An Adaptive Real-Time Multi-Tone Estimator and Frequency Tracker for Non-Stationary Signals

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I. Proposal

Real-Time estimation of instantaneous quantities
- Frequency
- Phase
- Amplitude

in non-stationary signals of the form

\[ S_k = \sum_{i=1}^{N} S_{i,k} + n_k \]

\[ S_{i,k} = A_{i,k} \sin(\phi_{i,k}) \]  

\[ k \rightarrow \text{discrete time sample}, \quad n \rightarrow \text{Gauss dist. Random noise}, \quad N \rightarrow \text{total number of tones} \]

II. Extended Kalman Filter Frequency Tracker [2]

- Predictor (phase rotation)

\[ \dot{x}_k = F_k x_{k-1}, \quad F = \begin{bmatrix} \cos(x_{2,k-1}) - \sin(x_{3,k-1}) & 0 \\ \sin(x_{2,k-1}) & \cos(x_{3,k-1}) \end{bmatrix} \]

- Corrector

\[ x_k = \hat{x}_k + K_k (z_k - H\hat{x}_k) \]

\[ K_k \text{ minimizes } E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T], \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix} \]

- Frequency Estimation

\[ f_k = \frac{x_{1,k}}{2\pi} \]

- Amplitude Estimation

\[ A_k = \sqrt{x_{1,k}^2 + x_{2,k}^2} \]

\[ x_{1,k}, x_{2,k} \rightarrow \text{in-phase signal estimation}, \quad x_{3,k}, x_{4,k} \rightarrow \text{quadrature signal estimation} \]

- Phase Distortion

\[ \Delta \phi_k = \arctan \frac{S_1, q_{i,k} - S_2, S_2, q_{i,k}}{S_1, q_{i,k} + S_2, S_2, q_{i,k}} \]

\[ S_1 \rightarrow \text{signal component}, \quad S_2 \rightarrow \text{quadrature of signal component} \]

- Multi-Tone Estimation (example with 2 tones)

<table>
<thead>
<tr>
<th>Tone 1</th>
<th>Tone 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>3.5</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- Tuning

- Performance is a compromise between response time and filtering performance

- Performance is tunable by balancing the weights given to the measurements and the theoretical model

- \( Q \rightarrow \text{model covariance matrix (inN x inN)} \) and \( R \rightarrow \text{measurement variance (scalar)} \)

- Consider \( Q \rightarrow \text{covariance matrix} \) and \( R \rightarrow \text{measurement variance (scalar)} \)

- \( Q \) determines the signal estimation bandwidth

- \( Q / R \) determines the frequency estimation bandwidth

- The EKFFT is a real-time, adaptive, band-pass filter

III. Performance

- Single-Tone

\[ \text{Noise StdDev} = 0.25 \]

\[ \text{Initial freq. guesses = 50 kHz, } \quad Q \rightarrow \text{array with 1, 1, 1, 1, 1, 1}, \quad R = 1 \]

- Remarks

- Successful Freq Tracking!
- Successful Amp Tracking!
- Phase dist. < 10°

- Amplitude Separation

\[ \text{Noise StdDev} = 0.25 \]

\[ \text{Initial freq. guesses = 15 and 20 kHz, } \quad R = 1 \]

- Frequency Separation

\[ \text{Noise StdDev} = 0.25 \]

\[ \text{Initial freq. guesses = 5, 10 and 15 kHz, } \quad R = 1 \]

- Multi-Tone Estimation

\[ \text{Noise StdDev} = 0.25 \]

\[ \text{Initial freq. guesses = 5, 10 and 15 kHz, } \quad R = 1 \]

- Real-time multi-tone estimation of non-stationary signal components in a noisy signal is achievable at the same sampling rate as the measurements!!!

IV. Conclusions & Future Work

- Multi-tone signal estimation and frequency tracking achievable with the EKFFT
- Instantaneous quantities (amplitude, phase and frequency) estimated at the same sampling rate as the measurements
- Successful estimation is also achievable in cases where the amplitude ratio of signal components is 10° or even 10°
- Correct tuning of the filter requires real-time knowledge of the amplitude of individual signal components
- Further studies required for assessing influence of initial frequency guesses in more complex component estimations
- Immediate application to real-time MHD mode detection in tokamak fusion research

References


Acknowledgements

This work was supported in part by the European Commission and the Instituto Superior Técnico under a contract of Association between Lisbon and IST and in part by Fundação para a Ciência e Tecnologia under a contract of Associated Laboratory. The views and opinions expressed herein do not necessarily reflect those of the European Commission. The authors would like to thank Dr. Jorge Sousa and Dr. Bernardo Carvalho for enlightening discussions.