Study of radial correlation reflectometry using a 2D full-wave code

E. Blanco, T. Estrada, and T. Happel.
Laboratorio Nacional de Fusión. Asociación Euratom-CIEMAT, Madrid, Spain

Abstract
A 2D full-wave code in O-mode with Gaussian beams propagation and normal incidence is used to simulate radial correlation reflectometry. The radial correlation lengths estimated from reflectometry signals are then compared with the true values under a wide range of turbulence conditions. The scan in the radial correlation length is $L_r / \lambda_0 \approx 0.4 - 2$ whereas the scan in the turbulence level is $\delta n_{rms} / n = 1 - 15$ %. Such scans allow us to study correlation reflectometry measurements both in linear and non-linear regimes. Full-wave results show that in the linear regime radial correlation lengths satisfying $L_r / \lambda_0 \geq 1$ can be estimated accurately from the amplitude signal. The transition between linear and non-linear regimes shows a marked decrease in the coherence of all reflectometry signals; close to the transition homodyne signal performs better. When non-linear effects become dominant radial correlation lengths are underestimated. Finally, an accurate measurement of radial correlation lengths shorter than the probing wavelength is not possible.

I. Introduction

The plasma is modelled as a slab with an average linear density profile in the radial direction (x-direction). Density fluctuations are added to the average density profile. The k-spectrum $\delta n(k_x, k_y)$ of density fluctuations is modelled as [1]:

$$\delta n(k_x, k_y) = \begin{cases} K & \text{if } 0 \leq k_x \leq k_{x,\max}\ ,\ 0 \leq k_y \leq k_{y,\max} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

where $k_{x,\max}$, $k_{y,\max}$ are the radial and perpendicular spectral widths of the turbulence, respectively, and the constant K determines the spectrum amplitude. The radial correlation length is related to the spectral width via $L_{cx} = 2.1 / k_{x,\max}$. The parameters K, $k_{x,\max}$, $k_{y,\max}$ determine the root mean square (rms) value of the density fluctuations via $\delta n_{rms}^2 = K^2 \cdot k_{x,\max} \cdot k_{y,\max}$. As shown in [2], the non-linear regime appears if the following criterion is satisfied:

$$\gamma \equiv \frac{\varpi_0^2}{c^2} l_{cx} x_c \left(\frac{\delta n_{rms}}{n_c} \right)^2 \ln \frac{x_c}{l_{cx}} \geq 1 \qquad (2)$$

where ϖ_0 is the probing frequency, c is the speed of light in vacuum, x_c is the cut-off distance from the plasma edge, l_{cx} is the radial correlation length of the turbulence, δn_{rms} is the rms value of the turbulence and n_c denotes the density at the cut-off layer. Our previous numerical results on Doppler reflectometry [1] confirmed the validity of criterion (2) to distinguish between linear and non-linear regimes. Doppler reflectometry simulations were performed for different radial correlation lengths, cut-off positions and density gradients. The numerical results showed a linear relationship between the Doppler peak amplitude and the turbulence level at low turbulence levels followed by a non-linear regime. The transition between linear and non-linear regimes occurred at different turbulence levels depending on the radial correlation length as expected from eq. (2). However, the influence of the radial correlation length on the Doopler peak amplitude was shown to be mainly a consequence of considering δn_{rms} as the relevant parameter. When the spectrum amplitude of the density fluctuations K was considered the transition between linear and non-linear regimes occurred close to $K = 10^{15}$ m^{-2} regardless of the radial correlation length. It was also shown that within the transition $\gamma \approx 1$ was obtained. In this work we use γ to study correlation reflectometry both in linear and non-linear regimes in the case of normal incidence.

II. Simulations on correlation reflectometry.

II. A. Linear regime

Fig. 1 shows the coherence of different reflectometry signals (amplitude A, homodyne $A\cos\phi$, complex amplitude $Ae^{i\phi}$, and complex phase $e^{i\phi}$) as a function of the radial separation between the two reflectometer channels. The solid line represents the coherence of the input density fluctuations. Three different radial correlation lengths are considered: $l_{cx}/\lambda_0 \approx 0.4$ (fig. 1a), $l_{cx}/\lambda_0 \approx 1.2$ (fig. 1b), and $l_{cx}/\lambda_0 \approx 2.5$ (fig. 1c). In all cases, the turbulence level is low enough to ensure that non-linear effects are negligible ($\gamma << 1$). Fig. 1 shows that the amplitude signal performs better than homodyne, complex amplitude, and/or complex phase signals in estimating the radial correlation length. Accurate measurements of the radial correlation length can be achieved provided that $l_{cx}/\lambda_0 \geq 1$ (figs. 1b and 1c). When $l_{cx}/\lambda_0 < 1$ (fig. 1a) the amplitude signal overestimates the true value and accurate measurements of radial correlation lengths are not possible. Homodyne, complex amplitude, and complex phase signals decay slowly and overestimate systematically the true value by a factor larger than 2; the error is especially large if $l_{cx}/\lambda_0 < 1$ (fig. 1a).

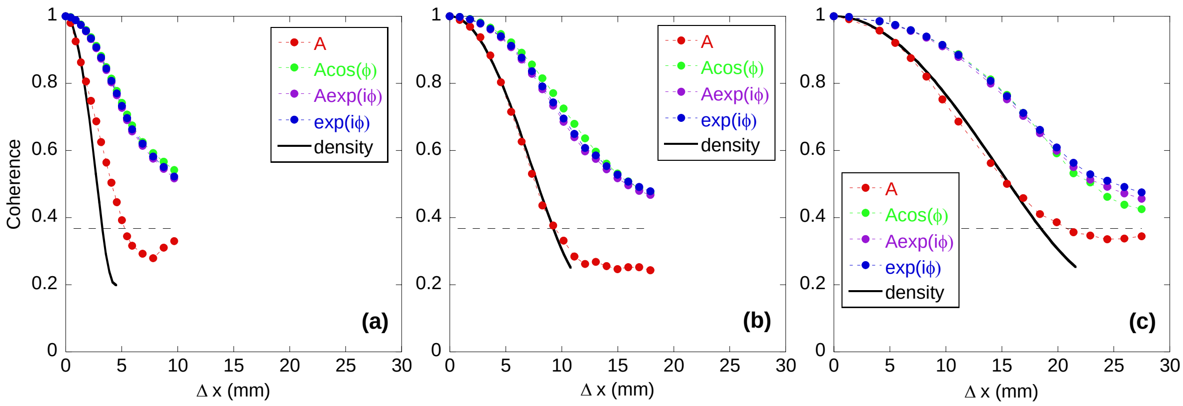

Figure 1. Coherence of different reflectometry signals in linear regime: amplitude (red), homodyne (green), complex amplitude (purple) and complex phase (blue) as a function of layer separation. The results are shown for radial correlation lengths (a) $l_{cx}/\lambda_0 \approx 0.4$ ($\gamma = 0.07$), (b) $l_{cx}/\lambda_0 \approx 1.2$ ($\gamma = 0.13$) and (c) $l_{cx}/\lambda_0 \approx 2.5$ ($\gamma = 0.18$). The solid line represents the coherence of density fluctuations.

The slow decay of the coherence has been observed with other numerical codes [3 - 5]. In [3] a 2D physical optics model predicts a very slow decay of the coherence for the phase signal at low turbulence levels. In [4] a 1D full-wave code confirms such slow decay of the coherence. The results are also compared with those expected from analytical theory showing a good agreement. The same behaviour is obtained in [5] using a 2D code and the paraxial approximation. Analytical theory [6] explains the slow decay of the coherence in terms of the small angle scattering due to density fluctuations having wavenumbers lower than the inverse density gradient length. When such wavenumbers are removed from the spectrum the coherence becomes closer to the correlation of the turbulence as it has been found in [4]. This result indicates that scattering process takes place along the probing wave trajectory leading to a degradation of diagnostic localization. Interestingly, amplitude signal seems to be less sensitive to low radial wavenumbers than the others signals. This result was also observed in [3] with a 2D physical optics code. It was shown that the coherence of the power signal (amplitude squared) showed a faster decrease with radial separation than the phase signal. More 2D full-wave simulations are needed to investigate the validity of this result in more general situations.

II. B. Transition from linear to non-linear regime.
The increase in the fluctuation level has a pronounced effect on the estimated correlation length as shown in fig. 2. The results are shown for three different γ values covering linear regime (fig. 2a), transition (fig. 2b), and non-linear regime (fig. 2c). All signals show a marked decrease in the coherence as the turbulence level increases. Note the abrupt change in the complex phase coherence when going through the transition (fig. 2a and fig. 2b). Close to the transition (fig. 2b) homodyne and complex amplitude signals perform better than the others signals and provide a good estimation for the correlation length of the turbulence. This result could be explained in terms of the destructive interference occurring at the receiving antenna due to the relatively high density fluctuations level. Destructive interference gives rise to low amplitude signals with incorrect phase values, not related with the movement of the density fluctuations. Homodyne and complex amplitude signals assign less weight to the incorrect phase values and their contribution to the coherence is smaller. In non-linear regime homodyne and complex amplitude signals still perform better than the others but the error is large and the correlation length is underestimated by a factor of 2.

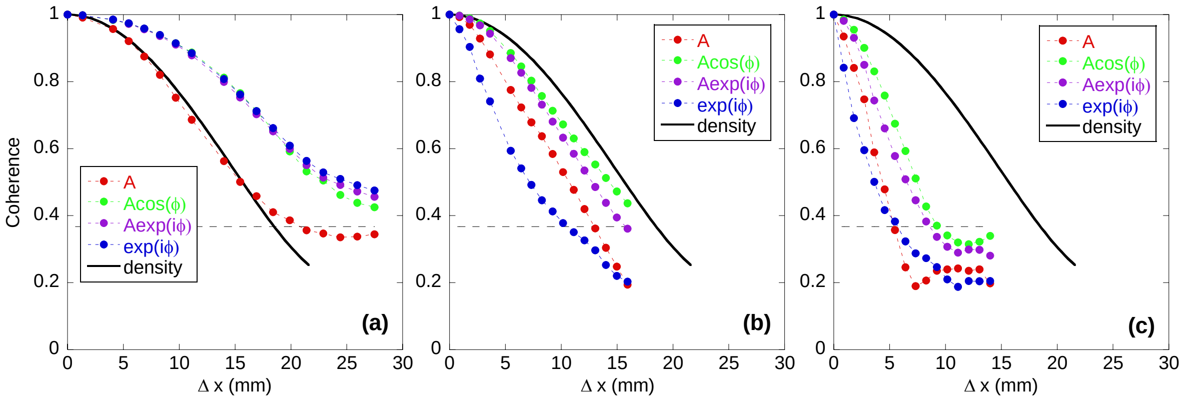

Figure 2. Coherence of amplitude, homodyne, complex amplitude, and complex phase signals as a function of layer separation. The results are shown for three turbulence level (a) $\delta n_{rms}/n = 1\%$ ($\gamma = 0.18$), (b) $\delta n_{rms}/n = 5\%$ ($\gamma = 4.62$), and (c) $\delta n_{rms}/n = 10\%$ ($\gamma = 18.49$). The solid line represents the coherence of density fluctuations.

The first attempt to study correlation reflectometry under high density fluctuation level was made in [7] using WKB approximation. This work showed a drastic decrease of the coherence when the turbulence level increases. In [3] a 2D physical optics code shows the collapse in correlation length with increasing the fluctuation amplitude. Same behaviour is observed in [4] with a 1D full-wave codes. The fast decay of the coherence in non-linear regime (fig. 2c) is expected from non-linear theory [2]. The role of poorly localized small angle scattering, which is dominant in linear regime, can be neglected in this situation. At high turbulence levels, the fast decay of the coherence is explained in terms of the strong interference produced by the multiple cut-offs that exist within the plasma. Above some turbulence level threshold the two signals coming from the two reflectometer channels becomes statistically independent. As a result a strong reduction in the measured correlation length is obtained. Furthermore, according to non-linear theory the decay of the coherence is proportional to the density fluctuation level in the cut-off layer. Therefore, radial correlation reflectometry provides highly localized information about turbulence characteristics.

Preliminary 2D full-wave results have shown that the estimated correlation length weakly depends on the perpendicular spectral width. However, more simulations are needed to improve our understanding of the perpendicular scale length effects on radial correlation measurements. In particular, 2D full-wave reflectometry simulations with a non-zero antenna tilt angle (perpendicular wavenumber selectivity) are underway.

III. Conclusions

Numerical simulations show that the measurement of radial correlation length $L_r / \lambda_0 \geq 1$ can be obtained accurately from the amplitude signal in the linear regime. Numerical simulations also suggest a possible method to help the experimentalist in determining if the measurements are performed in linear regime. Linear regime is characterized by a) the measured correlation lengths are larger than the probing wavelength and b) the correlation length of the amplitude signal is significantly shorter than the values obtained with other reflectometry signals. The transition from linear to non-linear regime shows a marked decrease in the correlation length estimated from all reflectometry signals. Close to the transition homodyne signal and/or complex amplitude will provide a better estimation. The transition has the following characteristics: a) the correlation length estimated from the complex phase signal is smaller than the estimated value from the amplitude signal and b) complex amplitude and homodyne signals give the largest values for the correlation length. In non-linear regime, radial correlation lengths are underestimated. This regime is characterized by similar values for the correlation length between amplitude and phase signals. Finally, the measurement of correlation lengths smaller than the probing wavelength is not possible.

References

[1] E. Blanco, and T. Estrada. Plasma Phys. Control. Fusion **50** (2008) 095011
[2] E Z Gusakov and A Yu Popov. Plasma Phys. Control. Fusion **44** (2002) 2327-2337
[3] G. D. Conway. Plasma Phys. Control. Fusion **39** (1997) 407-421
[4] G Leclert, S Heuraux, E Z Gusakov et al. Plasma Phys. Control. Fusion **48** (2006) 1389-1400
[5] G J Kramer, R Nazikian, and E Valeo. Rev. Sci. Instrum. **74** (2003) 1421-1425
[6] E Z Gusakov and B O Yakolev. Plasma Phys. Control. Fusion **44** (2002) 2125-2537
[7] E. Mazzucato, and R. Nazikian. Phys. Rev. Lett. 71 (1993) 1840