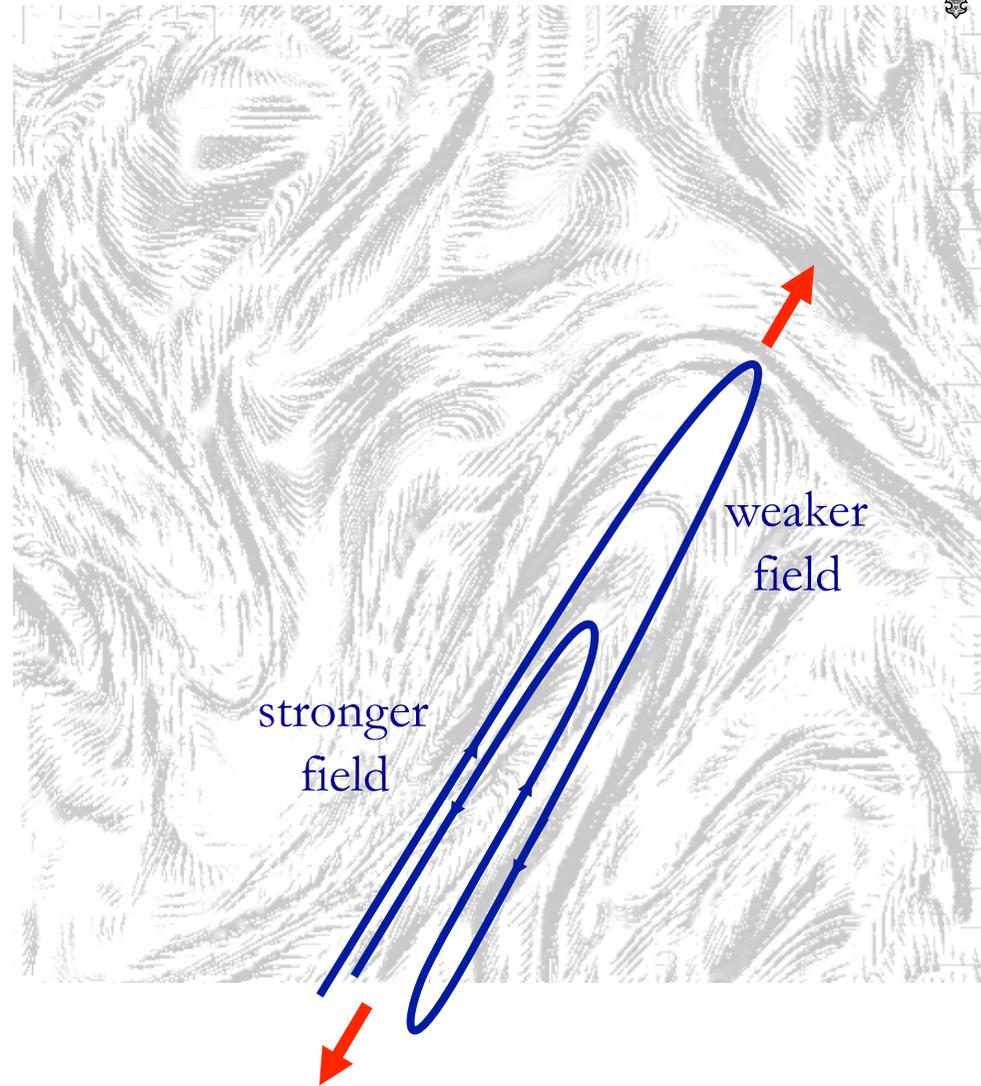


So, roughly, field in Lagrangian frame accumulates as random walk  
(in fact, situation more complex because of need to combat resistivity)

# Standard Turbulent MHD Dynamo



$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$



Key effect: a succession of random stretchings (and un-stretchings)

# Weak Collisions $\rightarrow$ Pressure Anisotropy



Changing magnetic field causes local pressure anisotropies:

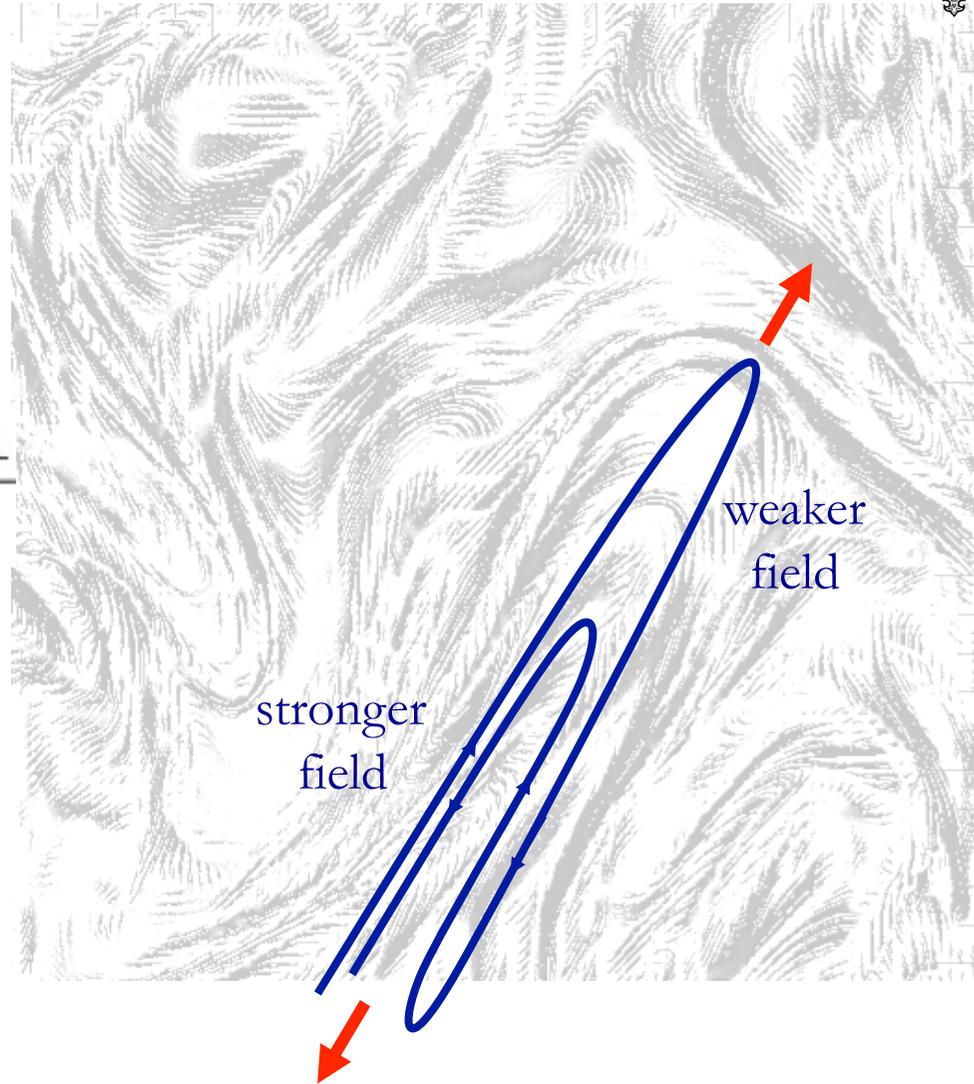
$$\frac{1}{p_{\perp}} \frac{dp_{\perp}}{dt} = \frac{1}{B} \frac{dB}{dt} - \nu \frac{p_{\perp} - p_{\parallel}}{p_{\perp}}$$

conservation of  $\mu = v_{\perp}^2/B$

$$\frac{1}{2p_{\parallel}} \frac{dp_{\parallel}}{dt} = -\frac{1}{B} \frac{dB}{dt} - \nu \frac{p_{\parallel} - p_{\perp}}{p_{\parallel}}$$

conservation of  $J = \oint dl v_{\parallel}$

$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$



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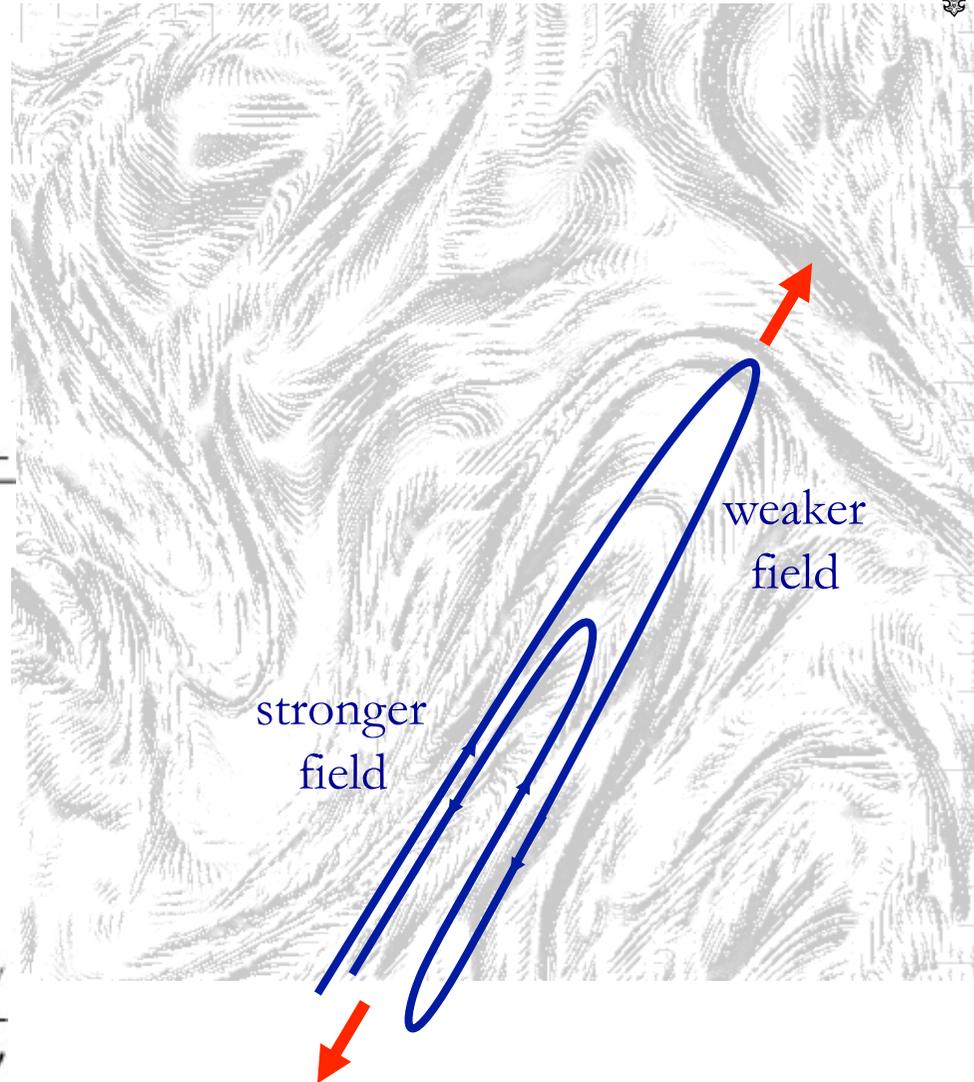
$$\frac{1}{2p_{\parallel}} \frac{dp_{\parallel}}{dt} = -\frac{1}{B} \frac{dB}{dt} - \nu \frac{p_{\parallel} - p_{\perp}}{p_{\parallel}}$$

conservation of  $J = \oint dl v_{\parallel}$

$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$

Typical pressure anisotropy:

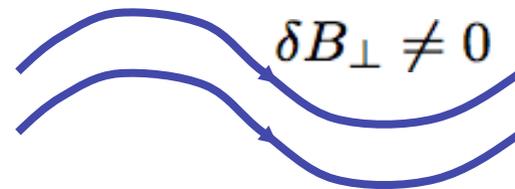
$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu}$$



# Pressure Anisotropy → Microinstabilities



Instabilities are fast, small scale.  
They are instantaneous compared to “fluid” dynamics.

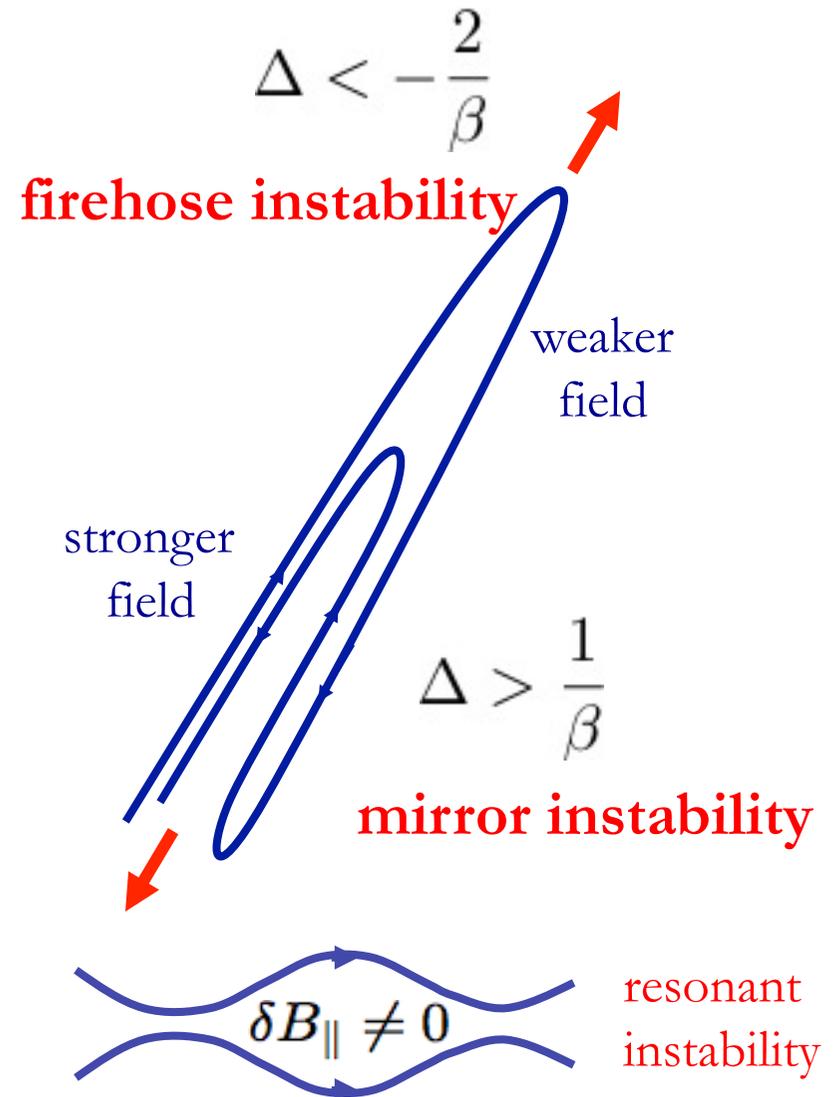


destabilised Alfvén wave

$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$

Typical pressure anisotropy:

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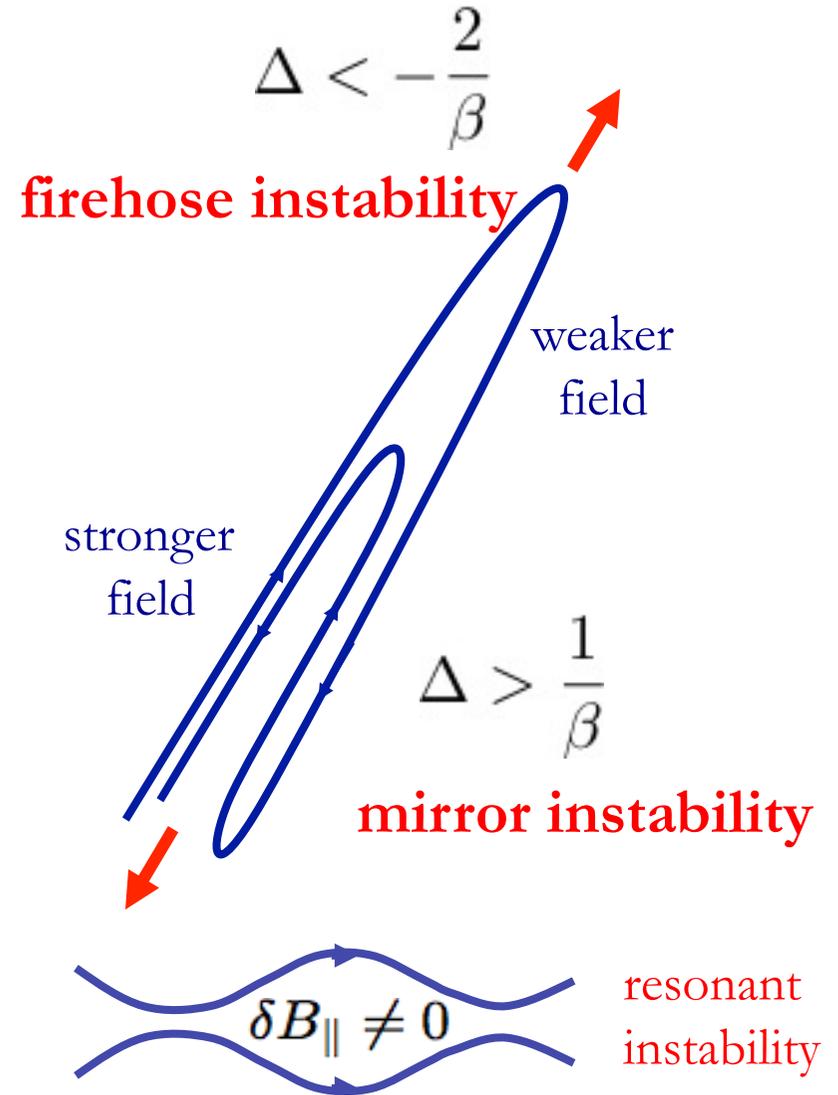
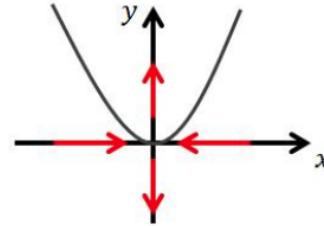


# Pressure Anisotropy → Microinstabilities



Scott Melville:

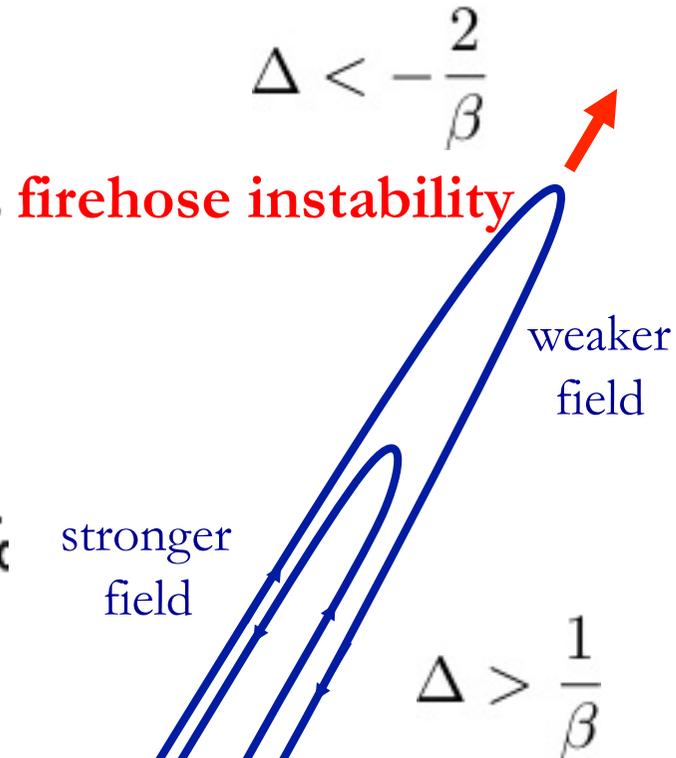
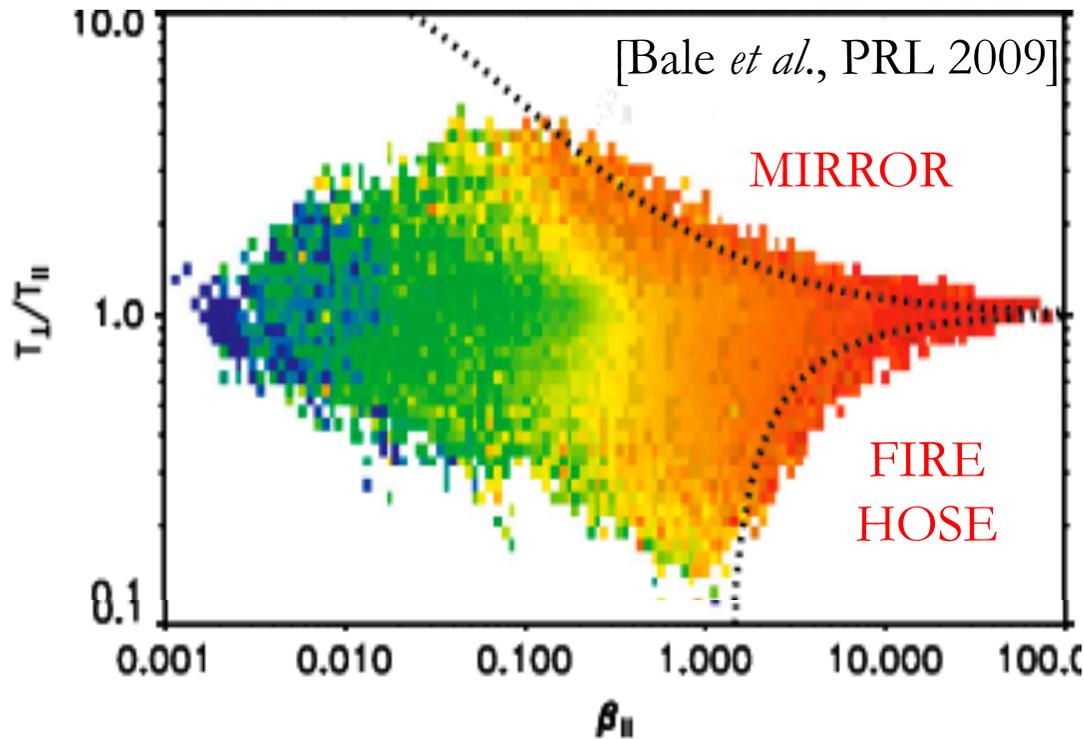
**folding field** goes **firehose-unstable**  
(in a 1D Braginskii model)



# Marginal State At All Times?



In the solar wind:



$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$

mirror instability

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

# Effective Closure Dilemma

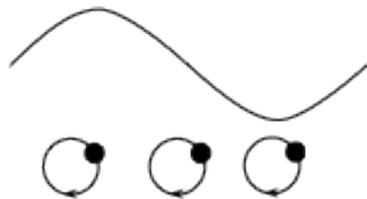


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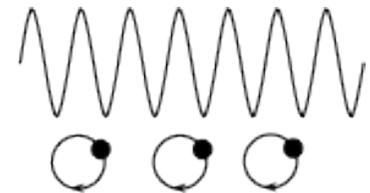
*Way to keep  
const rms B  
needed for this*

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Model I: Suppress stretching



Model II: Enhance collisionality



*Anomalous scattering  
of particles by Larmor  
scale fluctuations  
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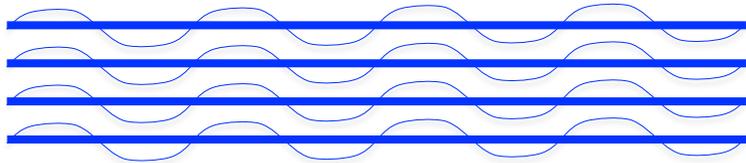
# Effective Closure Dilemma



How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

Example of mechanism to keep rate of change of  $B$  marginal when parallel-firehose unstable [Rosin et al., MNRAS 413, 7 (2011)]:

$$\begin{aligned} \frac{1}{B} \frac{dB}{dt} &= \overline{\hat{b}\hat{b} : \nabla u} = \overline{\hat{b}_0 \hat{b}_0 : \nabla u_0} + \overline{\hat{b}_0 \cdot (\nabla \delta u_{\perp})} \cdot \frac{\delta B_{\perp}}{B_0} \\ &= \frac{1}{B_0} \frac{dB_0}{dt} + \frac{1}{2} \frac{\partial}{\partial t} \frac{|\overline{\delta B_{\perp}}|^2}{B_0^2} \approx -\frac{2\nu_{ii}}{\beta} \end{aligned}$$



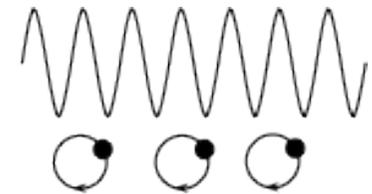
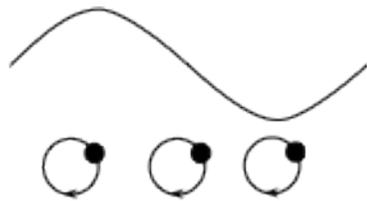
AAS et al., PRL **100**, 081301 (2008)

Rosin et al., MNRAS **413**, 7 (2011)

Way to keep  
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$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$



Anomalous scattering  
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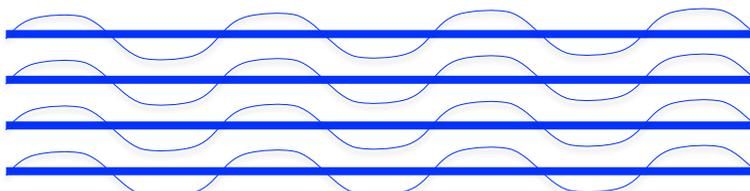
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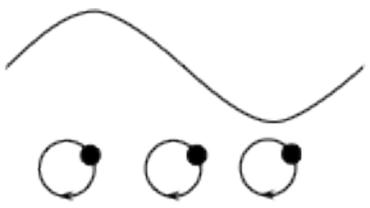
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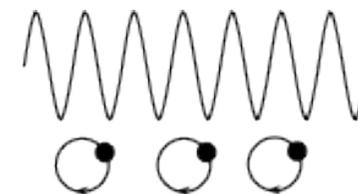
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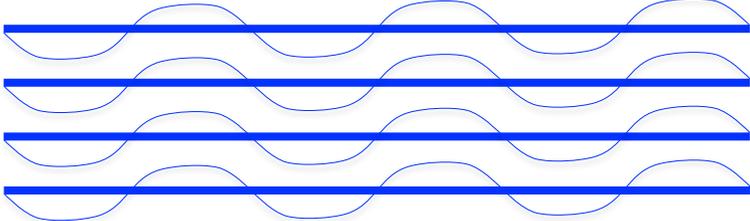
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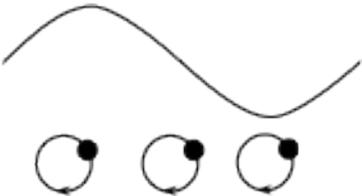
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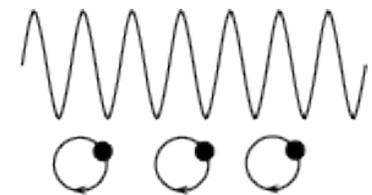
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Model II: Enhance collisionality



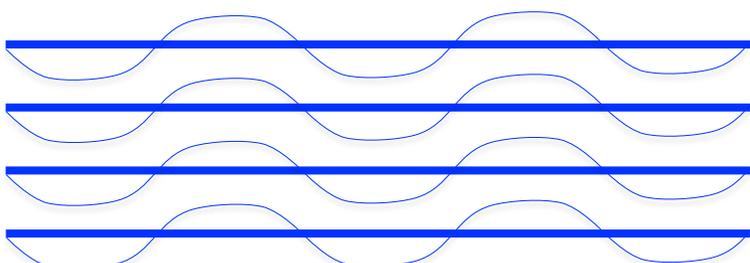
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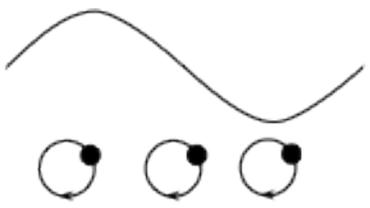
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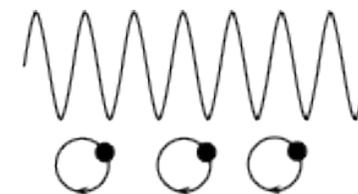
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# Dynamo under Model I (suppression of $\gamma$ )

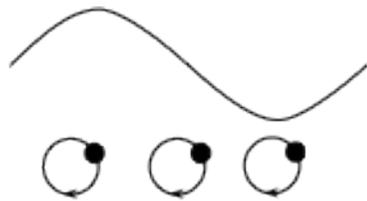


$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B \in \nu \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right] B$$

Suppose there is enough stirring to keep  $\Delta$  at the threshold:

$$\frac{dB}{dt} = \frac{\nu}{\beta} B = \frac{\nu}{8\pi p} B^3$$

Model I: Suppress stretching



$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$

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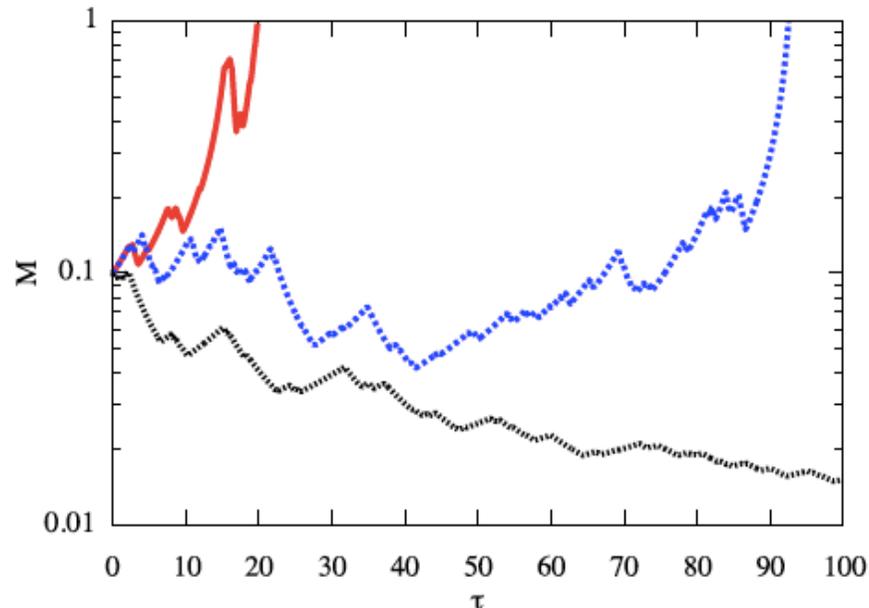


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Thus, **explosive growth**, but takes a **long time** to explode:  $t_c = \frac{\beta_0}{2\nu}$



for modeling details,  
caveats, complications,  
validity constraints,  
see



# Dynamo under Model I (suppression of $\gamma$ )



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Thus, **explosive growth**, but takes a **long time** to explode:  $t_c = \frac{\beta_0}{2\nu}$

For typical ICM parameters,

$$t_{\text{growth}} \sim \frac{\beta_0}{\nu} \sim \beta_0 \times 10 \left( \frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{-1} \left( \frac{T}{2 \text{ keV}} \right)^{3/2} \text{ yrs}$$

So this can efficiently restore fields from  $B \gtrsim 10^{-8} \text{ G}$   
to current values  $B \sim 10^{-5} \text{ G}$ ,

but for growth from a tiny seed, need a different mechanism

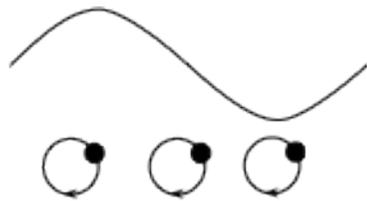
# ICM heating under Model I



Viscous heating rate ( $= Q_{\text{turb}}$  if we ignore energy cascade below  $l_{\text{visc}}$ )

$$Q_{\text{visc}} = \underbrace{(p_{\perp} - p_{\parallel})}_{p\Delta} \underbrace{\mathbf{b}\mathbf{b} : \nabla\mathbf{u}}_{\gamma \sim \nu\Delta} \sim p\Delta\gamma \sim p\nu\Delta^2 \sim \frac{p\nu}{\beta^2}$$

Model I: Suppress stretching



$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$

# ICM heating under Model I



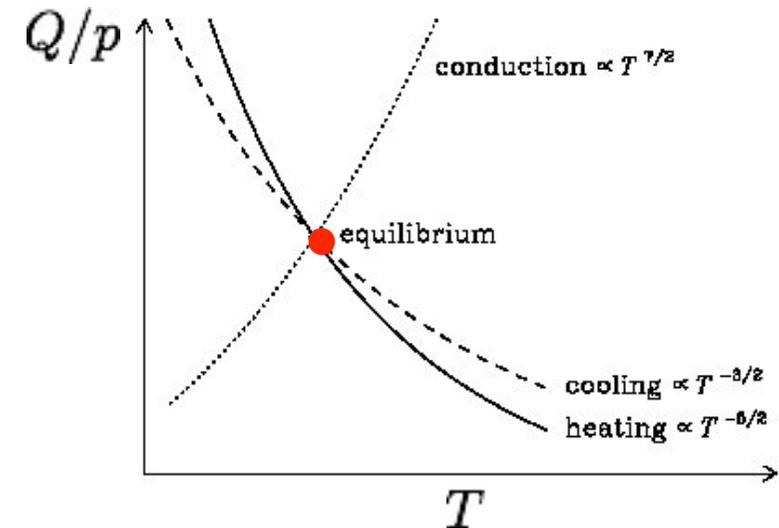
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$$\sim 10^{-25} \left( \frac{B}{10 \mu\text{G}} \right)^4 \left( \frac{T}{2 \text{ keV}} \right)^{-5/2} \frac{\text{erg}}{\text{s cm}^3}$$

$$Q_{\text{cool}} \sim 10^{-25} \left( \frac{n_e}{0.1 \text{ cm}^{-3}} \right)^2 \left( \frac{T}{2 \text{ keV}} \right)^{1/2} \frac{\text{erg}}{\text{s cm}^3}$$

➤ Thermally **stable** ICM



# ICM heating under Model I



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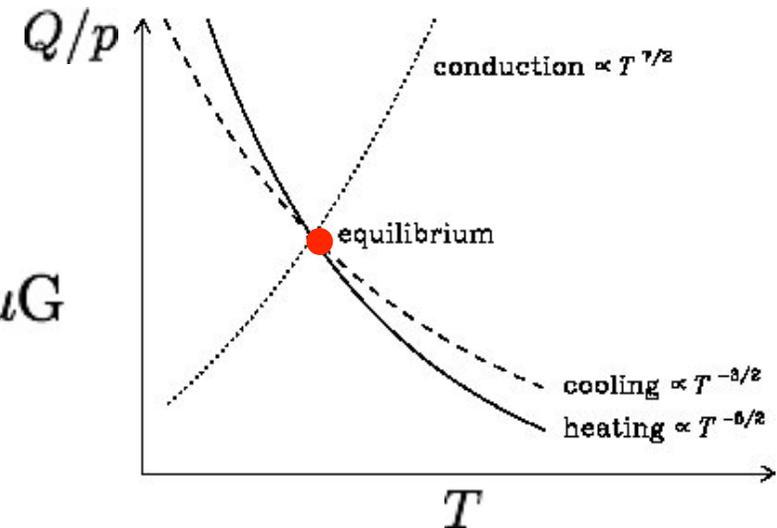
## ➤ Thermally stable ICM

➤ If  $Q_{\text{visc}} \sim Q_{\text{cool}}$ ,

$$B \sim 10 \left( \frac{n_e}{0.1 \text{cm}^{-3}} \right)^{1/2} \left( \frac{T}{2 \text{keV}} \right)^{3/4} \mu\text{G}$$

➤ If  $\rho u^2/2 \sim B^2/8\pi$ ,

$$u \sim 10^2 \left( \frac{T}{2 \text{keV}} \right)^{3/4} \frac{\text{km}}{\text{s}}$$



# Dynamo under Model II (enhancement of $\nu$ )



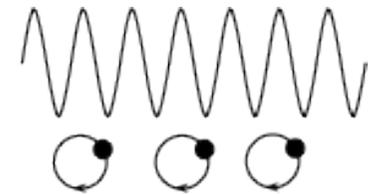
$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$

To stay at threshold, need effective collisionality  $\nu \sim \gamma\beta$

$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$



**Model II: Enhance collisionality**



*Anomalous scattering  
of particles by Larmor  
scale fluctuations  
needed for this*

# Dynamo under Model II (enhancement of $\nu$ )



$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$

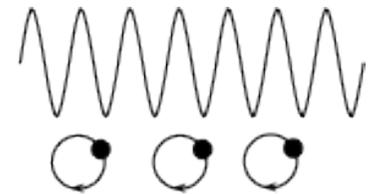
To stay at threshold, need effective collisionality  $\nu \sim \gamma\beta$

But collisionality determines viscosity  $\mu \sim p/\nu$

$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$



**Model II: Enhance collisionality**



*Anomalous scattering of particles by Larmor scale fluctuations needed for this*

# Dynamo under Model II (enhancement of $\nu$ )



$$\frac{dB}{dt} = (\mathbf{b}\mathbf{b} : \nabla\mathbf{u})B \equiv \gamma B$$

To stay at threshold, need effective collisionality  $\nu \sim \gamma\beta$

But collisionality determines viscosity  $\mu \sim p/\nu$

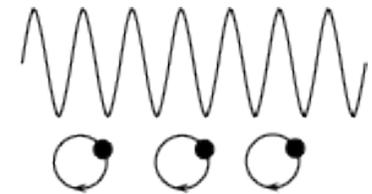
And viscosity determines maximal rate of strain:

$$\gamma \sim \left(\frac{\varepsilon}{\mu}\right)^{1/2} \sim \left(\frac{\varepsilon\nu}{p}\right)^{1/2} \sim \left(\frac{\varepsilon\gamma\beta}{p}\right)^{1/2} \Rightarrow \gamma \sim \frac{\varepsilon\beta}{p} \sim \frac{\varepsilon}{B^2}$$

$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$



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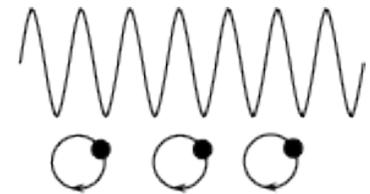
$$\gamma \sim \left(\frac{\varepsilon}{\mu}\right)^{1/2} \sim \left(\frac{\varepsilon\nu}{p}\right)^{1/2} \sim \left(\frac{\varepsilon\gamma\beta}{p}\right)^{1/2} \Rightarrow \gamma \sim \frac{\varepsilon\beta}{p} \sim \frac{\varepsilon}{B^2}$$

$$\frac{dB^2}{dt} = 2\gamma B^2 \sim \varepsilon$$

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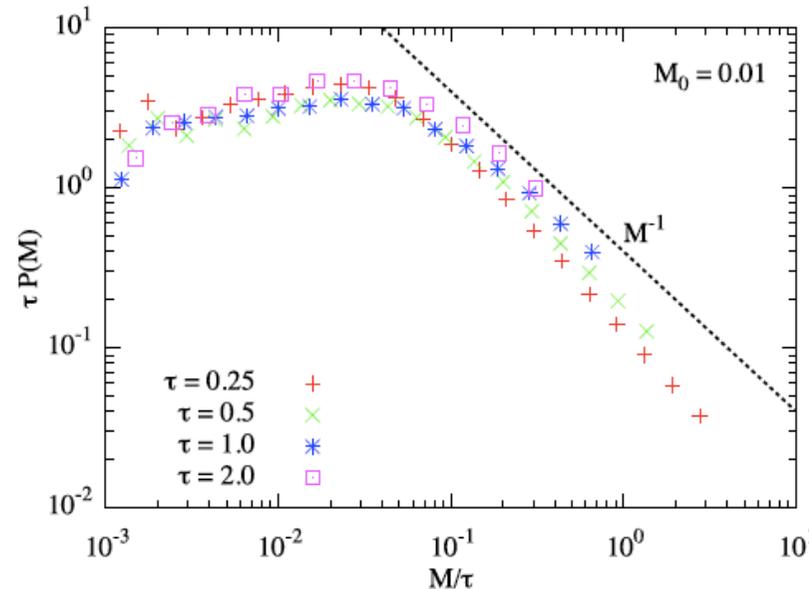
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$$\frac{dB^2}{dt} = 2\gamma B^2 \sim \varepsilon \Rightarrow B^2 \sim \varepsilon t$$

Thus, **secular growth**, but gets to dynamical strength very **quickly**:

$$t \sim \frac{B_{\text{sat}}^2}{\varepsilon} \sim \frac{u^2}{\varepsilon} \sim \frac{l}{u} \quad \text{one large-scale turnover rate}$$

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$$\gamma \sim \frac{\varepsilon \beta}{p} \sim \frac{\varepsilon}{B^2}$$

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Modeling gives **extremely intermittent, self-similar field distribution**; see

( $\rightarrow$  **intermittent viscosity, intermittent rate of strain**,

very hard to do right in “real” simulations with this effective closure!) ↓

# ICM heating under Model II



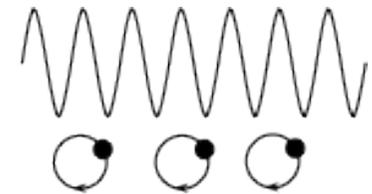
$$Q_{\text{visc}} = \underbrace{(p_{\perp} - p_{\parallel})}_{p\Delta} \underbrace{\mathbf{b}\mathbf{b} : \nabla\mathbf{u}}_{\gamma} \sim p\Delta\gamma \sim \varepsilon$$

$$\gamma \sim \frac{\varepsilon\beta}{p}$$

$$\Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{1}{\nu} \frac{1}{B} \frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[ -\frac{2}{\beta}, \frac{1}{\beta} \right]$$



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# ICM heating under Model II



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So we learn nothing new: all the turbulent power input, whatever it is, gets viscously dissipated

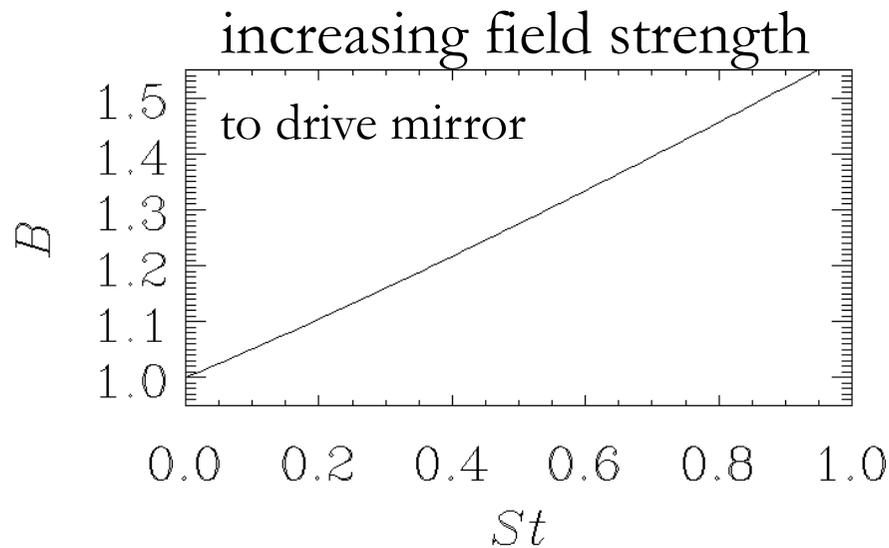
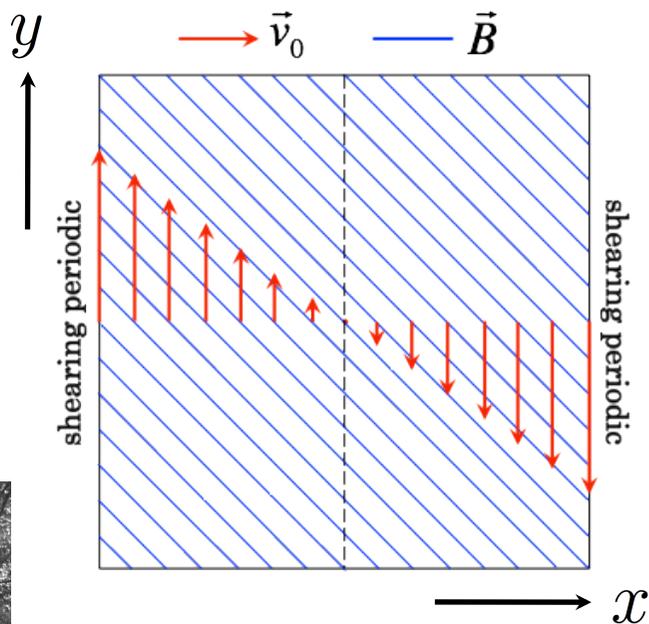
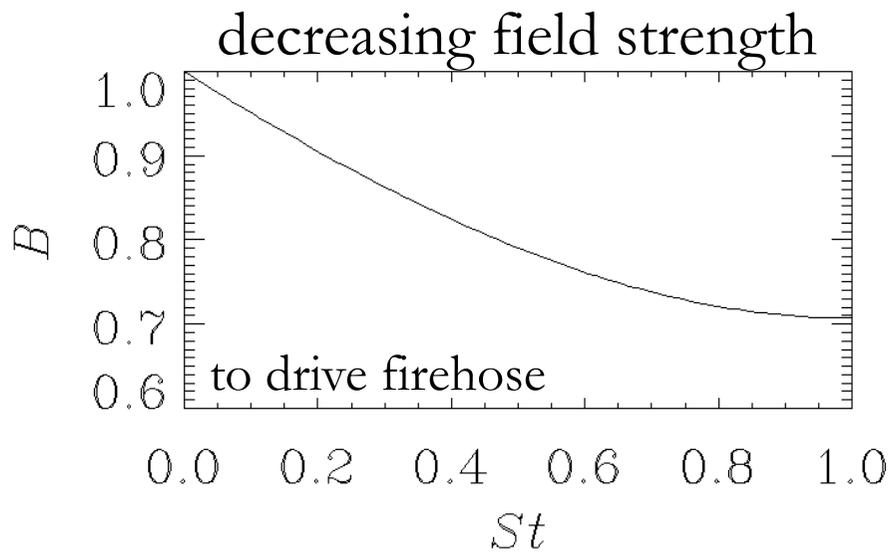
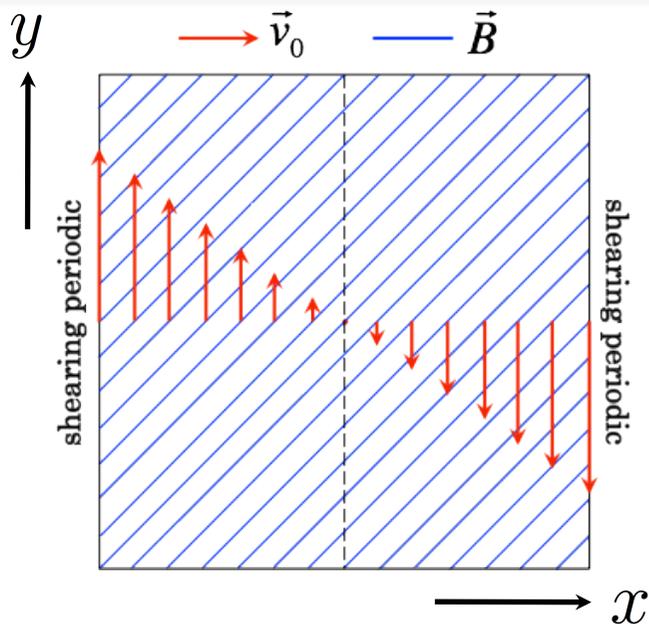
(in Model I,  $Q_{\text{visc}} \sim \varepsilon$  as well, but it allows one to fix the temperature profile in terms of other parameters, while in Model II it is hard-wired)

This would mean that whatever determines the thermal stability of the ICM has, under Model II, to do with large-scale energy deposition processes, not with microphysics:

*Rejoice all ye believers that microphysics should never matter!*

(although you need microphysics to know whether Model II is right)

# Instabilities in a Box (M. Kunz)



# Instabilities in a Box (M. Kunz)



Hybrid kinetic system solved by PEGASUS code:

$$\left( \frac{\partial}{\partial t} - Sx \frac{\partial}{\partial y} \right) f_i + \mathbf{v} \cdot \nabla f_i + \left[ \frac{Ze}{m_i} \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) + Sv_x \hat{\mathbf{y}} \right] \cdot \frac{\partial f_i}{\partial \mathbf{v}} = 0$$

$$\left( \frac{\partial}{\partial t} - Sx \frac{\partial}{\partial y} \right) \mathbf{B} = -c \nabla \times \mathbf{E} - SB_x \hat{\mathbf{y}}$$

$$\mathbf{E} = -\frac{\mathbf{u}_i \times \mathbf{B}}{c} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi Zen_i} - \frac{T_e \nabla n_i}{en_i}$$

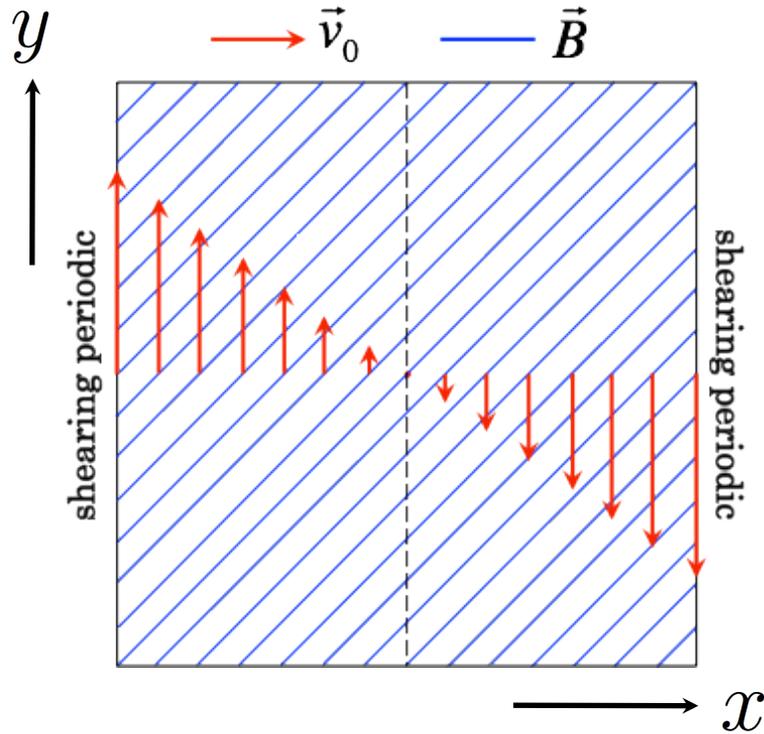
...in a shearing sheet  $\mathbf{u} = -Sx\hat{\mathbf{y}}$



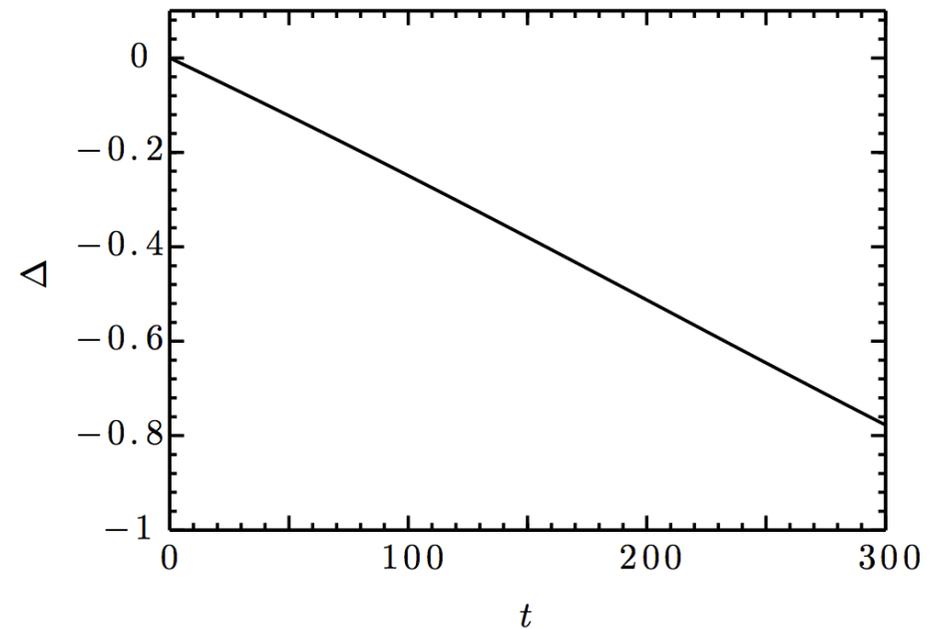
Kunz, Stone & Bai, *JCP* **259**, 154 (2014)



# Firehose Instability (M. Kunz)



$$\frac{dB}{dt} < 0 \Rightarrow \Delta = \frac{p_{\perp} - p_{\parallel}}{p} < 0$$

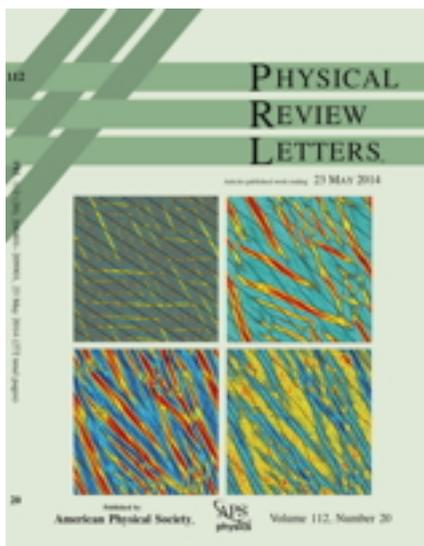
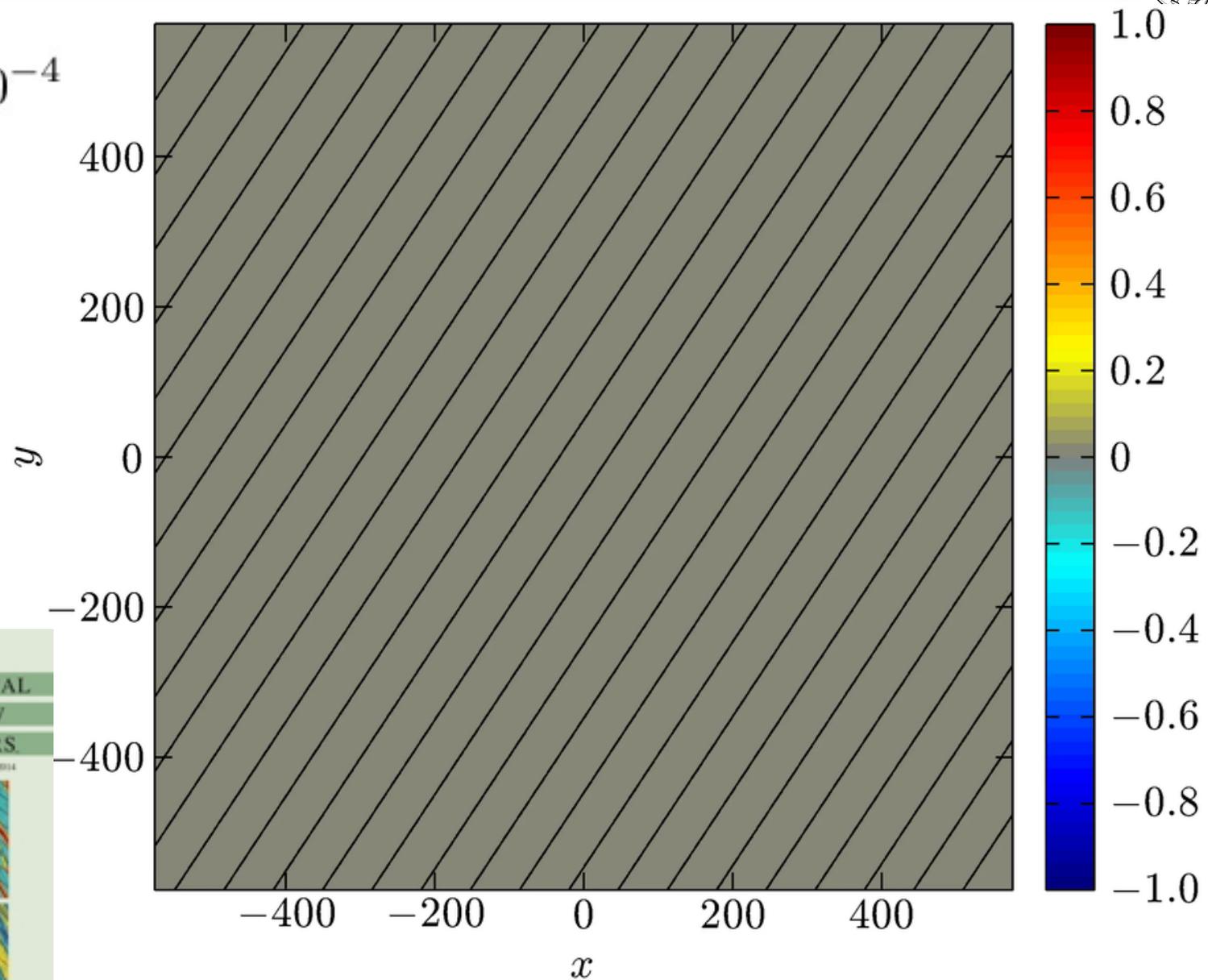


# Firehose Instability (M. Kunz)

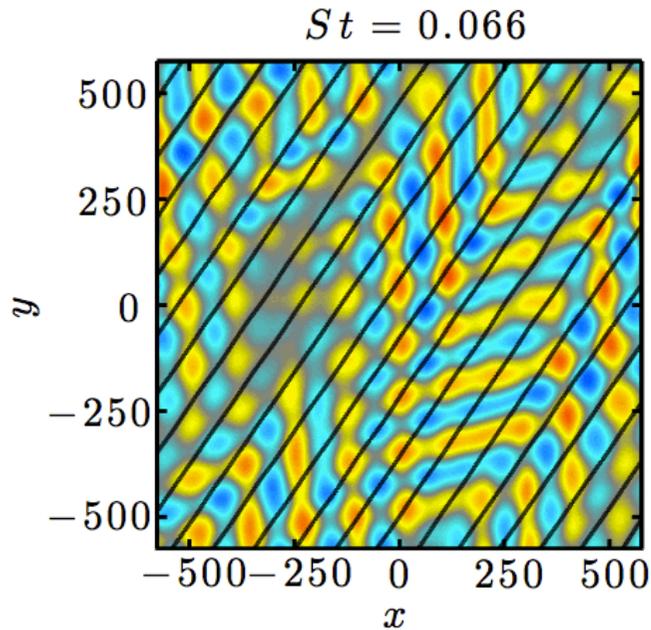


$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$

$$\beta_i = 200$$

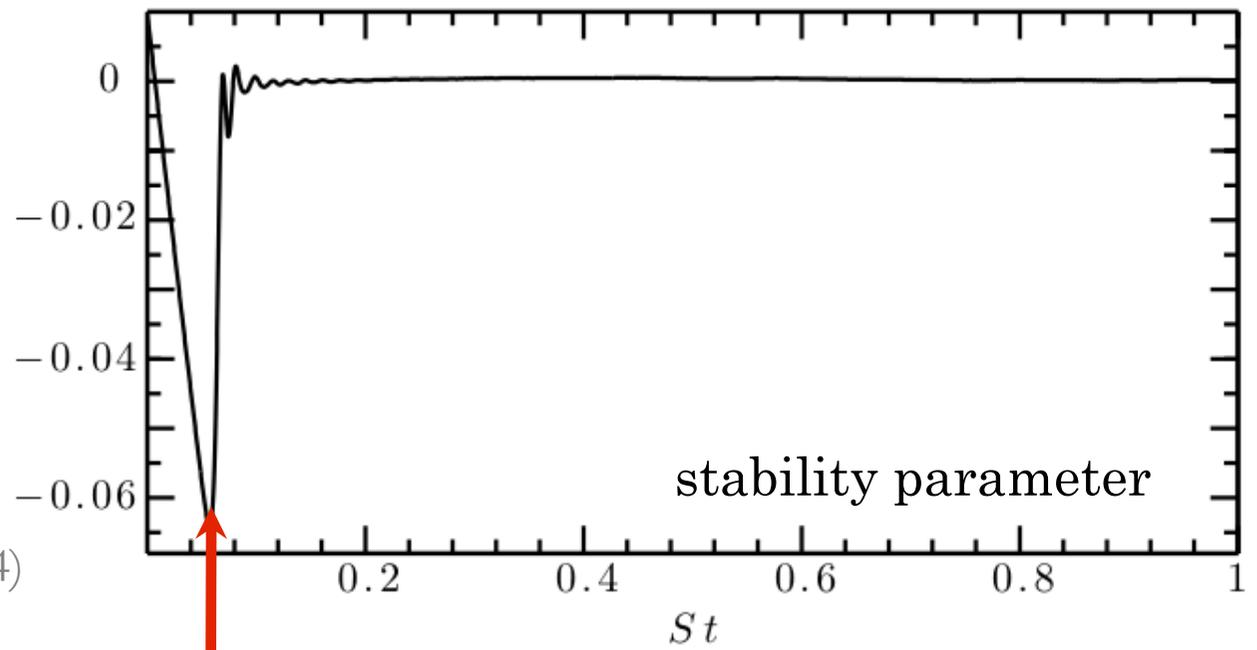
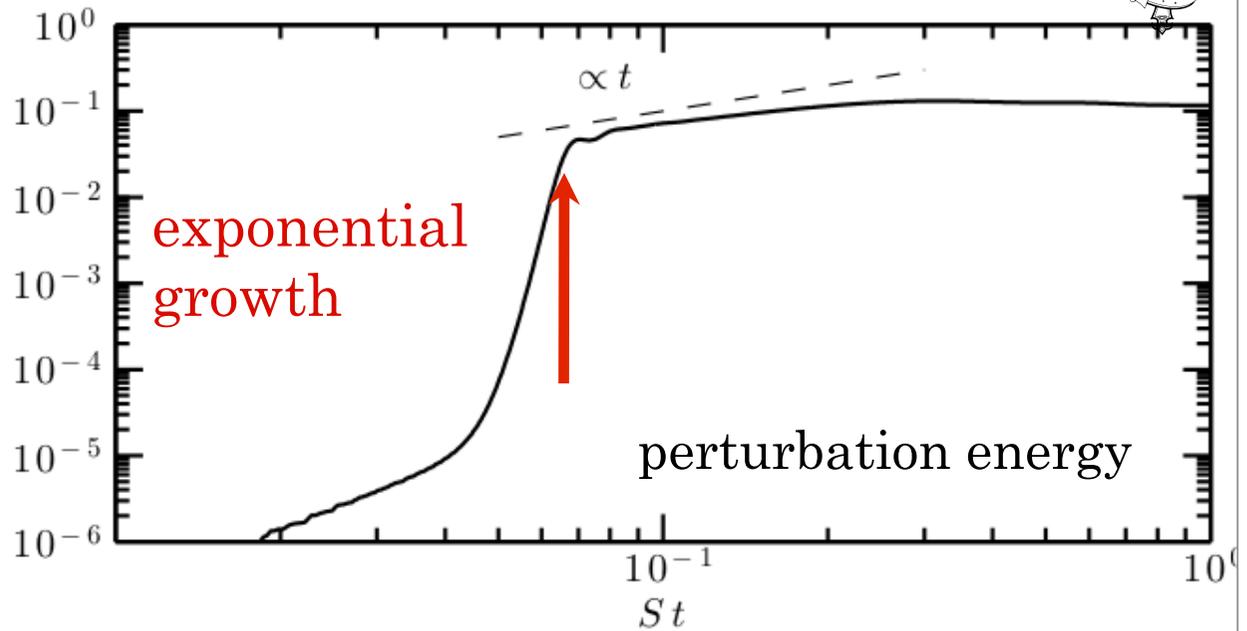


# Firehose Instability: Linear

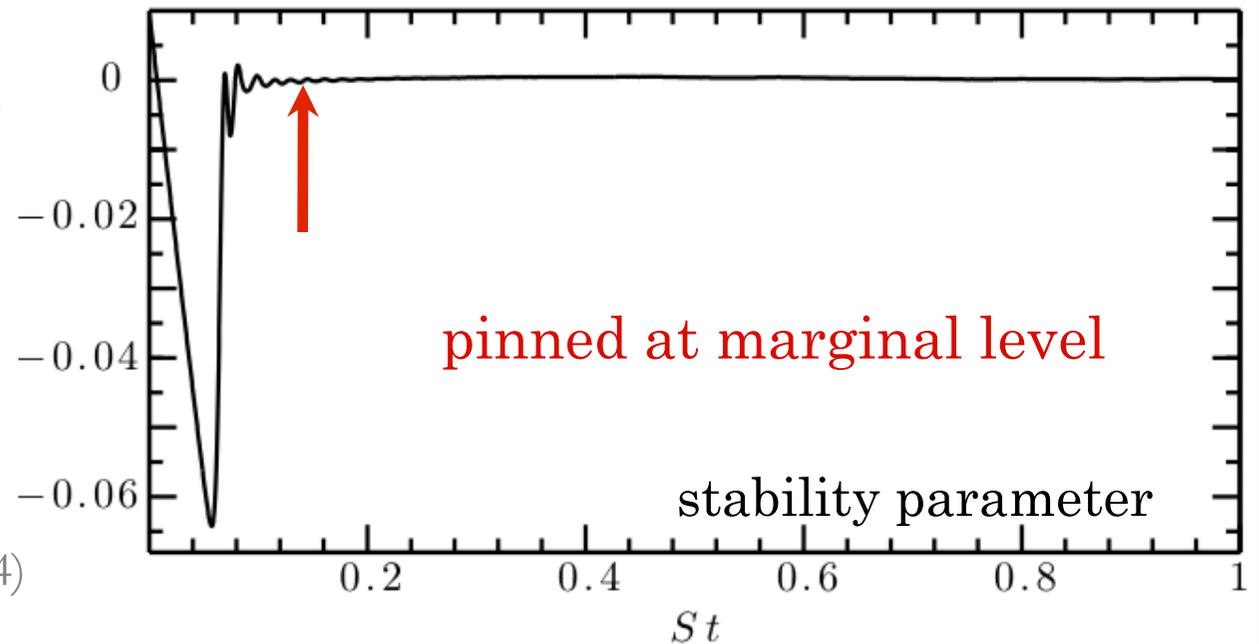
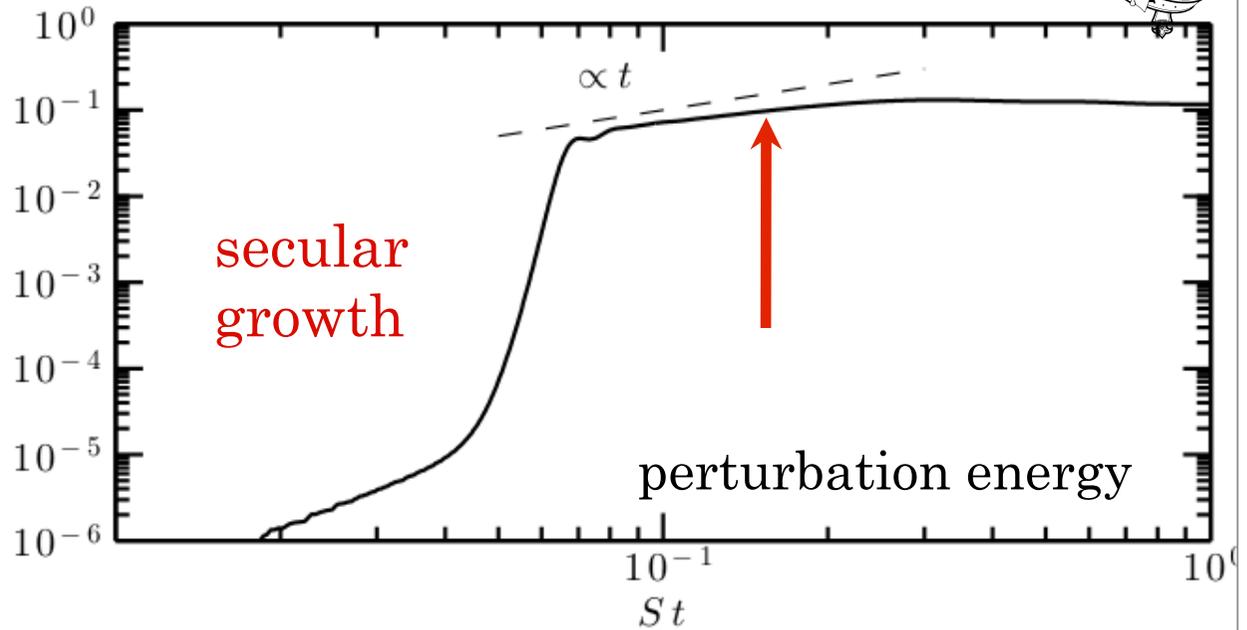
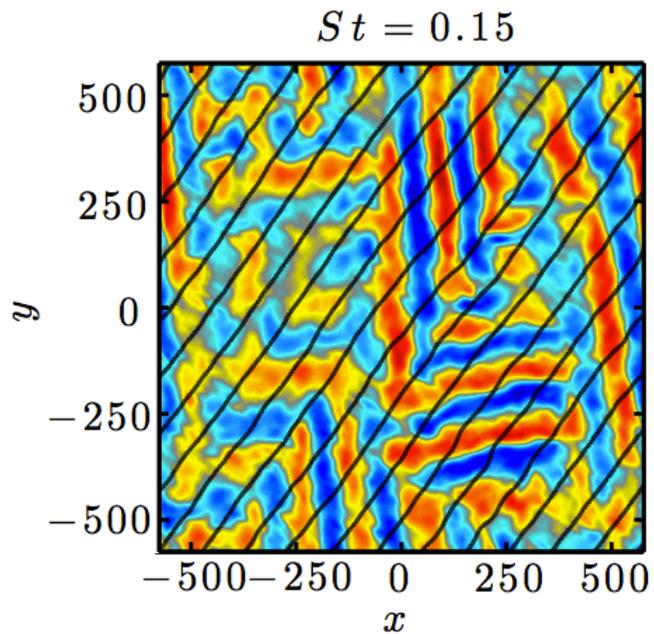


oblique modes

$$k_{\parallel} \rho_i \approx k_{\perp} \rho_i \approx 0.4$$



# Firehose Instability: Secular



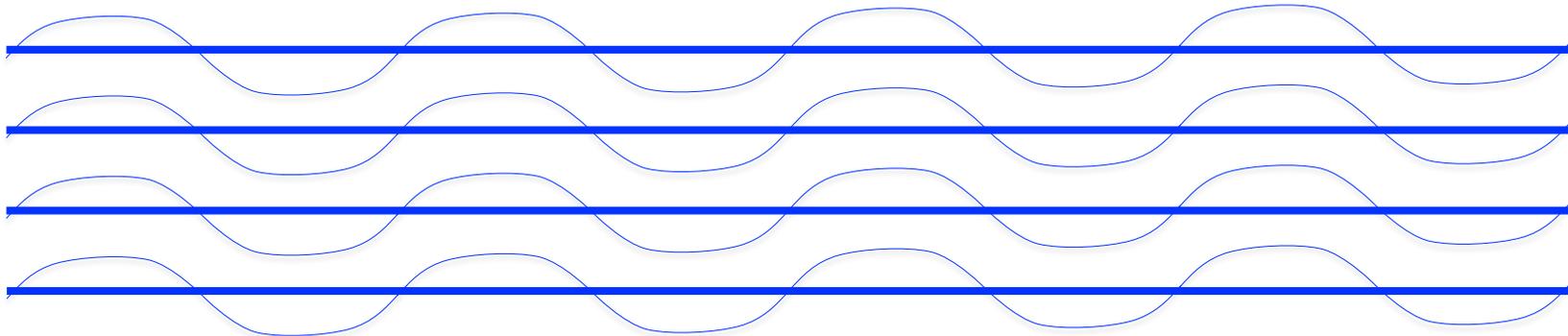
# Firehose Instability: Secular



$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3 \int^t dt' \frac{d \ln \overline{B}}{dt} = \int^t dt' \left( \underbrace{-3 \left| \frac{d \ln B_0}{dt} \right|}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{driven by shear}}} + \underbrace{\frac{3}{2} \frac{d \overline{|\delta\mathbf{B}_\perp|^2}}{dt} \frac{1}{B_0^2}}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{from firehose}}} \right) \rightarrow -\frac{2}{\beta}$$

marginal stability



AAS et al., *PRL* **100**, 081301 (2008) [arXiv:0709.3828]

Rosin et al., *MNRAS* **413**, 7 (2011) [arXiv:1002.4017]

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

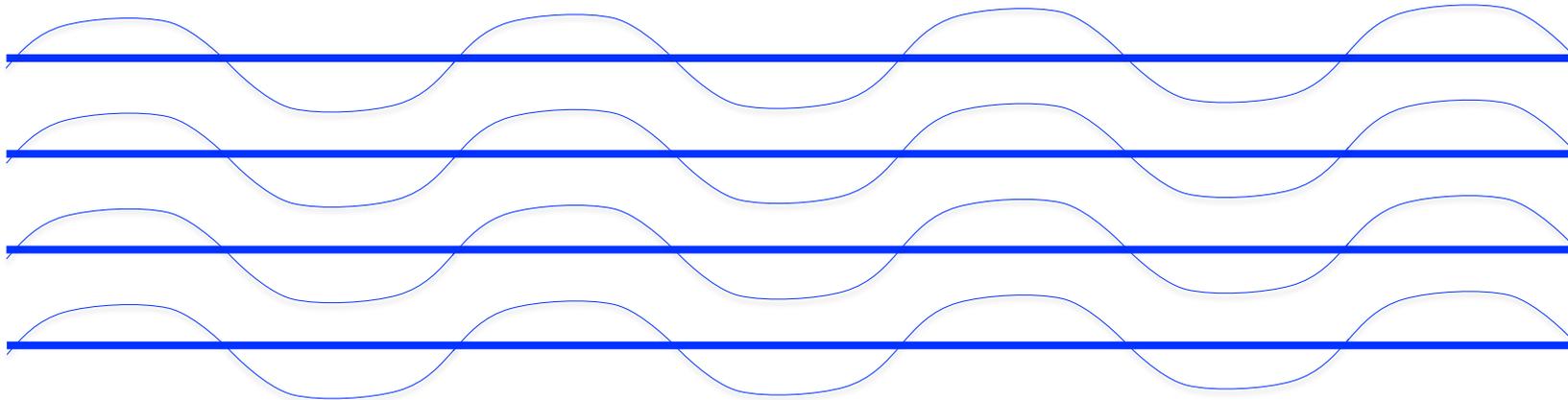
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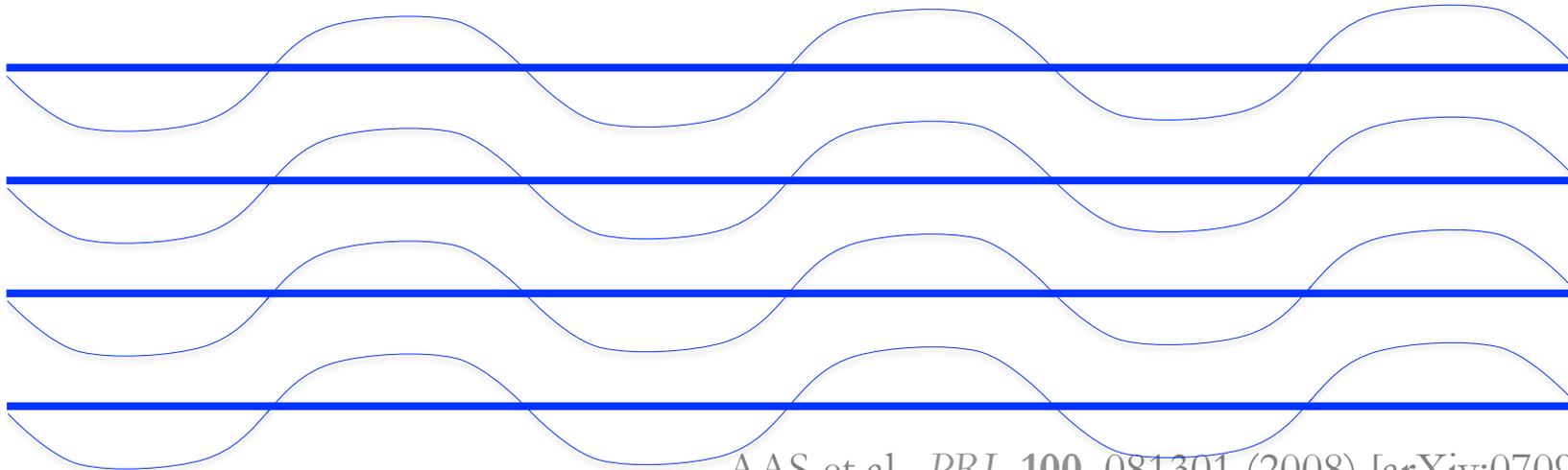
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Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

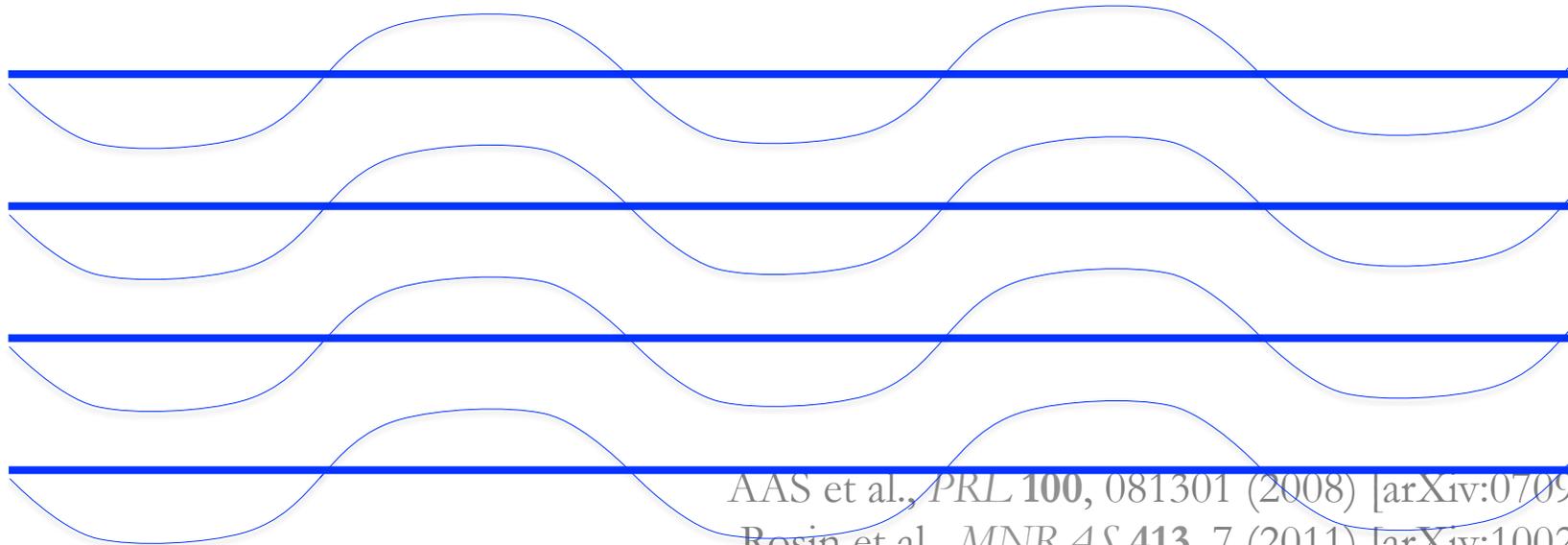
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marginal stability



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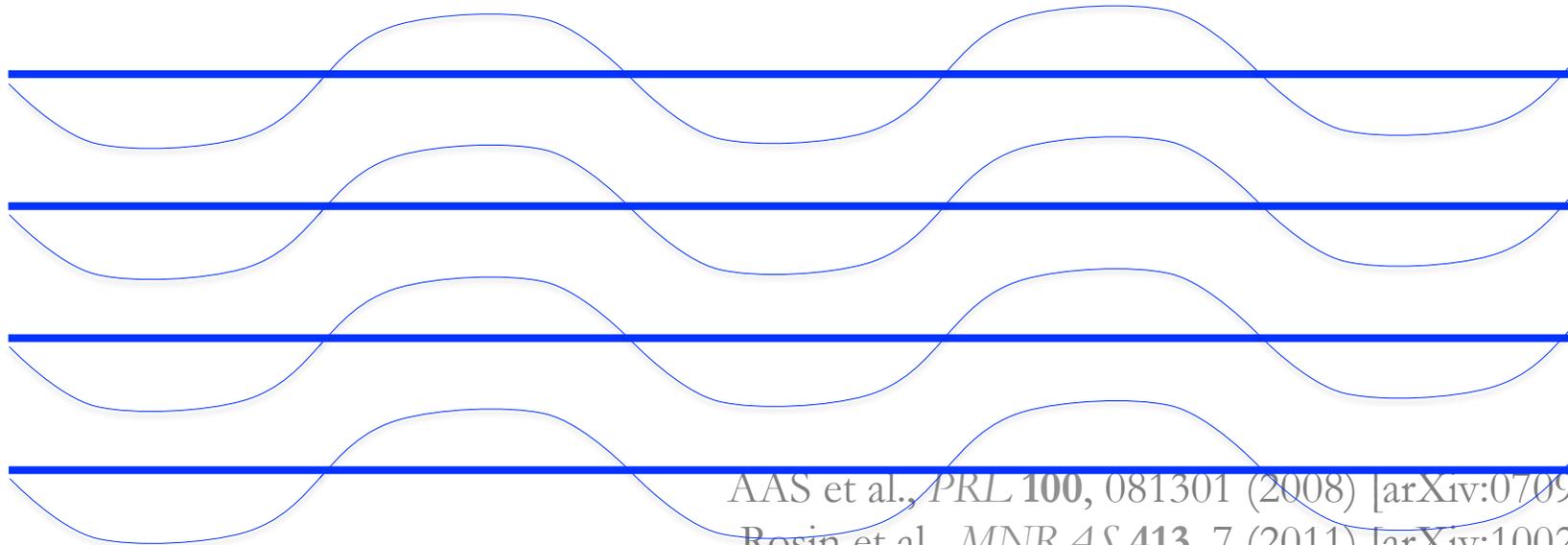
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$$\frac{3}{2} \frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} = 3S \int^t dt' \hat{b}_x(t') \hat{b}_y(t') - \frac{2}{\beta} \sim St \quad \text{secular growth}$$

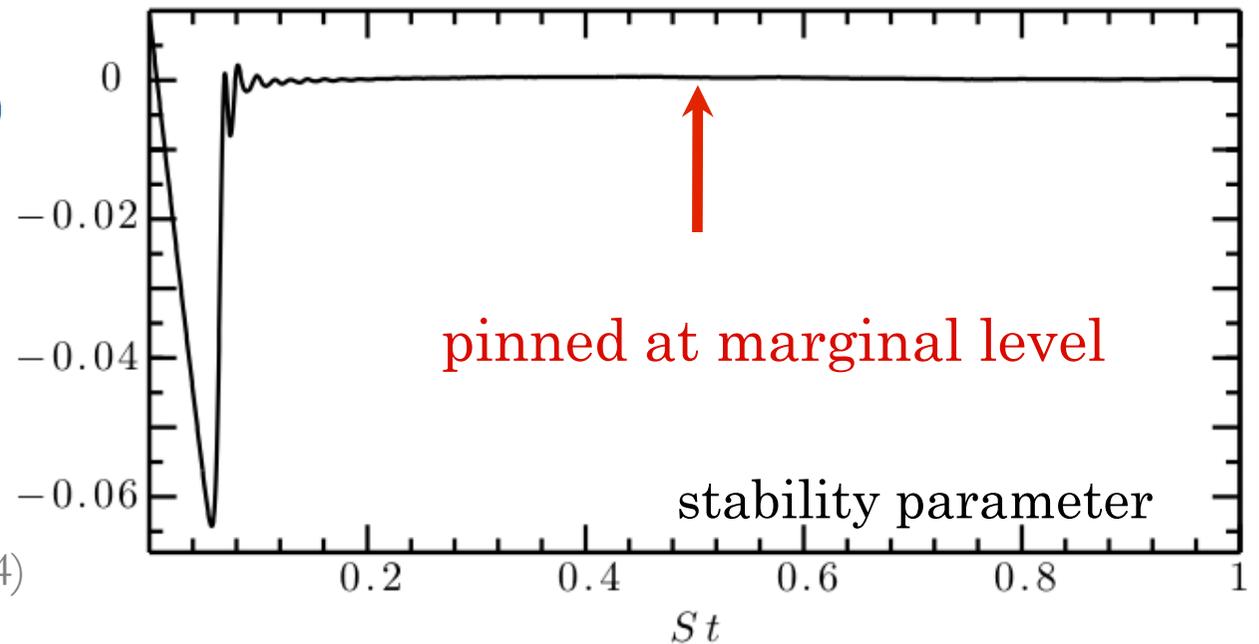
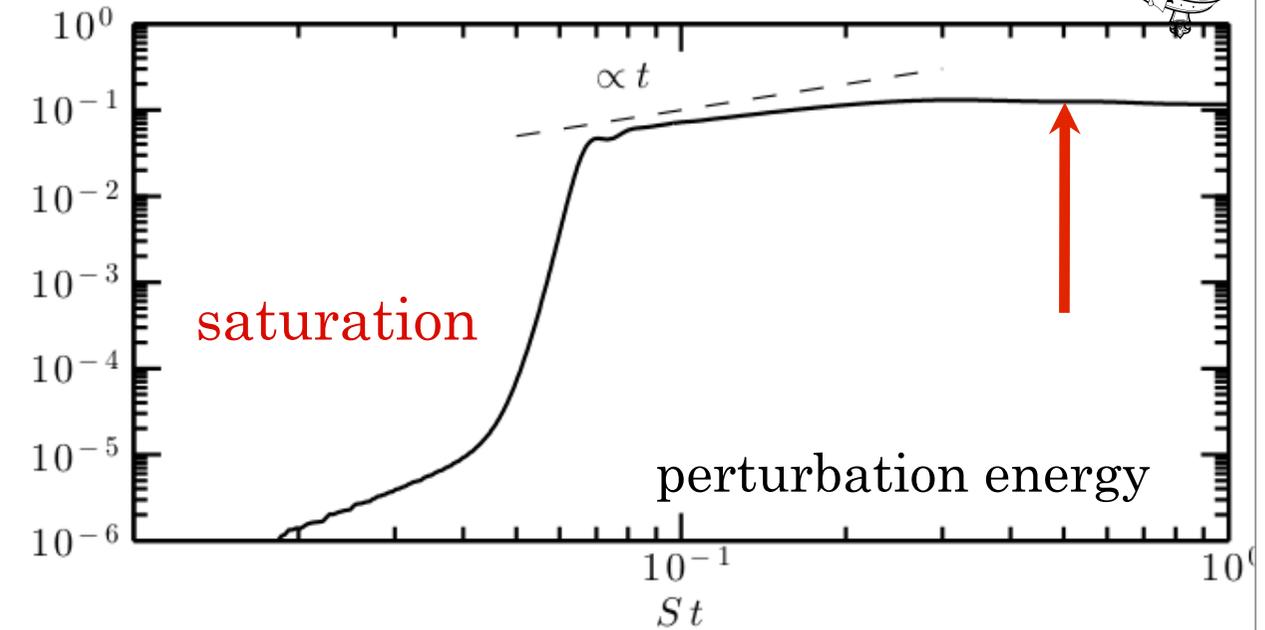
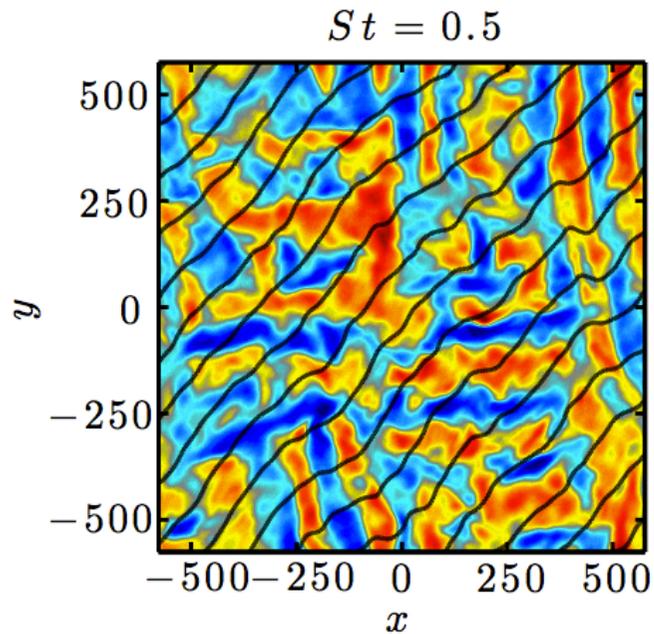


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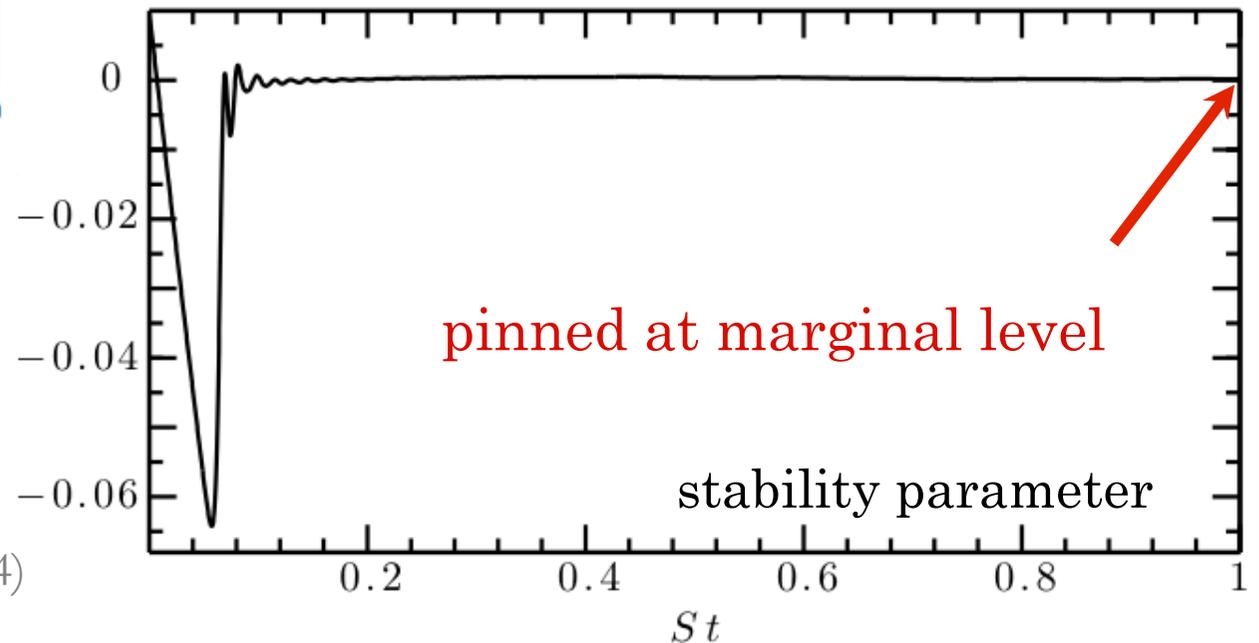
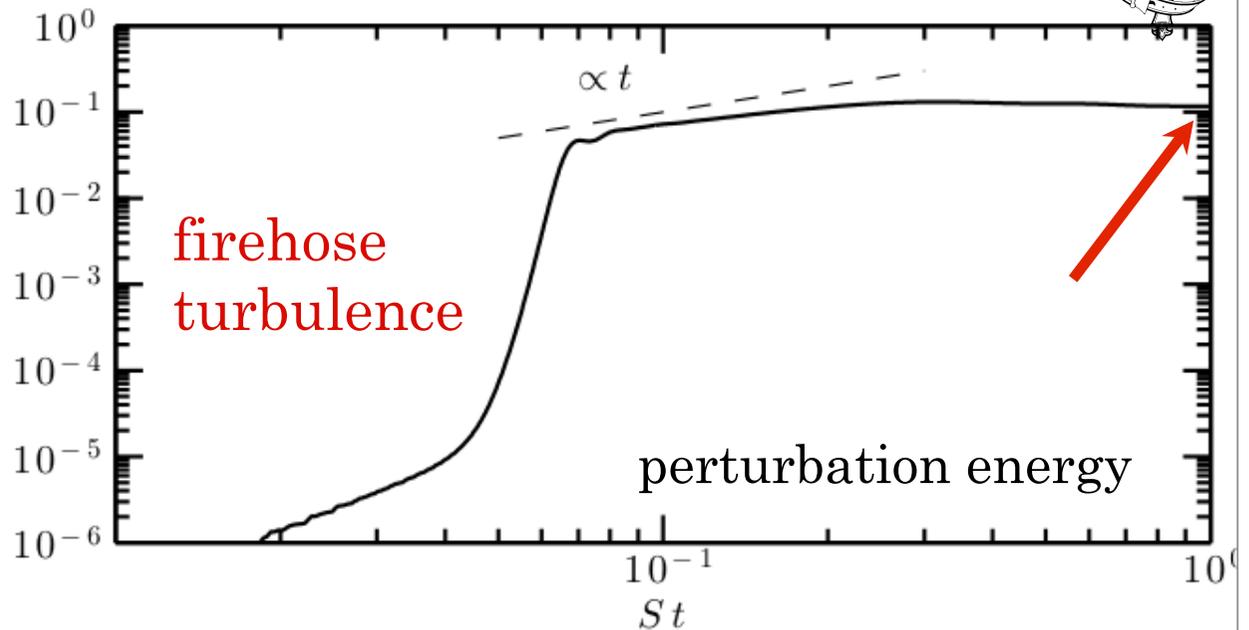
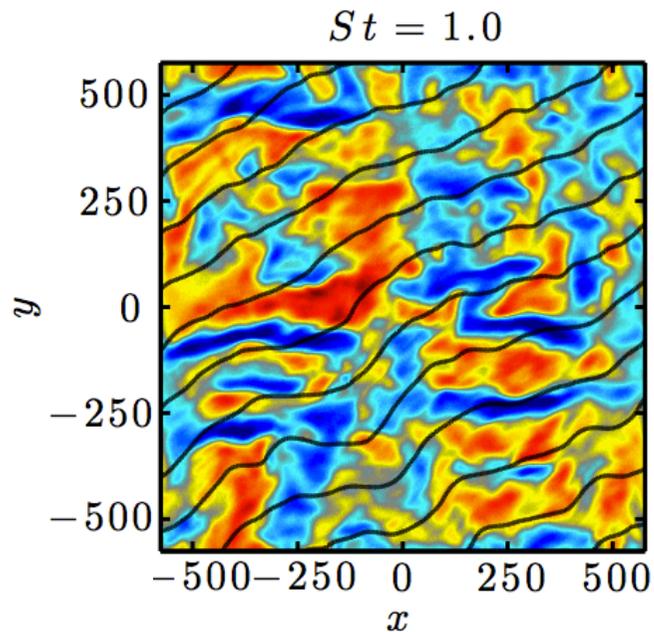
Rosin et al., *MNRAS* **413**, 7 (2011) [arXiv:1002.4017]

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

# Firehose Instability: Saturated



# Firehose Instability: Saturated

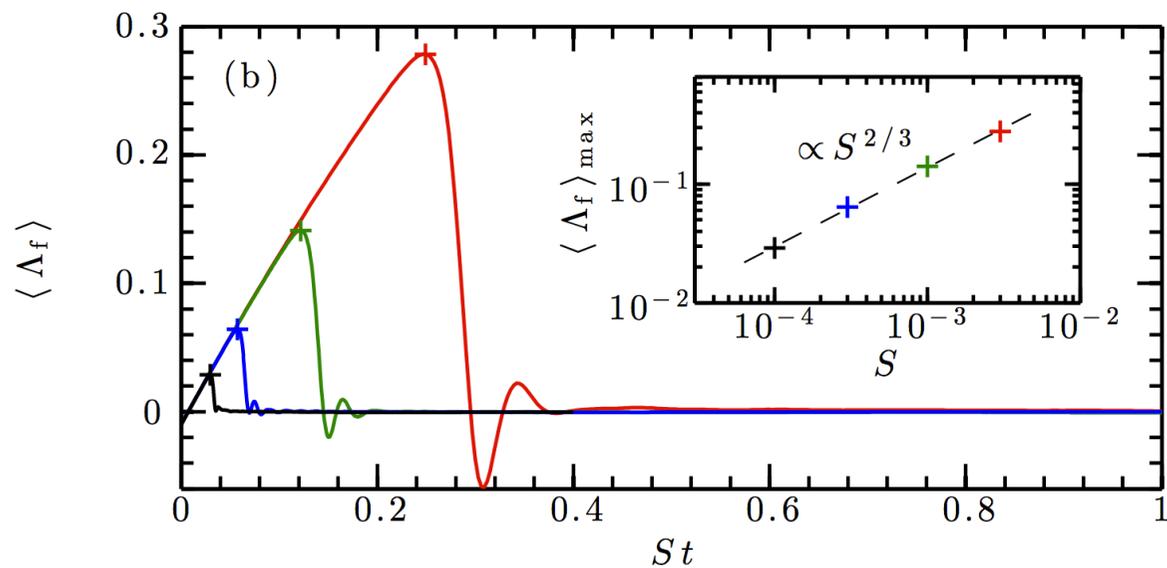
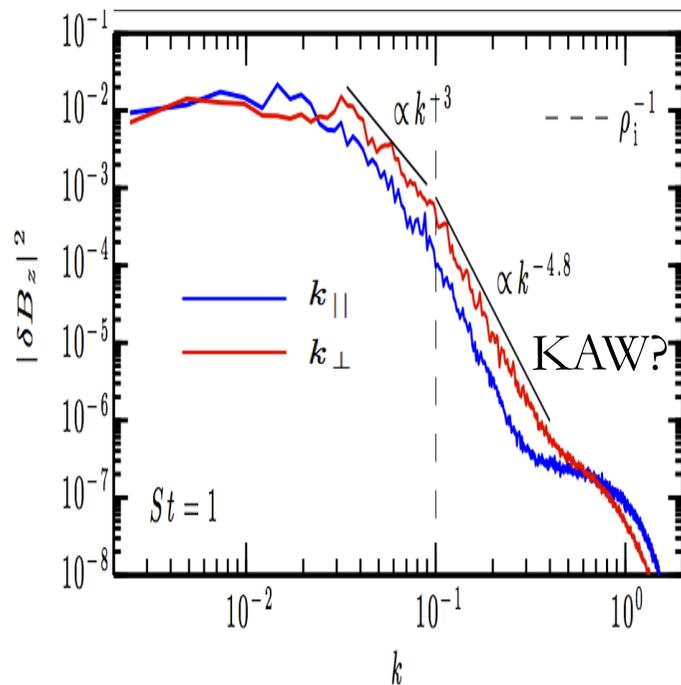
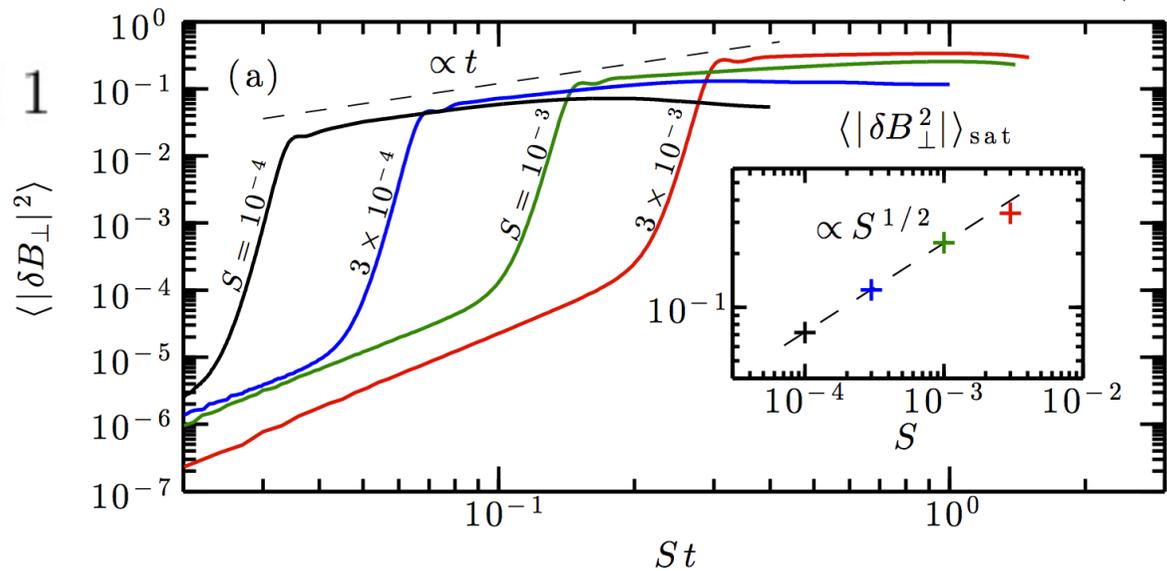


# Firehose Saturates at Small Amplitudes

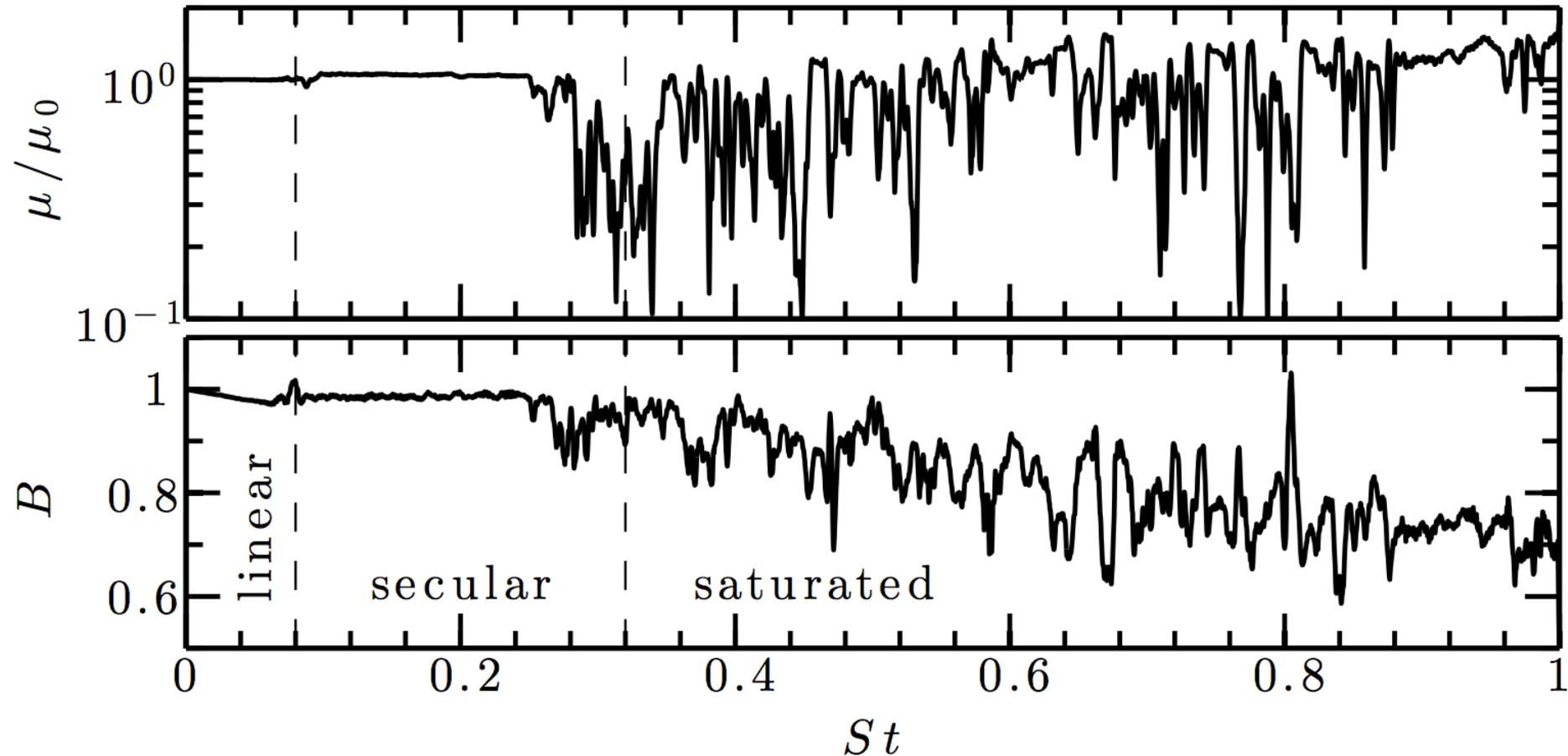


$$\frac{\langle |\delta \mathbf{B}_\perp|^2 \rangle}{B_0^2} \propto \left( \frac{S}{\Omega_i} \right)^{1/2} \ll 1$$

small-amplitude  
Larmor-scale  
firehose turbulence



# Saturated Firehose Scatters Particles

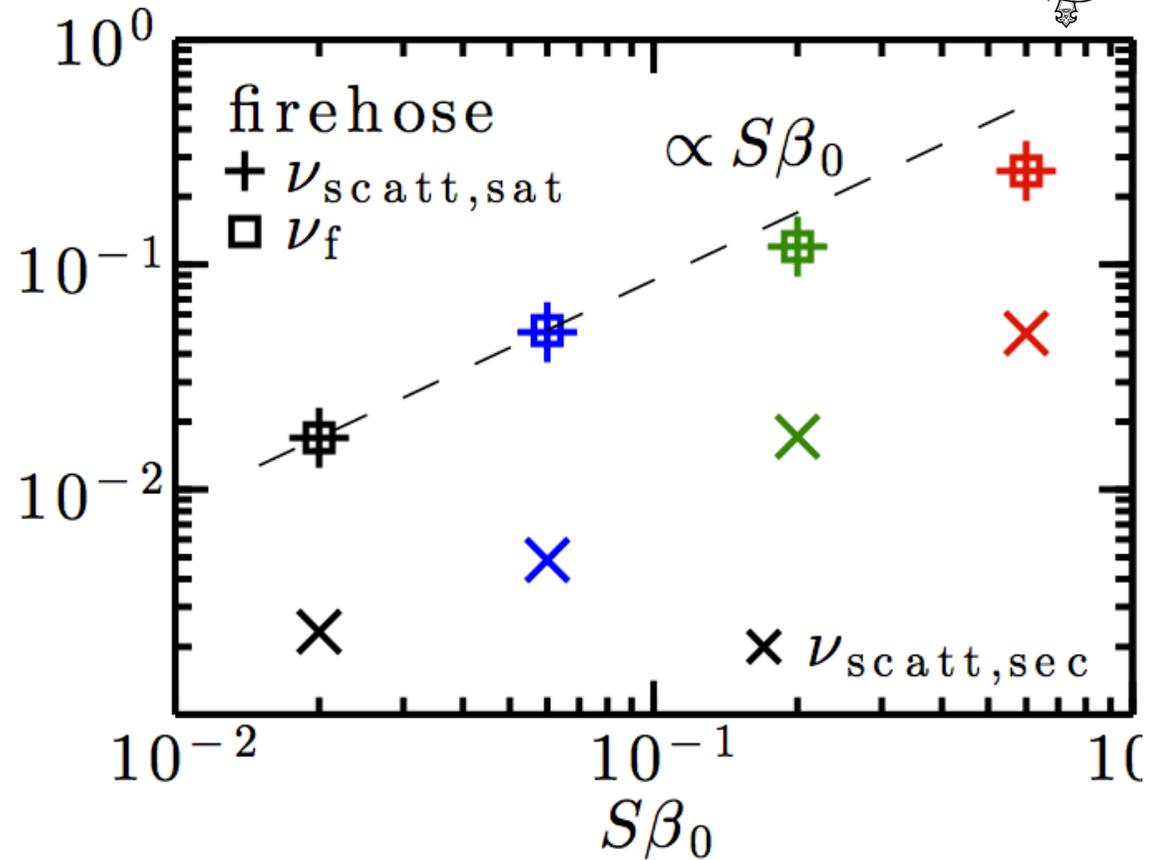


$\mu$  conservation is broken at long times, firehose fluctuations scatter particles to maintain pressure anisotropy at marginal level

# Saturated Firehose Scatters Particles

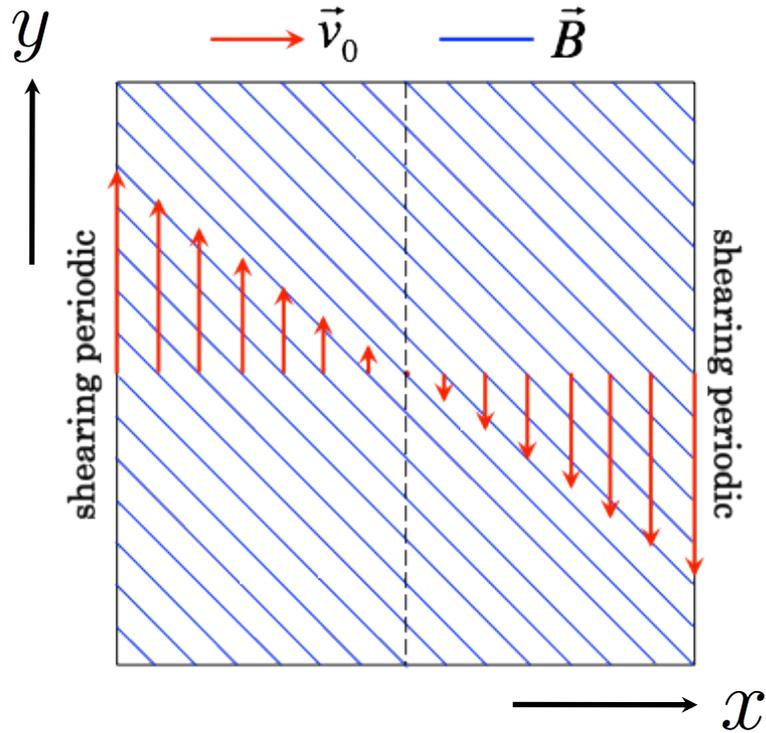


$$\begin{aligned} \frac{d\Delta}{dt} &= 3 \frac{d \ln |\langle \mathbf{B} \rangle|}{dt} - \nu_f \Delta \\ &= 0 \\ \nu_f &= \frac{3}{\Delta} \frac{d \ln |\langle \mathbf{B} \rangle|}{dt} \\ &= -\frac{3\beta}{2} \frac{d \ln |\langle \mathbf{B} \rangle|}{dt} \\ &\sim S\beta \end{aligned}$$

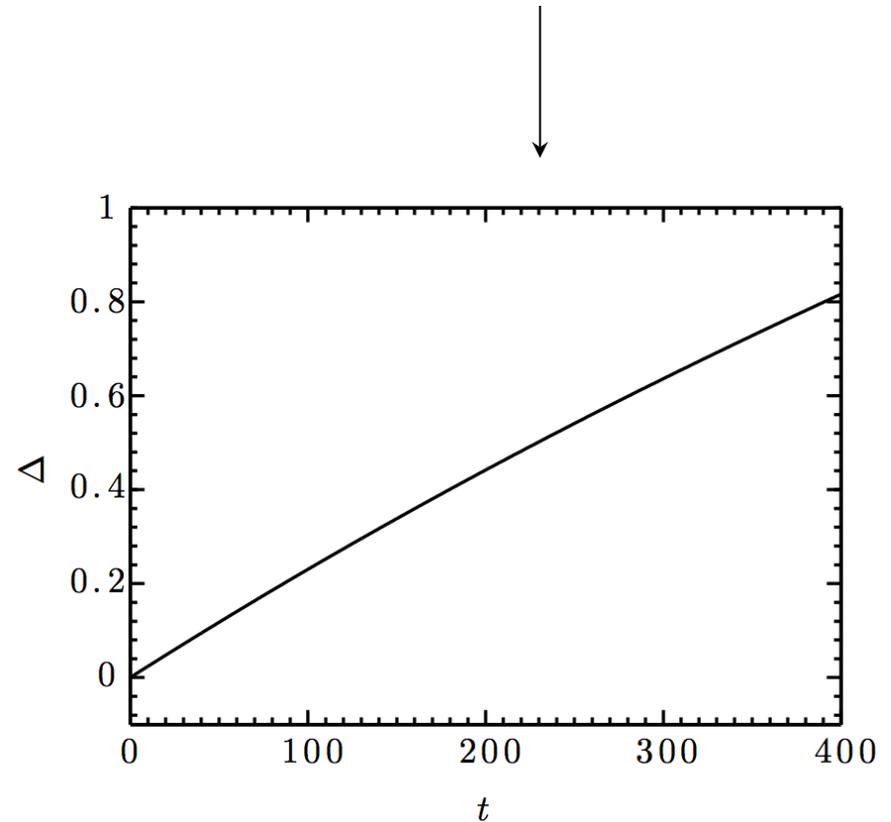


- effective collisionality required to maintain marginal stability
- + measured scattering rate during the saturated phase
- × measured scattering rate during the secular phase

# Mirror Instability (M. Kunz)



$$\frac{dB}{dt} > 0 \Rightarrow \Delta = \frac{p_{\perp} - p_{\parallel}}{p} > 0$$

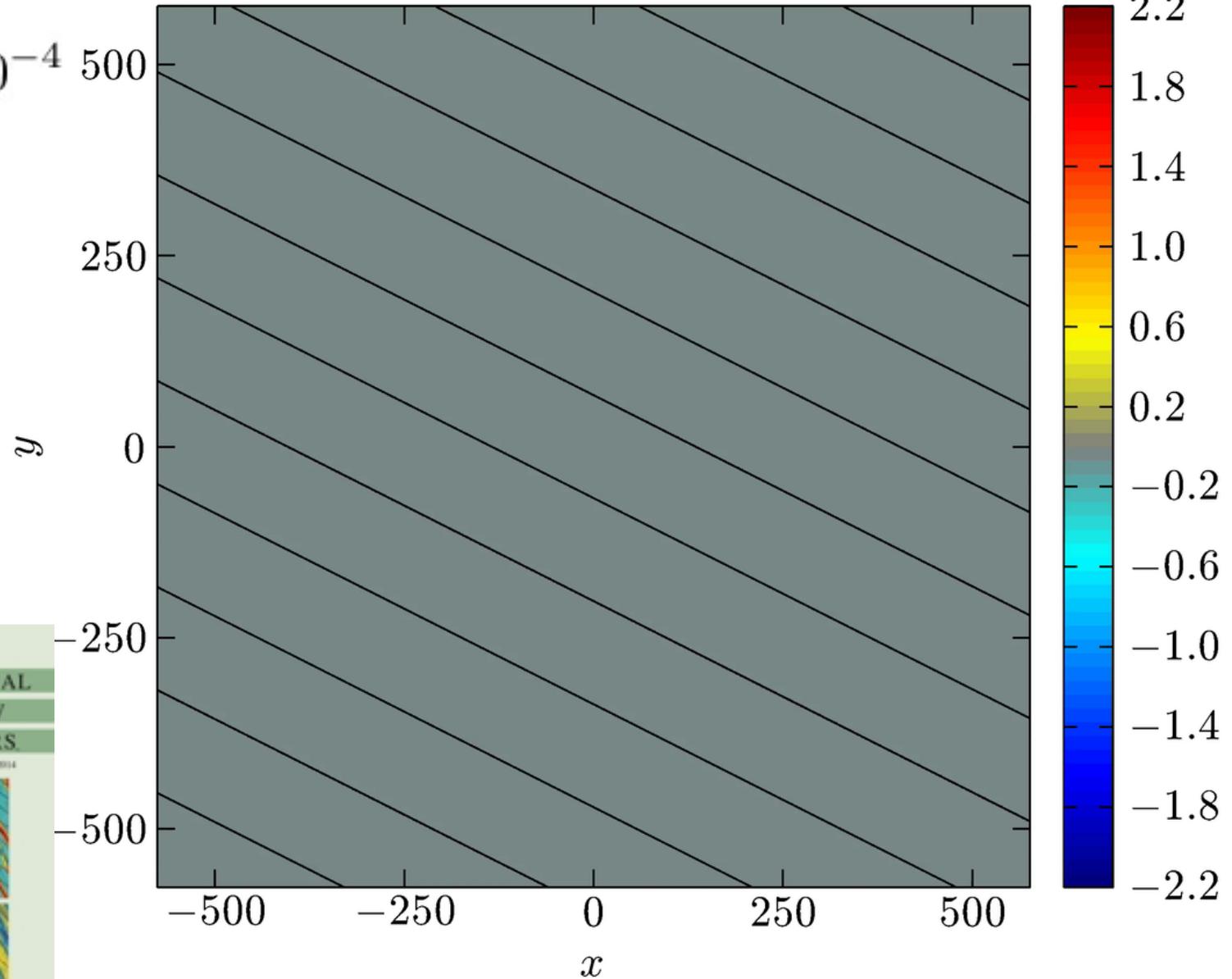


Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]  
Riquelme, Quataert & Verscharen, arXiv:1402.0014 (2014)

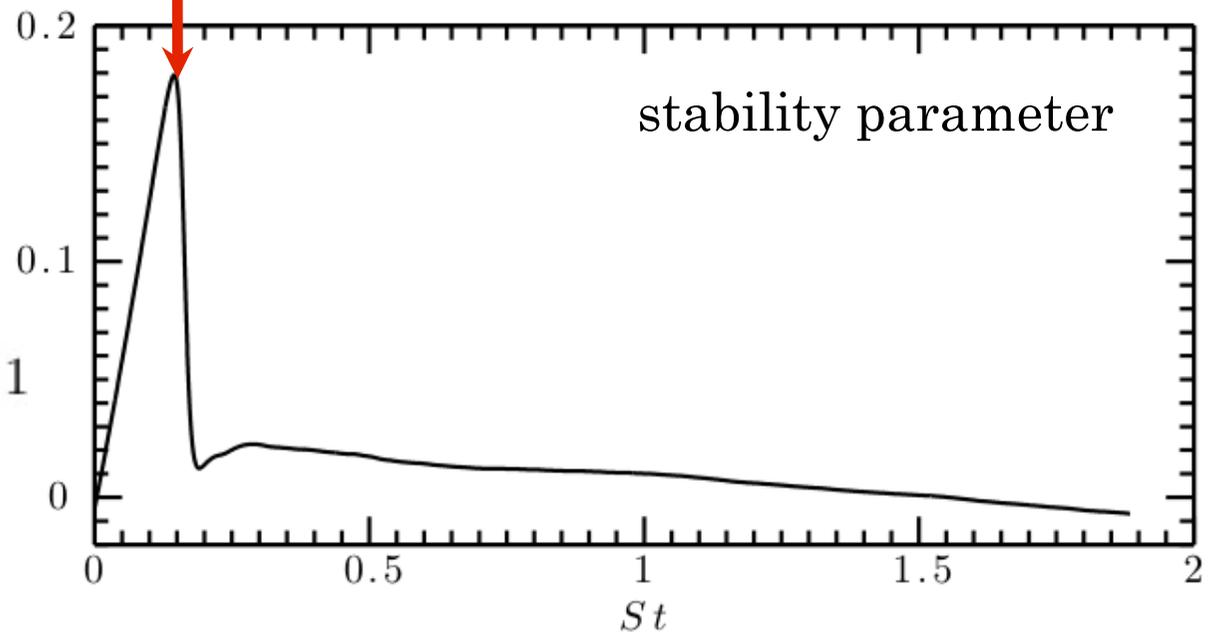
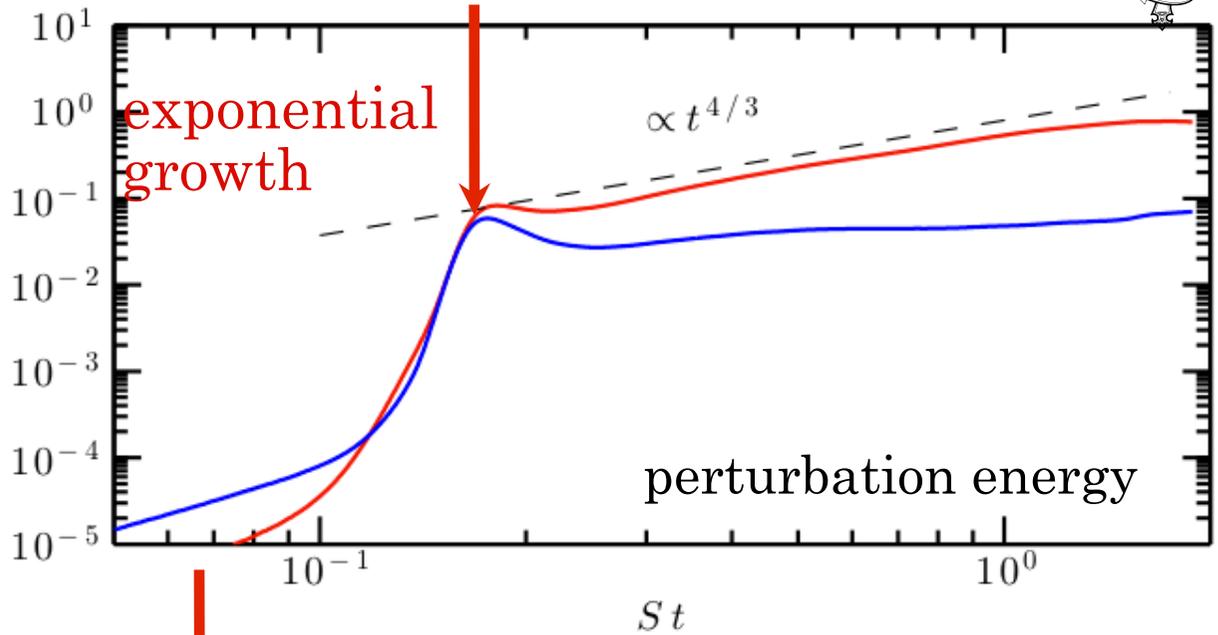
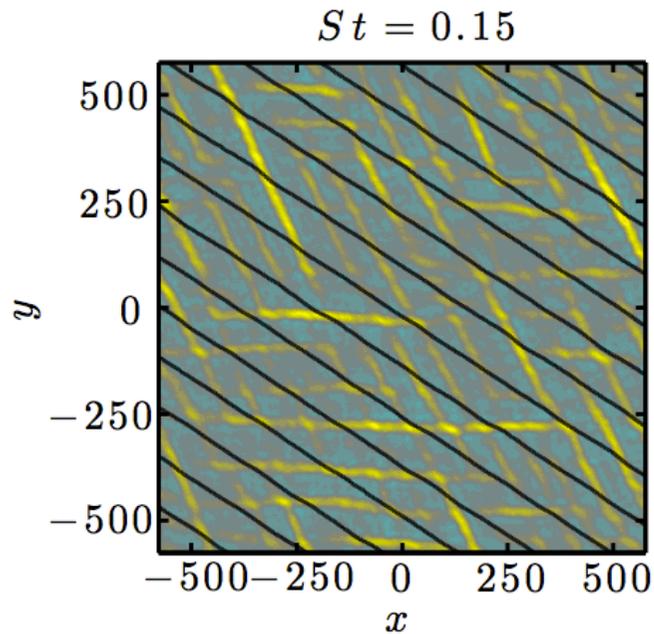
# Mirror Instability (M. Kunz)



$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$
$$\beta_i = 200$$



# Mirror Instability: Linear



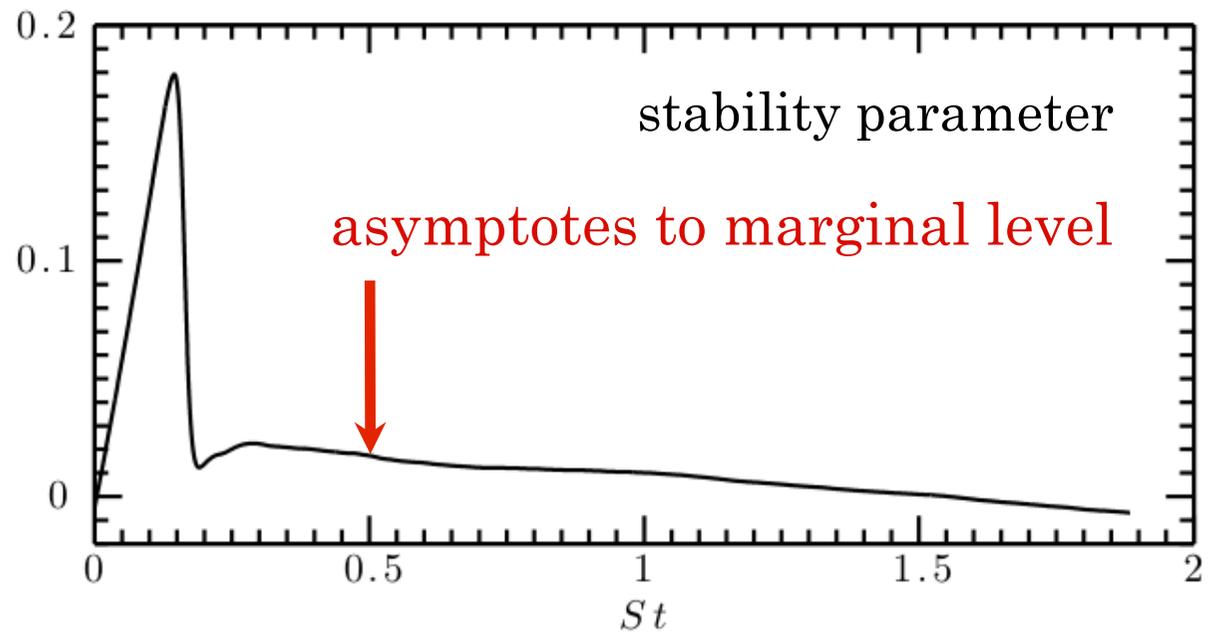
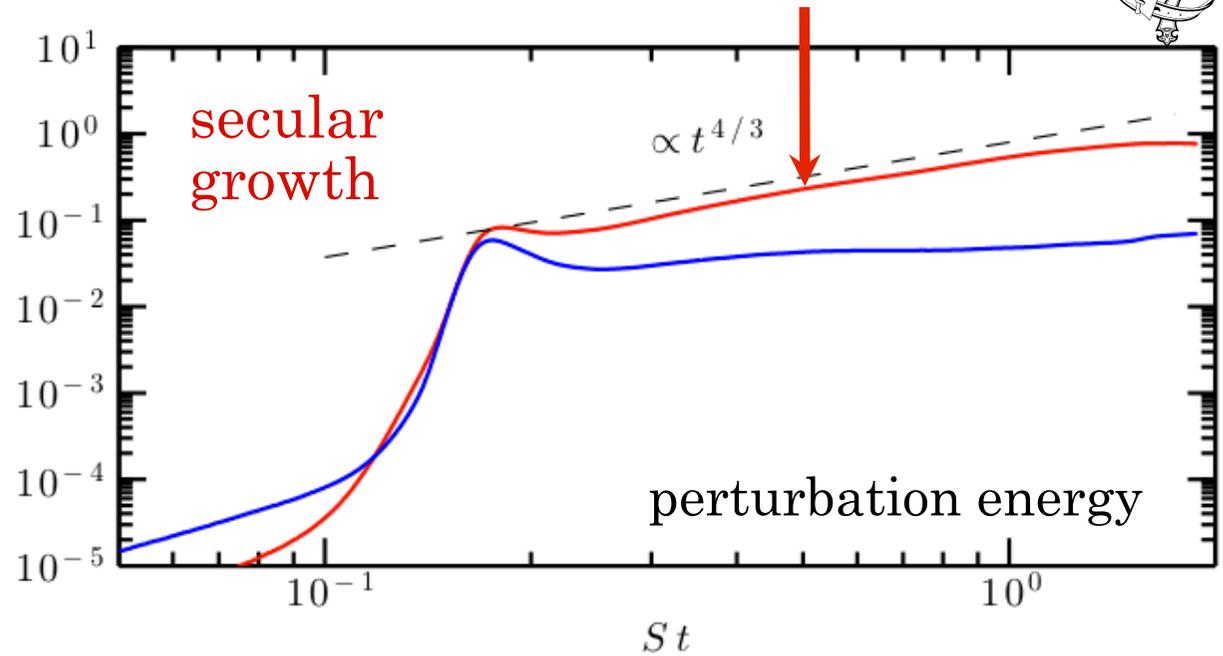
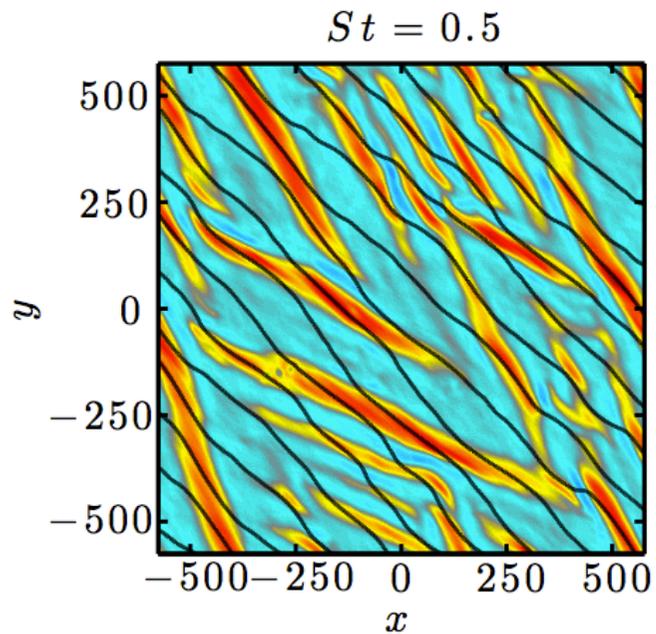
long, oblique modes

$$k_{\parallel} \rho_i \sim \left( \Delta - \frac{1}{\beta} \right)^{1/2} \quad k_{\perp} \rho_i \ll 1$$

Kunz et al., *PRL* 112, 205003 (2014)

[arXiv:1402.0010]

# Mirror Instability: Secular

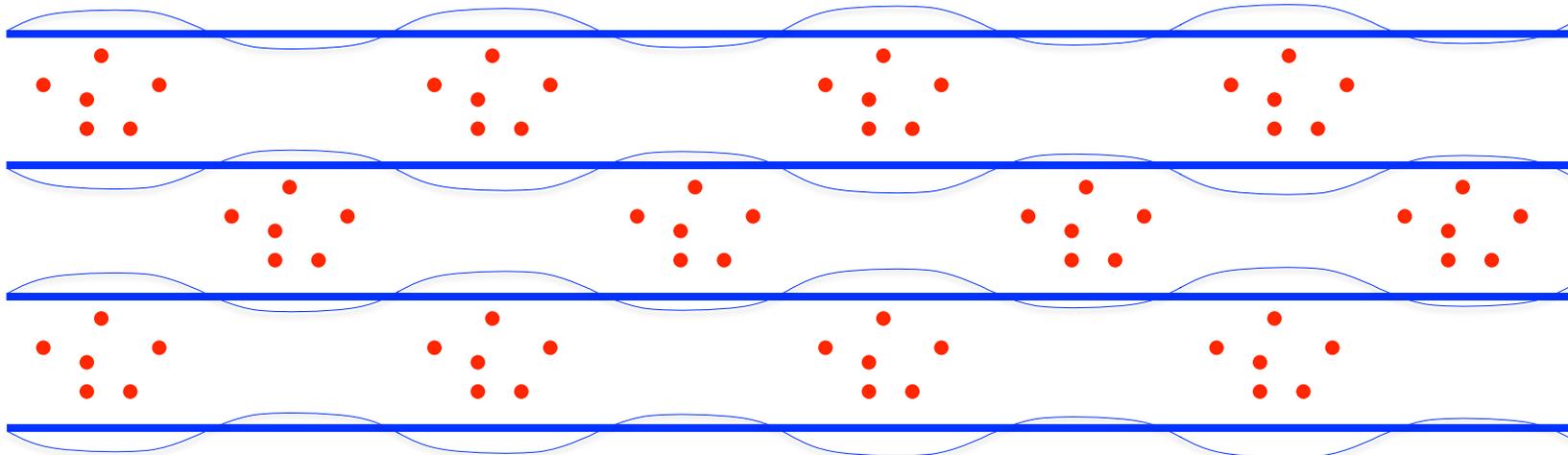


# Mirror Instability: Secular



$$\overline{B} = B_0 + \overline{\delta B_{\parallel}}$$

$$\Delta = 3 \int^t dt' \frac{d \ln \overline{B}}{dt} = 3 \int^t dt' \left( \underbrace{\frac{d \ln B_0}{dt}}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{driven by shear}}} + \underbrace{\frac{d \overline{\delta B_{\parallel}}}{dt B_0}}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{from} \\ \text{mirror-trapped} \\ \text{particles in holes} \\ \text{(fraction } \sim |\delta B_{\parallel}/B_0|^{1/2} \text{)}}} \right) \rightarrow \frac{1}{\beta}$$



Rincon, AAS & Cowley, arXiv:1407.4707

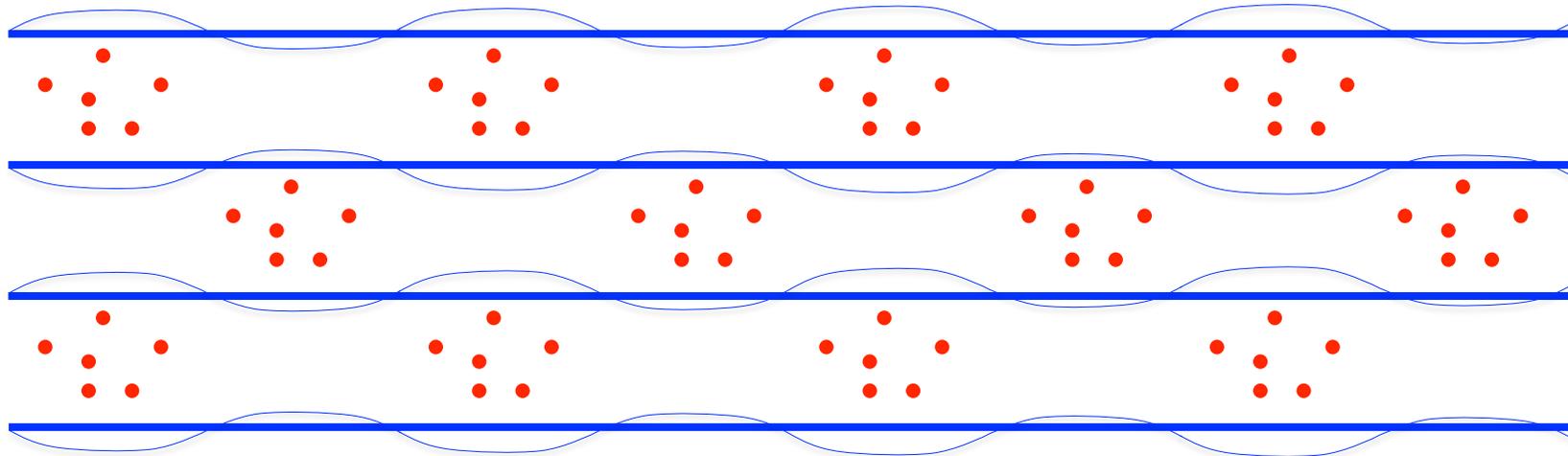
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

# Mirror Instability: Secular



$$\bar{B} = B_0 + \overline{\delta B_{\parallel}}$$

$$\Delta = 3 \int^t dt' \frac{d \ln \bar{B}}{dt} \sim 3 \int^t dt' \left( \underbrace{\frac{d \ln B_0}{dt}}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{driven by shear}}} - \underbrace{\frac{d}{dt} \left| \frac{\delta B_{\parallel}}{B_0} \right|^{3/2}}_{\substack{\text{pressure} \\ \text{anisotropy} \\ \text{from} \\ \text{mirror-trapped} \\ \text{particles in holes} \\ \text{(fraction } \sim |\delta B_{\parallel}/B_0|^{1/2} \text{)}}} \right) \rightarrow \frac{1}{\beta}$$



Rincon, AAS & Cowley, arXiv:1407.4707

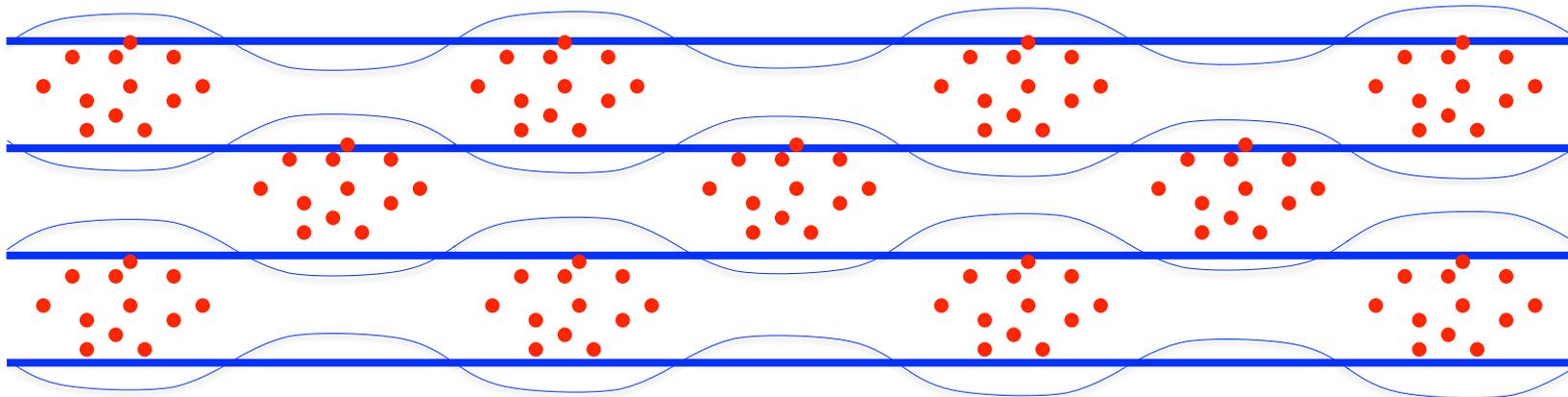
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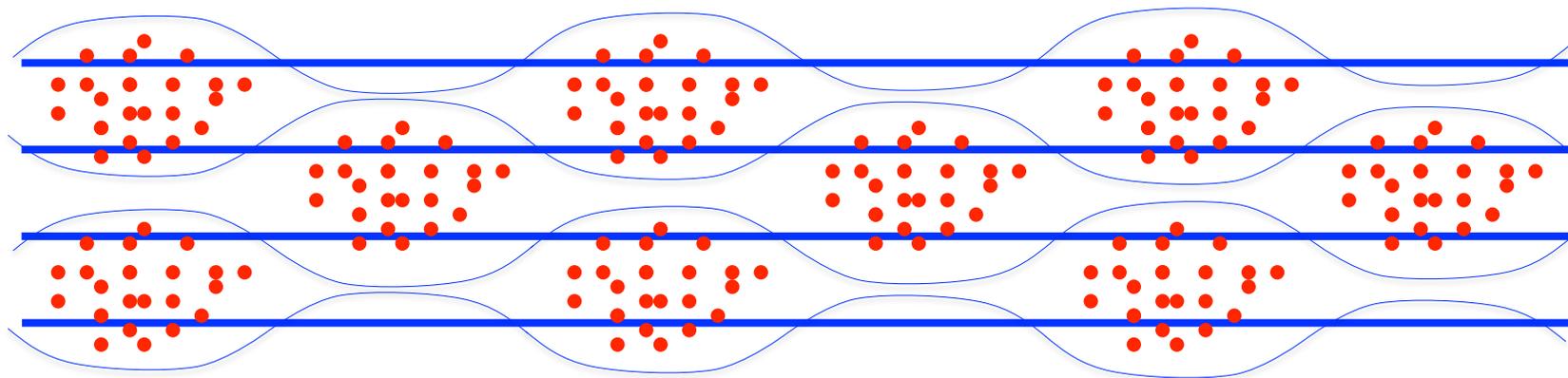
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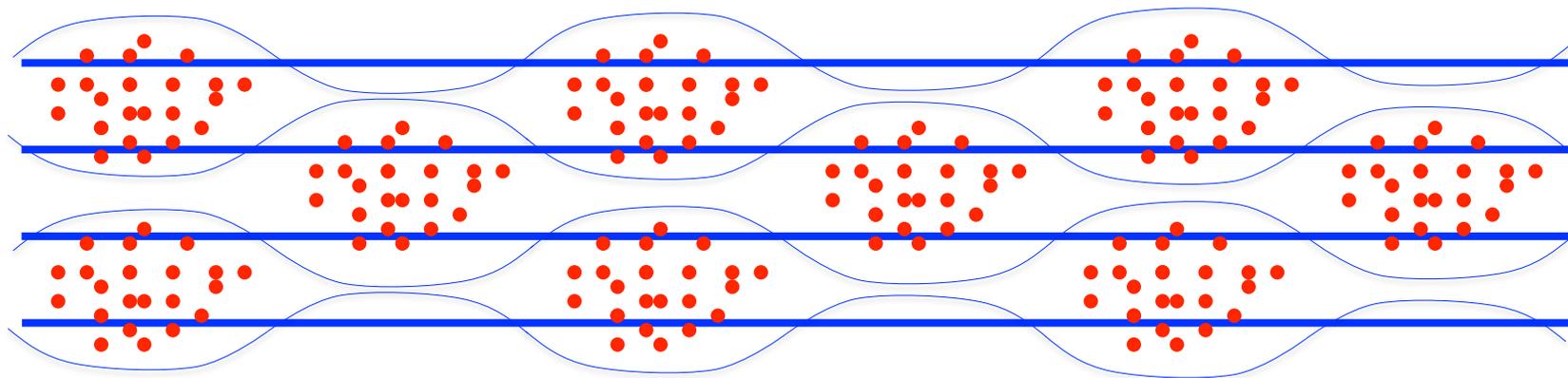


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$$\left| \frac{\delta B_{\parallel}}{B_0} \right|^{3/2} = S \int^t dt' \hat{b}_x(t') \hat{b}_y(t') - \frac{1}{\beta} \Rightarrow \frac{\delta B_{\parallel}^2}{B_0^2} \sim (St)^{4/3}$$

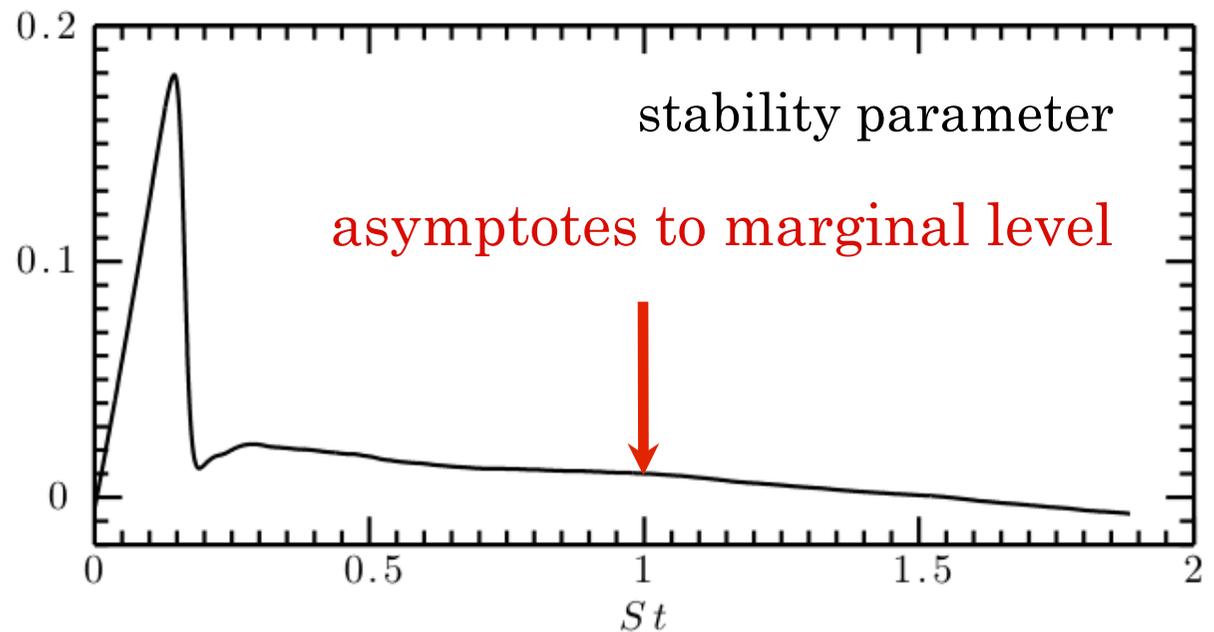
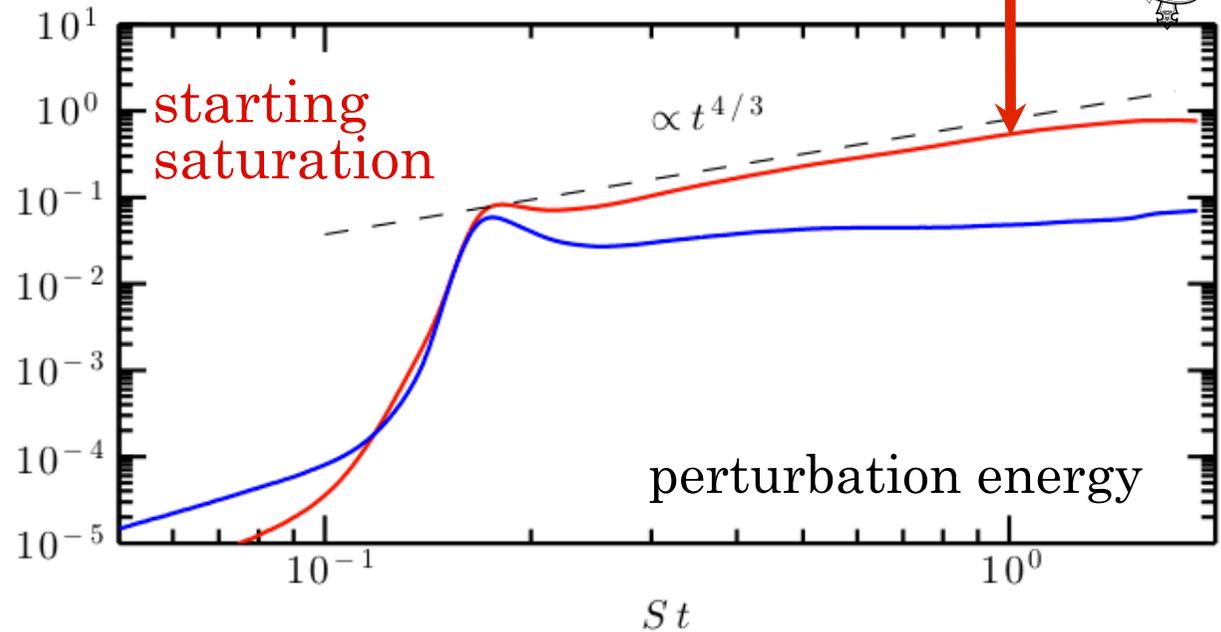
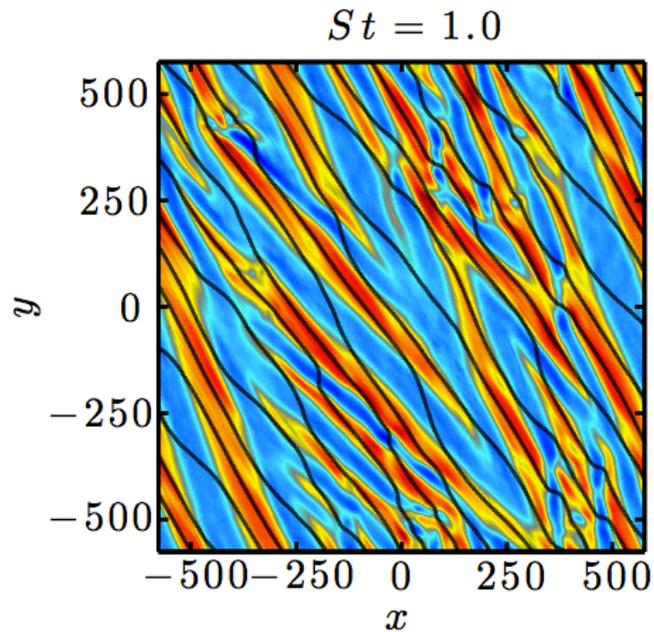
secular growth



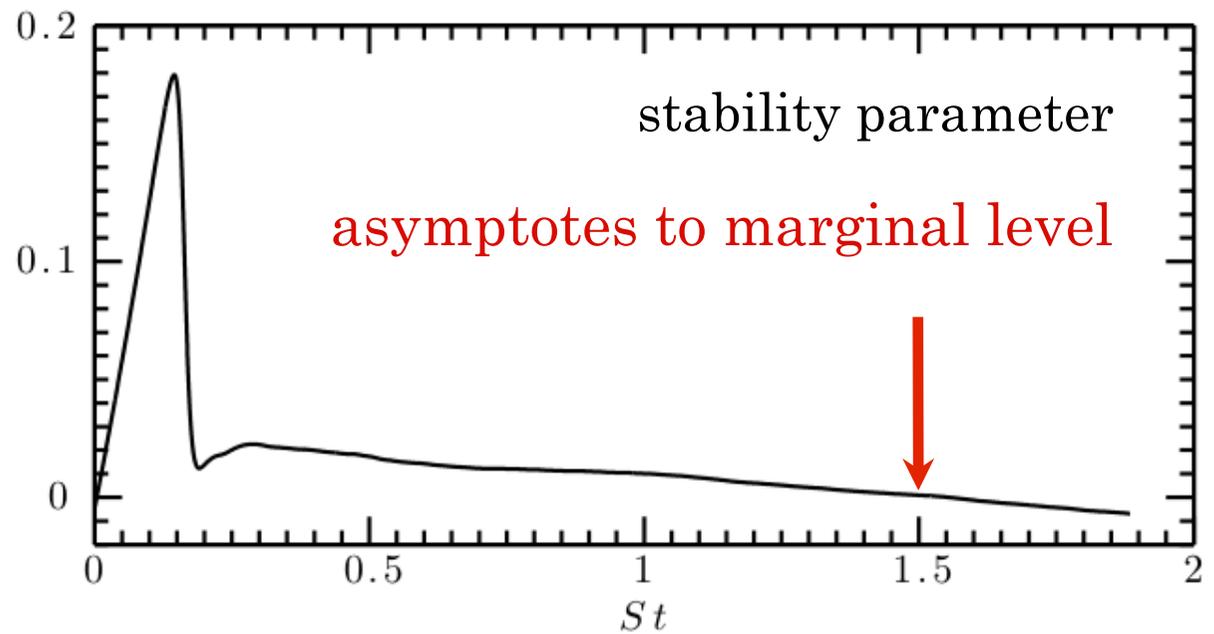
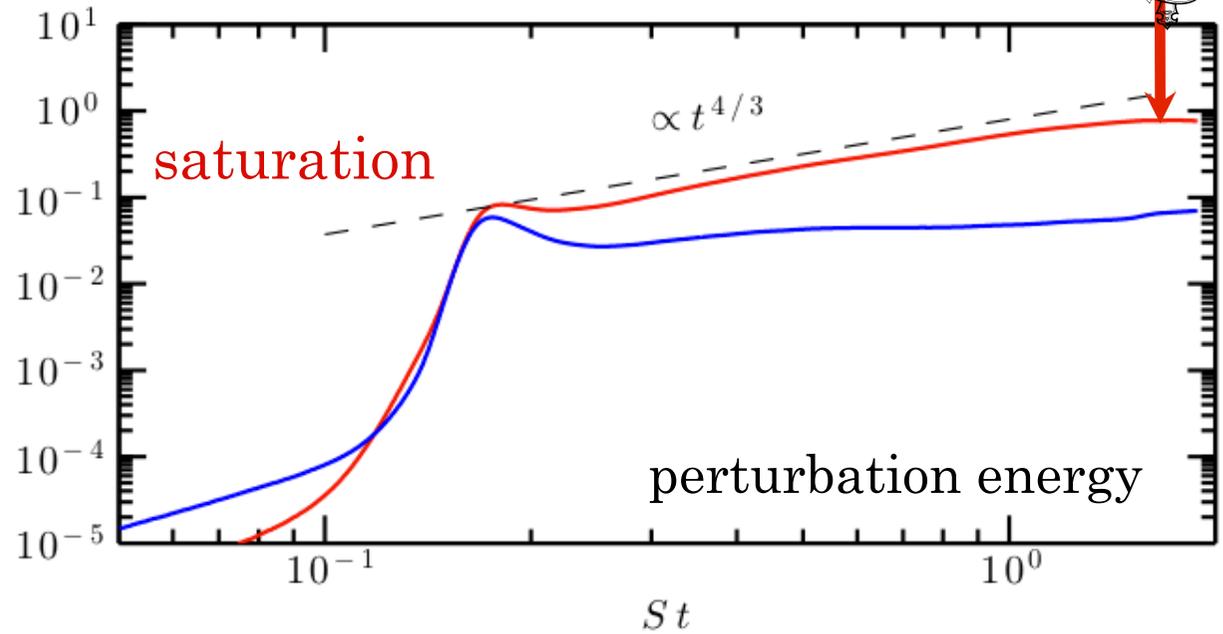
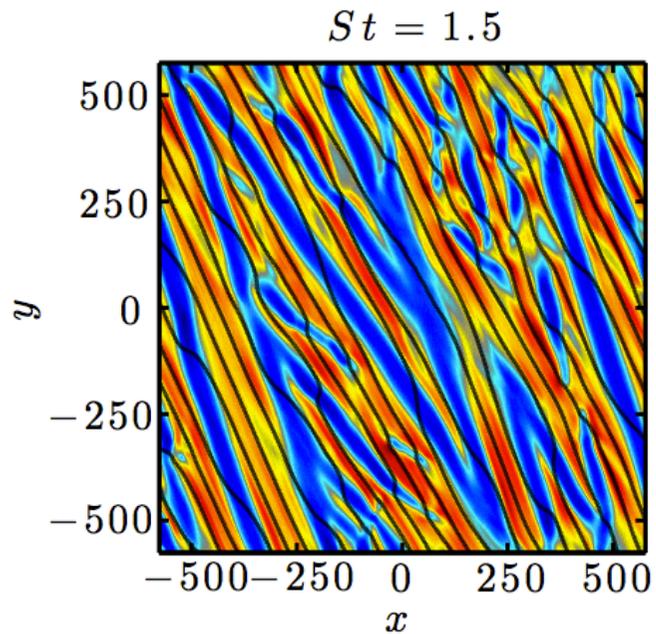
Rincon, AAS & Cowley, arXiv:1407.4707

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# Mirror Instability: Secular



# Mirror Instability: Saturated

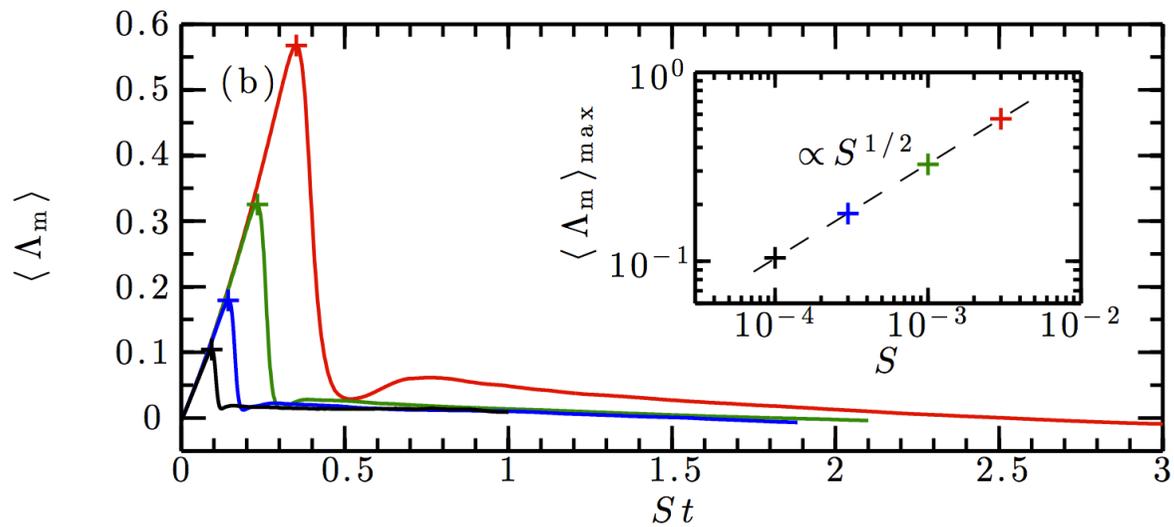
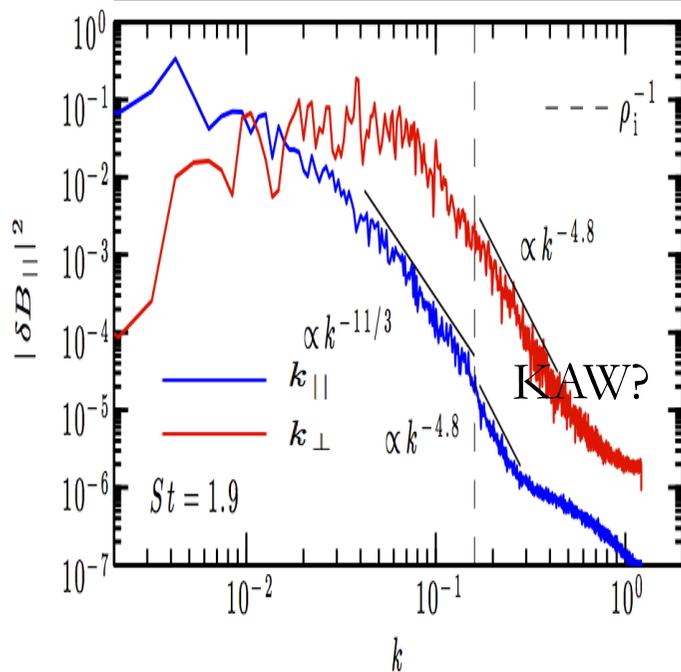
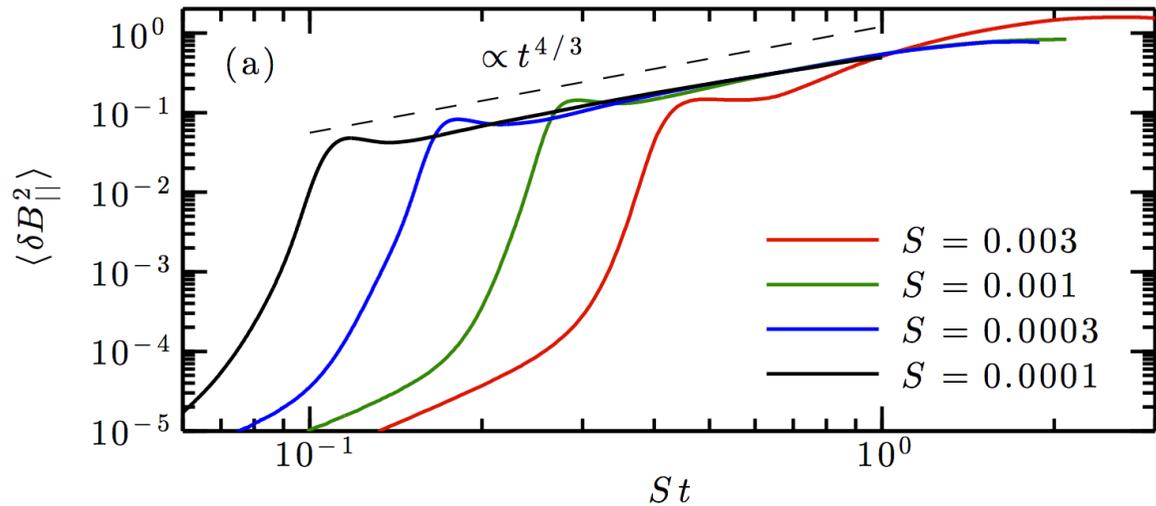


# Mirror Saturates at Order-Unity Amplitudes

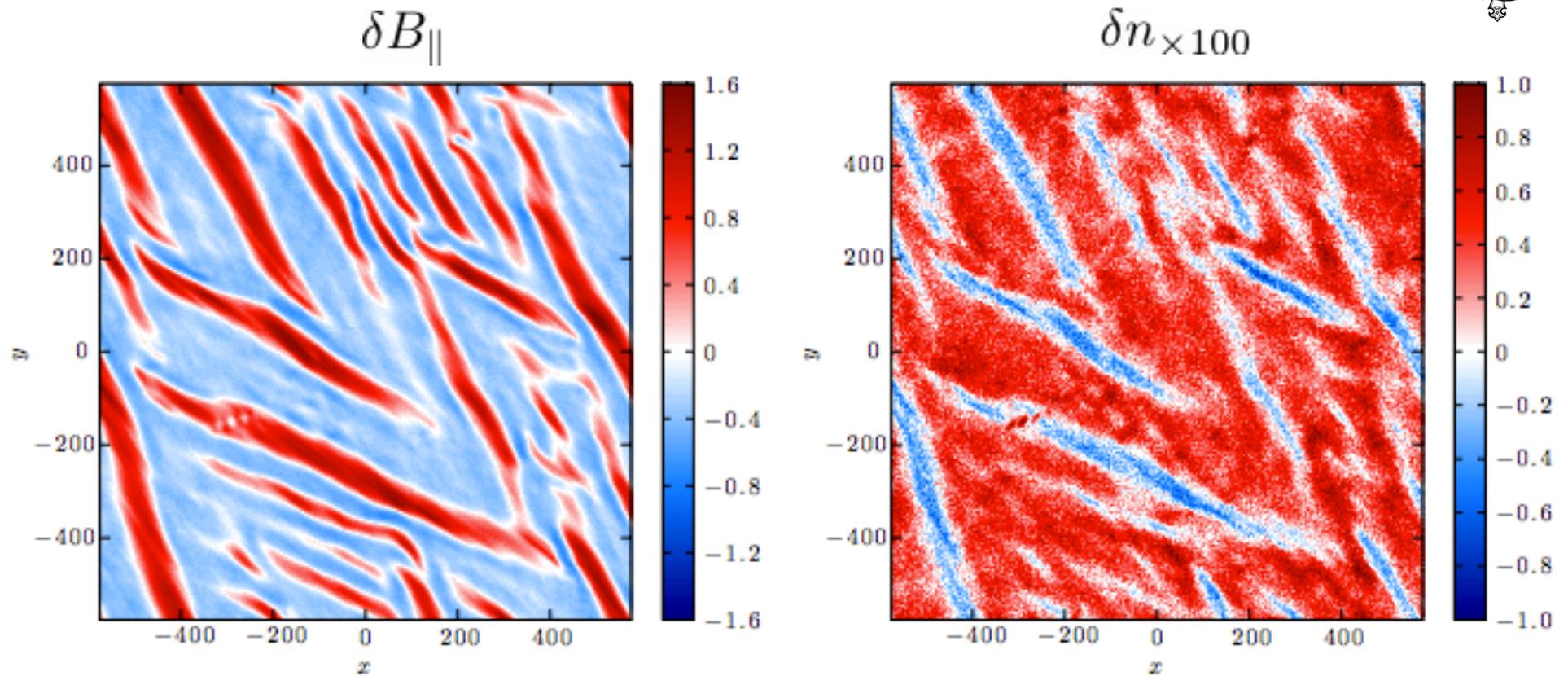


$$\frac{\langle \delta B_{\parallel}^2 \rangle}{B_0^2} \sim 1$$

order-unity-amplitude  
(independent of  $S$ )  
long-parallel-scale  
mirror turbulence

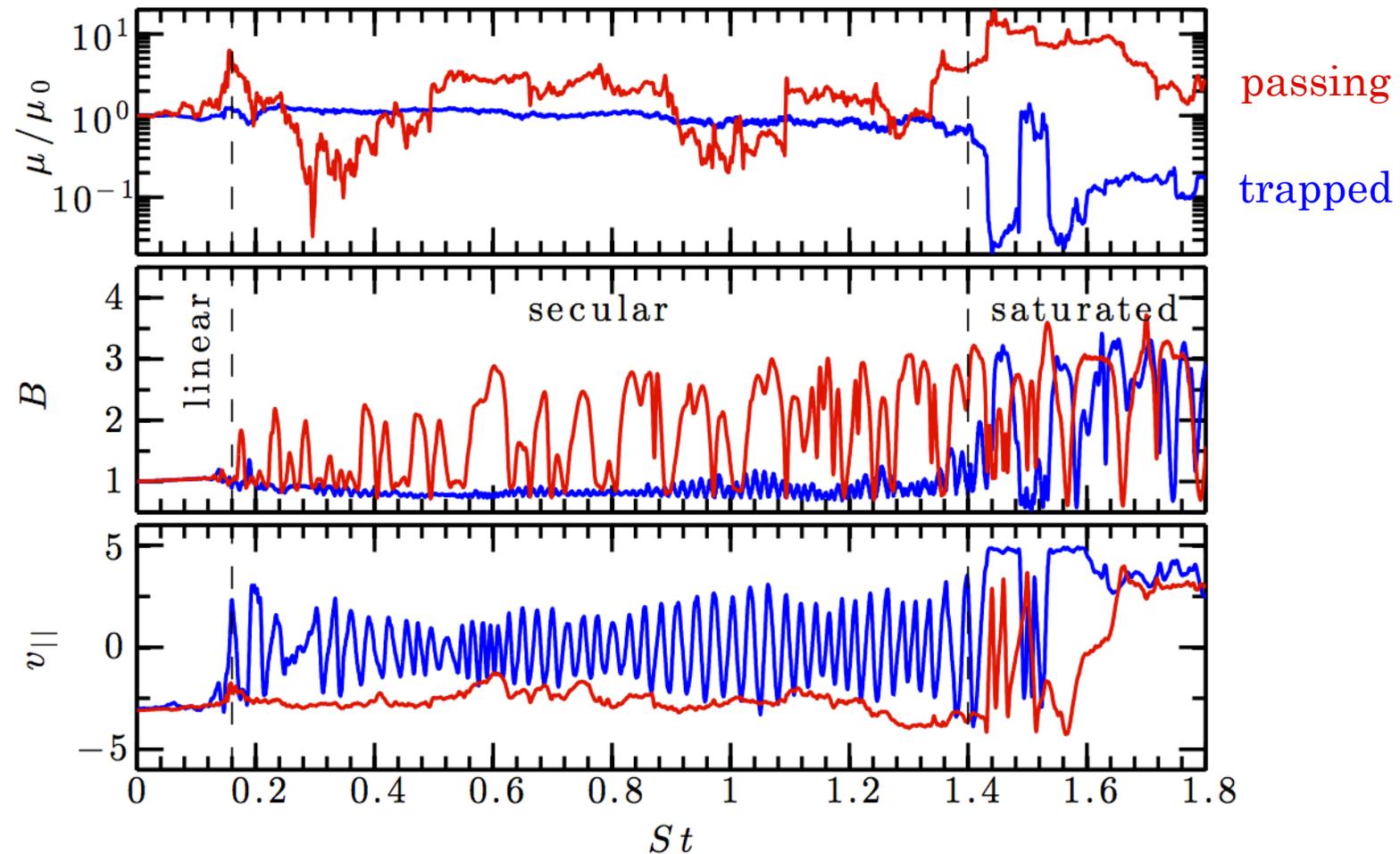


# Mirror Instability: Trapped Particles



pressure anisotropy is regulated by **trapped particles** in magnetic mirrors, where field strength stays constant on average...

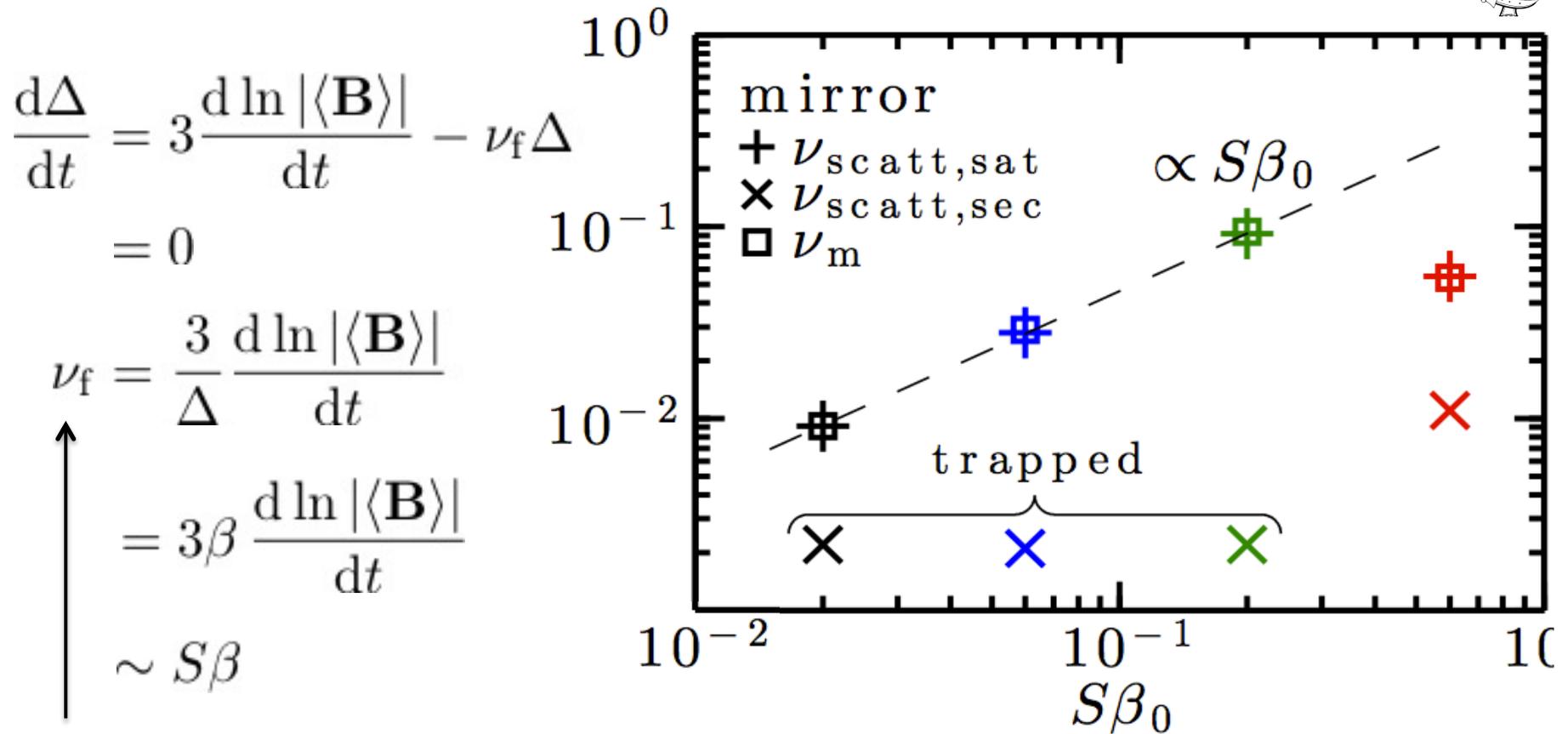
# Secular Mirror Doesn't Scatter Particles



pressure anisotropy is regulated by **trapped particles** in magnetic mirrors,  
where field strength stays constant on average...

**no particle scattering** until (late) saturation (off mirror edges)

# Secular Mirror Doesn't Scatter Particles



- effective collisionality required to maintain marginal stability
- + measured scattering rate during the saturated phase
- × measured scattering rate during the secular phase

# Conclusions So Far



➤ *Very different scenarios for plasma dynamo depending on whether nonlinear firehose and mirror fluctuations regulate pressure anisotropy by scattering particles or by adjusting rate of change of the magnetic field:*

- **No scattering** → explosive growth, but long time to get going

$$t \sim \beta_0 / 2\nu$$

scales with collision time  
and initial field

- **Efficient scattering** → secular growth, but very fast

$$t \sim l/u$$

one large-scale  
turnover time

- Driven **firehose** saturates at low amplitudes, scatters particles
- Driven **mirror** grows to  $\delta B/B \sim 1$  without doing much scattering (marginal state achieved via trapped population in mirrors)
- [Both instabilities have a sub-Larmor tail, which appears to be KAW turbulence with the usual spectrum]
- **Plasma Dynamo: the race is on**

Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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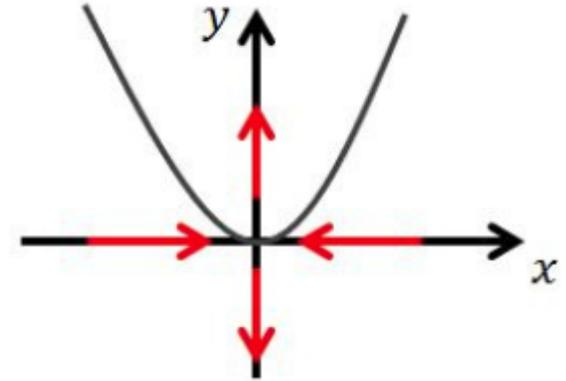
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**WE DON'T REALLY KNOW (YET) HOW MAGNETISED, HIGH  $\beta$  PLASMA MOVES**

# Effects of Magnetic Field



Initially parabolic magnetic field line subject to Braginskii viscosity (by Scott Melville)

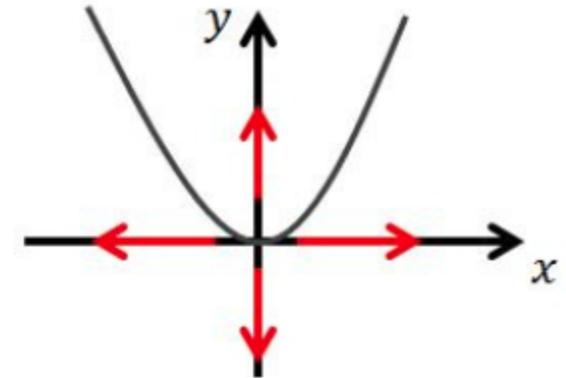


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